MA 161 EXAM I **Fall 2005**

Name ____

ten–digit Student ID number __________________________

Division and Section Numbers __________________________

Recitation Instructor ________________________________

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 16 problems, each worth 6 points. You get 2 points for coming and 2 if you fully comply with instruction 1. The maximum score is 100 points.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

Key: abda ccbe dabd ecbb

1. For which functions is it true that $\lim_{x\to 1} f(x) = 2$?

I. $f(x) = \dfrac{4x-4}{x^2-1} \qquad x \neq \pm 1.$ $\qquad f(x) = \frac{4}{x+1} \quad x \neq \pm 1$

II. $f(x) = \begin{cases} x+1 & x > 1, \\ x-1 & x \leq 1. \end{cases}$ $\qquad \lim_{x \to 0^+} f(x) = 0$

III. $f(x) = x^2 + 1.$

a. Just I and III

b. Just II and III

c. Just I and II

d. Just III

e. All three

2 If $-x^2 + 1 - x \leq g(x) \leq x^2 - x + 1$ for all x, then $\lim_{x\to 0} g(x)$

$f(x) \qquad h(x)$

a. equals 0 $\qquad$ pinching <u>theorem</u>

b. equals 1 $\qquad \lim_{x\to 0} f(x) = \lim_{x \to 0} h(x) = 1$

c. does not exist

d. cannot be determined

e. equals 2

3. $\lim_{x\to 1} \sqrt{\dfrac{x^2+2x-3}{x-1}}$ $\qquad \sqrt{\dfrac{(x+3)(x-1)}{(x-1)}} = \sqrt{x+3} \qquad x \neq 1$

a. $= 4$

b. $= 1$

c. $= 3$

d. $= 2$

e. does not exist

4. $\lim_{x \to 3} \dfrac{x-3}{\sqrt{x}-\sqrt{3}}$

$$\frac{x-3}{\sqrt{x}-\sqrt{3}} \cdot \frac{\sqrt{x}+\sqrt{3}}{\sqrt{x}+\sqrt{3}} = \frac{(x-3)}{(x-3)}(\sqrt{x}+\sqrt{3})$$

$$= \sqrt{x}+\sqrt{3} \quad \text{for } x \neq 3$$

a. $= 2\sqrt{3}$

b. $= \dfrac{\sqrt{3}}{3}$

c. $= \dfrac{2\sqrt{3}}{3}$

d. $= \sqrt{3}$

e. does not exist

5. Let $h(x) = \begin{cases} \dfrac{1}{x}, & 0 < x < 1, \\ x, & 1 < x. \end{cases}$

Which of the following are true?

I. $\lim_{x \to 1^+} h(x)$ exists ✓

II. $\lim_{x \to 1^-} h(x)$ exists ✓

III. $\lim_{x \to 1} h(x)$ exists ✓

IV. h is continuous at $x = 1$ h(1) is not defined

a. only I

b. only I and II

c. only I, II, and III

d. only IV

e. all four

6. Determine the total number of vertical and horizontal asymptotes for

$$f(x) = \frac{x^2 + 4x + 4}{x^2 + 3x + 2}.$$

$$= \frac{(x+2)^2}{(x+2)(x+1)} = \frac{x+2}{(x+1)} \qquad x \neq -1, -2$$

a. none

b. 1

c. 2

d. 3

e. 4

V asy. @ $x = -1$

h asy. is $y = 1$

7. Let $f(x) = x^2 + cx$. Determine c so that $f'(1) = 0$.

a. -1

b. -2

c. 0

d. 1

e. 2

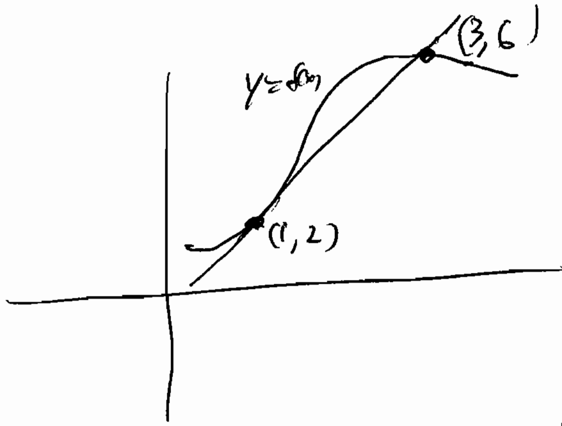
$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + c(x+h) - x^2 - cx}{h}$$

$$= \frac{\cancel{h}(2x + h + c)}{\cancel{h}}$$

$$f'(x) = 2x + c$$

$$0 = f'(1) = 2 + c$$

8.



Slope $= \frac{6-2}{3-1} = 2$

point $(1,2)$

The equation of the tangent line T to the graph of $f(x)$ at $x = 1$ is

a. $y = x - 2$

b. $y = x + 2$

c. $y = 2x + 2$

d. $y = 2x + 4$

e. $y = 2x$

$$\frac{y-2}{x-1} = 2$$

$$y = 2x$$

9. A function $y = f(x)$ is both even and odd. Then,

a. Its graph is symmetric with respect to the line $y = 1$.

