MA 16100

Fall 2011

1. If $\sec x = 3$ and $\frac{3\pi}{2} < x < 2\pi$, then $(\sin x + \cos x) =$

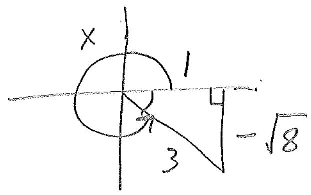
A. $\frac{1}{3}(1+\sqrt{8})$

B. $\frac{1}{3}(1-\sqrt{8})$

C. $\frac{1}{2}(1+\sqrt{8})$

D. $\frac{1}{3}(-1+\sqrt{8})$

E. $\frac{1}{2}(-1-\sqrt{8})$



$\Rightarrow \sin x + \cos x = -\frac{\sqrt{8}}{3} + \frac{1}{3}$

$= \frac{1}{3}(1-\sqrt{8})$

2. Find the equation of the line which passes through the centers of these two circles:

$$(x-3)^2 + (y+1)^2 = 1 \quad and \quad x^2 + y^2 - 4y = 1\,.$$

center: $(3,-1)$

$x^2 + (y^2 - 4y + 4) = 1 + 4$

$(x-0)^2 + (y-2)^2 = 5$

center: $(0, 2)$

A. $y = x + 2$

B. $y = -2 - x$

C. $y = 2 - x$

D. $y = -3x + 2$

E. $y = -3x$

Slope of line: $\frac{\Delta y}{\Delta x} = \frac{2-(-1)}{0-3} = -1$

Line: $y - 2 = -1(x - 0)$

$y = -x + 2$

3. If $f(x) = x^2 + e^{(x+1)}$ and $g(x) = 4x - 1$, then $(f \circ g)(2t) =$

A. $(2t)^2 + e^{(2t-1)}$

B. $(4t+1)^2 + e^{4t}$

C. $(4t+1)^2 + e^{(4t+1)}$

D. $(8t-1)^2 + e^{(8t-2)}$

E. $(8t-1)^2 + e^{8t}$

$$(f \circ g)(2t) = f(g(2t))$$
$$= f(8t-1)$$
$$= (8t-1)^2 + e^{((8t-1)+1)}$$
$$= (8t-1)^2 + e^{(8t)}$$

4. The equation of the function $g(x)$ obtained by shifting the graph of $f(x) = \log_{10} x$ three units vertically down and then reflecting it across the x-axis is given by

A. $g(x) = 3 - \log_{10} x$

B. $g(x) = -3 - \log_{10} x$

C. $g(x) = -3 + \log_{10} x$

D. $g(x) = -\log_{10}(x-3)$

E. $g(x) = -\log_{10}(x+3)$

$f(x) = \log_{10} x$

Shift 3 units vertically down

$\rightarrow h(x) = (\log_{10} x) - (3)$

Then reflect across the x-axis

$\rightarrow g(x) = -[(\log_{10} x) - (3)]$

$= 3 - \log_{10} x$

5. Solve for x: $e^{|2x-1|} = 2$.

A. $x = \frac{1}{2}\ln 2$ and $x = -\frac{1}{2}\ln 2$

B. $x = 1 + \ln 2$ and $x = \frac{1}{2}(1 + \ln 2)$

C. $x = \frac{1}{2}(1 + \ln 2)$ and $x = -\frac{1}{2}(1 + \ln 2)$

D. $x = \frac{1}{2}(1 - \ln 2)$ and $x = \frac{1}{2}(1 + \ln 2)$

E. $x = \frac{1}{2}\ln 2$

$$e^{|2x-1|} = 2$$

$$\to |2x-1| = \ln 2$$

$$\to \begin{cases} 2x-1 = \ln 2 & \text{if } 2x-1 \geq 0 \\ -2x+1 = \ln 2 & \text{if } 2x-1 < 0 \end{cases}$$

$$\to \begin{cases} 2x = \ln 2 + 1 & \text{if } 2x-1 \geq 0 \\ -2x = \ln 2 - 1 & \text{if } 2x-1 < 0 \end{cases}$$

$$\to \begin{cases} x = \frac{1}{2}(\ln 2 + 1) & \text{if } 2x-1 \geq 0 \\ x = -\frac{1}{2}(\ln 2 - 1) & \text{if } 2x-1 < 0 \end{cases}$$

$$\to \begin{cases} x = \frac{1}{2}(1 + \ln 2) & \text{if } 2x-1 \geq 0 \\ x = \frac{1}{2}(1 - \ln 2) & \text{if } 2x-1 < 0 \end{cases}$$

6. The domain of $\ln\left(\dfrac{4x^2}{x+1}\right)$ is

A. $(1, \infty) \cup (-\infty, -1)$

B. $(0, \infty) \cup (-\infty, -1)$

C. $(-1, 0) \cup (0, \infty)$

D. $(-1, 0]$

E. All real numbers except $x = 0$ and $x = -1$

domain of $\ln\left(\frac{4x^2}{x+1}\right)$ is $\frac{4x^2}{x+1} > 0$

$$\left.\begin{array}{l} \text{note } 4x^2 > 0 \text{ for all } x \neq 0 \\ x+1 > 0 \text{ for } x > -1 \end{array}\right\} (-1, 0) \cup (0, \infty)$$

7. If $f(x) = \ln(3x - 1)$, find the domain of f^{-1}

A. $(\frac{1}{3}, \infty)$

B. $(0, \infty))$

C. $(-\frac{1}{3}, \infty)$

D. $(1, \infty)$

E. $(-\infty, \infty)$

The range of $f(x) = \ln(3x-1)$ is $(-\infty, \infty)$.
Therefore the domain of f^{-1} is $(-\infty, \infty)$.

