1. The graph of $x^2 - 6x + 8 - y = 0$ is obtained from the graph of $y = x^2$ by

$y = (x-3)^2 - 1$

3R 1D

A. Moving it 4 units to the right and 3 units down

B. Moving it 3 units to the left and 1 unit up

C. Moving it 3 units to the right and 1 unit down

D. Moving it 4 units to the left and 3 units down

E. Moving it 1 unit to the right and 3 units up

2. The solution to the inequality $x \leq 5x - 3 < 8x - 2$ is

$x \leq 5x - 3 \Downarrow 3 \leq 4x \Downarrow \frac{3}{4} \leq x$ AND $5x - 3 < 8x - 2 \Downarrow -1 < 3x$, $-\frac{1}{3} < x$

A. $x \geq -\dfrac{1}{3}$

B. $x \geq \dfrac{3}{4}$

C. $-\dfrac{1}{3} \leq x < \dfrac{3}{4}$

D. $-\dfrac{1}{3} < x \leq \dfrac{3}{4}$

E. $x > \dfrac{3}{4}$

3. Given that $\sin x = \dfrac{2}{5}$ and $\cos x < 0$, it follows that $\tan x$ is equal to

$\cos x = -\sqrt{1 - \frac{4}{25}}! = -\frac{\sqrt{21}}{5}$

$\tan x = \frac{\sin x}{\cos x} =$

A. $-\dfrac{2}{\sqrt{21}}$

B. $-\dfrac{\sqrt{21}}{25}$

C. $-\dfrac{5}{\sqrt{21}}$

D. $-\dfrac{4}{25}$

E. $-\dfrac{4}{\sqrt{21}}$

4. The center C and radius r of the circle given by $x^2 + y^2 - 10x + 3y = 5$ are

$$(x-5)^2 + \left(y+\frac{3}{2}\right)^2 = 5 + 25 + \frac{9}{4}$$

C

$$= \frac{129}{4} = \left(\frac{\sqrt{129}}{2}\right)^2 = r^2$$

A. $C = \left(-\frac{3}{2}, 5\right), \quad r = \frac{\sqrt{129}}{2}$

B. $C = \left(5, -\frac{3}{2}\right), \quad r = \frac{\sqrt{129}}{2}$

C. $C = (5, -3), \quad r = 7$

D. $C = (-5, 3), \quad r = 7$

E. $C = \left(\frac{3}{2}, -5\right), \quad r = \frac{\sqrt{129}}{2}$

5. An equation of the line through $(-2, 2)$ and parallel to $4x + 3y - 7 = 0$ is

$$\downarrow$$

$$y = -\frac{4}{3}x + \frac{7}{3}$$

// through $(-2,2)$ is

$$y - 2 = -\frac{4}{3}(x+2)$$

$$\Downarrow$$

$$3y + 4x + 2 = 0$$

A. $3y + 4x + 2 = 0$

B. $2x + 3y + 8 = 0$

C. $4x + 3y - 14 = 0$

D. $4y + 3x + 2 = 0$

E. $2x + 3y - 2 = 0$

6. Given that $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{x^2 + 1}$, the domain of $g \circ f$ is

$$\text{dom}(g) = (-\infty, \infty)$$

$$\text{dom}(f) = \{x : 4 - x^2 \geq 0\} = [-2, 2]$$

Thus, $\text{dom}(g \circ f) = \text{dom}(f) = [-2, 2]$

A. $[-\sqrt{5}, -2] \cup [2, \sqrt{5}]$

B. $[-\sqrt{5}, \sqrt{5}]$

C. $(-\infty, -2] \cup [2, \infty)$

D. $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$

E. $[-2, 2]$

7. Which of the following statements are true?

T I. $5^x \cdot 5^y = 5^{x+y}$

F II. $(4 \cdot 3)^x = 4^x + 3^x$ for $x=1$, $12 \neq 7$

F III. $8^x + 8^y = 8^{x+y}$ for $x=y=1$, $16 \neq 64$

A. Only I

B. Only II

C. Only I and II

D. Only III

E. I, II, and III

8. The inverse of the function $f(x) = \dfrac{3x-2}{2x+5}$ is $f^{-1}(x) =$

$$x = \frac{3y-2}{2y+5} \Rightarrow 2xy + 5x = 3y - 2$$

$$\Rightarrow (2x-3)y = -2-5x$$

$$\Rightarrow y = \frac{5x+2}{3-2x}$$

A. $\dfrac{5x-2}{3-2x}$

B. $\dfrac{2x-5}{3-2x}$

C. $\dfrac{2x+3}{5-2x}$

D. $\dfrac{5x+2}{3-2x}$

E. $\dfrac{3x-2}{3-5x}$

9. $\lim\limits_{x \to 1} \dfrac{\sqrt{2x+5} - \sqrt{7}}{x-1} = \lim\limits_{x \to 1} \dfrac{(\sqrt{2x+5} - \sqrt{7})(\sqrt{2x+5} + \sqrt{7})}{(x-1)(\sqrt{2x+5} + \sqrt{7})}$

$$= \lim_{x \to 1} \frac{2x+5-7}{(x-1)(\sqrt{2x+5} + \sqrt{7})}$$

$$= \lim_{x \to 1} \frac{2\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{2x+5} + \sqrt{7})}$$

$$= \frac{2}{\sqrt{7} + \sqrt{7}} = \frac{1}{\sqrt{7}}$$

A. $\sqrt{5} - \sqrt{7}$

B. $\dfrac{2}{\sqrt{5}}$

C. $\dfrac{2}{\sqrt{7}}$

D. $\dfrac{1}{\sqrt{5} - \sqrt{7}}$

E. $\dfrac{1}{\sqrt{7}}$

10. If f and g are continuous at $x = 2$ with $g(2) = 3$ and $\lim\limits_{x \to 2} \dfrac{2f(x) - 3g(x)}{2g(x) - f(x)} = 7$, then f(2) is