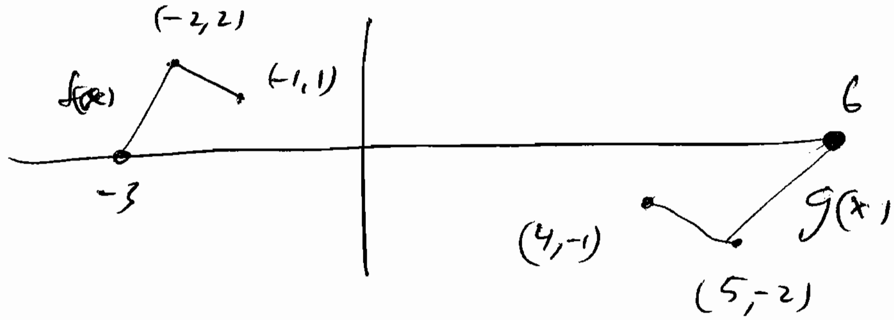
b. It cannot be a linear function.

c. It must be a logarithmic function.

d. It must be constant. In fact $f(x)$ must be $f(x) \equiv 0$

e. Its domain must be $\{0\}$.

10. Given the graphs of f and g below, we conclude that $g(x) =$



a. $g(x) = -f(3-x)$

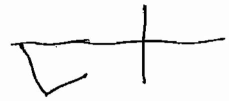
b. $-f(x-7)$

c. $f(3-x)$

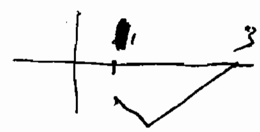
d. $3 - f(x)$

e. $f(-x) - 4$

$f \to -f$,

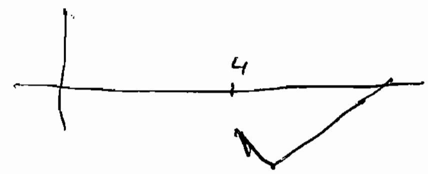


$-f(x) \to -f(-x)$



$x \to x-3$

$-f(-x) \to -f(3-x)$



11. Under certain conditions a population of certain bacteria doubles every 4 hours. If there are initially 100 bacteria, how many will there be after 6 hours? Assume exponential growth.

a. 200

b. $200\sqrt{2}$

c. 300

d. $300\sqrt{3}$

e. none of the above

$P(t) = 100\, 2^{t/4}$

$P(6) = 100\, 2^{6/4}$

12. The solution of $\ln(5-2x)=-3$ is

a. $\dfrac{e^{-3}-5}{2}$

b. e^4

c. $e^{-3/2}-5$

d. $\dfrac{5-e^{-3}}{2}$

e. $5-e^{-3/2}$

$5-2x=e^{-3}$

$5-e^{-3}=2x$

13. $\cos(\csc^{-1} t)$, for $|t|\geq 1$, equals

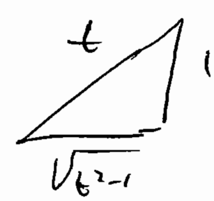
a. $\sqrt{t^2-1}$

b. $\dfrac{1}{\sqrt{t^2-1}}$

c. $\dfrac{t}{\sqrt{t^2-1}}$

d. $\dfrac{1}{t}$

e. $\dfrac{\sqrt{t^2-1}}{t}$



14. Given the 1–to–1 function $f(x) = -7+3x^3+\tan\dfrac{\pi x}{2}$, $|x| < 1$, the value of $f(f^{-1}(2\pi))$ is

a. undefined

b. -7

c. 2π

d. $-7 + (2\pi^2)$

e. 0

15. If $\sec\theta = \dfrac{5}{4}$, $\dfrac{3\pi}{2} < \theta < 2\pi$, then $\cot\theta =$

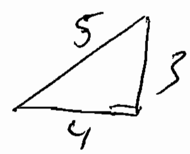
a. $\dfrac{4}{3}$

b. $-\dfrac{4}{3}$

c. $\dfrac{3}{4}$

d. $-\dfrac{3}{4}$

e. none of the above



$\cot\theta < 0$ if $\frac{3\pi}{2} < \theta < 2\pi$

$-\frac{4}{3}$

16. Find the center and radius of the circle represented by the equation $16x^2 - 16x + 16y^2 + 24y = 19$.

a. $(1, -\frac{3}{2})$, $r = \sqrt{19}$

b. $(\frac{1}{2}, -\frac{3}{4})$, $r = \sqrt{2}$

c. $(-1, \frac{3}{2})$, $r = \dfrac{\sqrt{19}}{4}$

d. $(-\frac{1}{2}, \frac{3}{4})$, $r = \sqrt{7}$

e. $(-8, 12)$, $r = \sqrt{3}$

$16(x^2 - x + \frac{1}{4}) + 16(y^2 + \frac{3}{2}y + \frac{9}{16})$

$= 19 + 4 + 9$

$= 32$

$(x - \frac{1}{2})^2 + (y - \frac{3}{4})^2 = 2$