Also note that interchanging x and y gives
$x = \ln(3y-1)$, and solving for y gives
$e^x = 3y - 1 \to y = \frac{e^x + 1}{3} = f^{-1}(x)$.
So, again, the domain of f^{-1} is $(-\infty, \infty)$, since that is the domain of e^x.

8. Compute $\lim_{x \to 2^-} \frac{x^2 - x - 2}{(x-2)^2}$

$$= \lim_{x \to 2^{-1}} \frac{(x-2)(x+1)}{(x-2)^2}$$

$$= \lim_{\substack{x \to 2^{-1} \\ (x<2)}} \frac{x+1}{x-2}$$

$$= \frac{3}{0^-}$$

$$= -\infty$$

A. ∞

B. $-\infty$

C. 0

D. 1

E. -1

9. Compute $\lim_{t \to 0} \frac{\sqrt{2+t} - \sqrt{2-t}}{t}$ $= \frac{0}{0}$

A. 2

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{2}}$

E. $\sqrt{2}$

$$= \lim_{t \to 0} \frac{\left(\sqrt{2+t} - \sqrt{2-t}\right)\left(\sqrt{2+t} + \sqrt{2-t}\right)}{t\left(\sqrt{2+t} + \sqrt{2-t}\right)}$$

$$= \lim_{t \to 0} \frac{(2+t) - (2-t)}{t\left(\sqrt{2+t} + \sqrt{2-t}\right)}$$

$$= \lim_{t \to 0} \frac{2\cancel{t}}{\cancel{t}\left(\sqrt{2+t} + \sqrt{2-t}\right)} = \frac{2}{\sqrt{2}+\sqrt{2}} = \frac{1}{\sqrt{2}}$$

10. Let $G(x) = \begin{cases} 1-x & \text{if } x < 0 \\ x + x^2 & \text{if } 0 \le x < 1 \\ 2-x & \text{if } x \ge 1 \end{cases}$. Then G is discontinuous

A. Only at 0

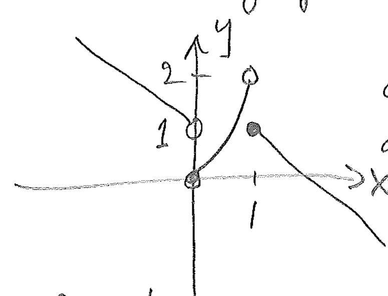
B. Only at 1

C. Only at 0 and 1

D. Only at $-1, 0,$ and 1

E. The function is continuous everywhere

Consider the graph of G.



discontinuous at $x=0$ and $x=1$.

G is continuous at all values of x except possible at $x=0$ and $x=1$ since G is made up of polynomials. Consider one-sided limits at $x=1$ and $x=0$.

$\lim_{x \to 0^-} G(x) = \lim_{x \to 0^-} 1-x = 1$

$\lim_{x \to 0^+} G(x) = \lim_{x \to 0^+} x + x^2 = 0$

$\rightarrow \lim_{x \to 0} G(x)$ does not exist

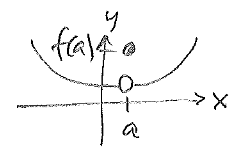
$\lim_{x \to 1^-} G(x) = \lim_{x \to 1^-} x + x^2 = 2$

$\lim_{x \to 1^+} G(x) = \lim_{x \to 1^+} 2-x = 1$

$\rightarrow \lim_{x \to 0} G(x)$ does not exist.

11. Consider the statements

I. If $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$, then f is continuous. false



II. If f is continuous at b, then $f(b)$ does not have to be defined. false $\lim_{x \to a} f(x) = f(a)$.

III. The function $g(x) = \sqrt{1 - x^2}$ is continuous only on $(-1, 1)$. false

Which are true?

domain of g is $1-x^2 \geq 0 \to 1 \geq x^2 \to -1 \leq x \leq 1$.

g is continuous on $(-1,1)$ and has one-sided continuity at -1 and 1 because for example $\lim_{x \to -1^+} \sqrt{1-x^2} = \sqrt{1-(-1)^2} = 0 = g(-1)$

A. I

B. I,II

C. II, III

D. II

E. None are true

12. Compute $\lim_{x \to \infty} \dfrac{2x - 5x^2}{\sqrt{4x^2 + 9}}$

$$\frac{x^2\left(\frac{2}{x} - 5\right)}{x\sqrt{4 + \frac{9}{x^2}}} = \frac{(x)\left(\frac{2}{x} - 5\right)}{\sqrt{4 + \frac{9}{x^2}}}$$

$$\to \frac{\infty(-5)}{2} \text{ as } x \to \infty.$$

Therefore the limit is $-\infty$

A. $-\dfrac{5}{2}$

B. 1

C. $-\dfrac{5}{4}$

D. $\dfrac{1}{2}$

E. $-\infty$

13. What is the total number of horizontal and vertical asymptotes for the function $\dfrac{x^2-x}{4-x^2}$?

A. 3

B. 4

C. 2

D. 1

E. 0

$\lim_{x \to \pm\infty} \dfrac{x^2-x}{-x^2+4} = -1 \Rightarrow$ 1 horizontal asymptote

$4-x^2=0 \rightarrow x=\pm 2$ (and numerator $x^2-x \neq 0$ for either $x=2$ or $x=-2$)

$\Rightarrow$ 2 vertical asymptotes.

14. Compute $\lim_{x\to 2} e^{\left(\frac{x^2+1}{2x+1}\right)}$ $= e^{\lim_{x\to 2}\left(\frac{x^2+1}{2x+1}\right)} = e^{\frac{4+1}{4+1}} = e,$

A. $e^{\frac{3}{5}}$

B. ∞

C. e

D. $e^{\frac{4}{5}}$

E. $\dfrac{4e}{5}$