A. undefined

B. $= \dfrac{17}{3}$

C. $= \dfrac{7}{3}$

D. $= 1$

E. impossible to determine

$$2f(2) - 3g(2) = 7\left[2g(2) - f(2)\right]$$
$$\Rightarrow 9f(2) = 17\,g(2) = 17 \times 3$$
$$\Rightarrow f(2) = \frac{17 \times 3}{9} = \frac{17}{3}$$

11. $\lim\limits_{x \to -\infty} \sqrt{\dfrac{1 - 4x^2 + 7x^3}{28x^3 - \pi x + e}}$

A. $\dfrac{1}{2}$

B. 2

C. $\dfrac{1}{4}$

D. $\dfrac{1}{e}$

E. $-\infty$

$$= \lim_{x \to -\infty} \sqrt{\frac{\frac{1}{x^3} - \frac{4}{x} + 7}{28 - \frac{\pi}{x^2} + \frac{e}{x^3}}} = \sqrt{\frac{7}{28}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

12. The total number of asymptotes, vertical and horizontal, for the graph of $f(x) = \dfrac{x - 2}{\sqrt{2x^2 + 7x + 3}}$ is

A. 0

B. 1

C. 2

D. 3

E. 4

$$f(x) = \frac{x-2}{\sqrt{(2x+1)(x+3)}}$$ has vertical asymptotes at $x = -\frac{1}{2}$ and $x = -3$.

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{1 - \frac{2}{x}}{\sqrt{2 + \frac{7}{x} + \frac{3}{x^2}}} = \frac{1}{\sqrt{2}}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{1 + 2/x}{-\sqrt{2 + 7/x + 3/x^2}} = -\frac{1}{\sqrt{2}}$$

13. If a ball is thrown directly up from the ground with a velocity v_0, then its height above ground at time t is given by $H(t) = v_0 t - \frac{g}{2}t^2$ until is falls back to the ground. Here g is the acceleration of gravity. Then, the velocity of the ball when it hits the ground is

Hits ground when $H(t) = 0 = v_0 t - \frac{g}{2}t^2 = v_0 t\left(1 - \frac{gt}{2v_0}\right)$

i.e. when $t = \frac{2v_0}{g}$. Thus, the velocity is

$$\lim_{t \to \frac{2v_0}{g}} \frac{H(t) - H\left(\frac{2v_0}{g}\right)}{t - \frac{2v_0}{g}} = \lim_{t \to \frac{2v_0}{g}} \frac{v_0 t\left(1 - \frac{gt}{2v_0}\right) - 0}{t - \frac{2v_0}{g}}$$

$$= \lim_{t \to \frac{2v_0}{g}} \frac{2gtv_0 - g^2t^2}{2gt - 4v_0} = \lim_{t \to \frac{2v_0}{g}} \frac{-gt\cancel{(gt - 2v_0)}}{2\cancel{(gt - 2v_0)}} = -\frac{\cancel{g}}{\cancel{2}}\frac{\cancel{2}v_0}{\cancel{g}} = -v_0$$

A. v_0

B. $\frac{v_0}{2g}$

C. 0

D. $-\frac{2g}{v_0}$

E. $-v_0$

14. $f'(a) = \lim_{h \to 0} \frac{32(2^h - 1)}{h}$ represents the derivative of a certain function f at a number a in its domain. Determine f and a.

$$f'(a) = \lim_{h \to 0} \frac{2^5 \cdot 2^h - 2^5}{h}$$

$$= \lim_{h \to 0} \frac{2^{h+5} - 2^5}{h}$$

A. $f(x) = 32$ and $a = 0$

B. $f(x) = 32 \cdot 2^x$ and $a = 2$

C. $f(x) = 2^x$ and $a = 5$

D. $f(x) = 2^x$ and $a = 32$

E. $f(x) = 32\frac{2^x - 1}{x}$ and $a = 0$

15. If $r + 3s + 1 = 0$ is the tangent line to $r = g(s)$ at $(-1, 2)$, then

$r = -3s - 1$

$r - 2 = g'(-1)(s+1) \Rightarrow r = g'(-1)s + [g'(-1) + 2]$

this is $g(-1)$

Thus, $g'(-1) = -3$ and $g'(-1) + 2 = -1$

A. $g(-1) = 2$ and $g'(-1) = 3$

B. $g(2) = -1$ and $g'(2) = 3$

C. $g(-1) = 2$ and $g'(-1) = -\frac{1}{3}$

D. $g(2) = -1$ and $g'(-1) = 3$

E. $g(-1) = 2$ and $g'(-1) = -3$