MA 161, EXAM 1, FALL 2015

1. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{3 - \underbrace{\sqrt{x+1}}}}$.

$X+1 \geq 0$

$X \geq -1$

$3 - \sqrt{x+1} > 0$

$3 > \sqrt{x+1}$

$9 > x+1$

$8 > x$

A. $(-10, 8)$

B. $x \neq 8$

C. $\boxed{[-1, 8)}$

D. $(-\infty, -1) \cup (8, \infty)$

E. $(-\infty, -10) \cup (-10, 8) \cup (8, \infty)$

2. Suppose the graph of the exponential function $f(x) = ca^x$ passes through the points $(-1, 9)$ and $(1, 4)$ in the xy-plane. Find the value of c.

A. $\frac{1}{9}$

B. 9

C. 4

D. $-\frac{5}{2}$

E. $\boxed{6}$

$c a^{-1} = 9$

$c a^{1} = 4 \rightarrow a = \frac{4}{c} \rightarrow a^{-1} = \frac{c}{4}$

$c\left(\frac{c}{4}\right) = 9$

$c^2 = 36$

e^{x-3} $2e^{x-3}$

3. The graph of $y = e^x$ is translated to the right by 3 units, stretched vertically by a factor of 2, and finally vertically translated up by 5 units to obtain the graph of $y = h(x)$. Find a formula for $h(x)$.

$2e^{x-3} + 5$

A. $e^{2x-6} + 5$

B. $\boxed{2e^{x-3} + 5}$

C. $e^{\frac{x}{2} - \frac{3}{2}} + 5$

D. $2e^{x+3} + 10$

E. $e^{\frac{x}{2} - \frac{3}{2}} - 5$

4. If the displacement (in cm) of a particle moving back and forth on a line is given by

$$s(t) = \sin(\pi t) + 2\cos(\pi t)$$

where t is given in seconds, then the average velocity during the time interval $[3, 6]$ is

A. $\boxed{4/3 \text{ cm/sec}}$

B. 6 cm/sec

C. $-\pi/2$ cm/sec

D. 0 cm/sec

E. -3 cm/sec

$$\frac{s(6) - s(3)}{6-3} = \frac{2-(-2)}{3} = \frac{4}{3}$$

5. Find the inverse of the function

$$f(x) = \frac{3^x}{2 + \pi 3^x}.$$

A. $f^{-1}(x) = \log_3(2x) - \log_3(1 + \pi x)$

B. $\boxed{f^{-1}(x) = \log_3(2x) - \log_3(1 - \pi x)}$

C. $f^{-1}(x) = \log_3\left(2x - \frac{2}{\pi}\right)$

D. $f^{-1}(x) = \log_3\left(2x + \frac{2}{\pi}\right)$

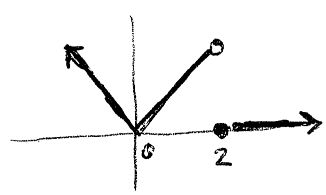
E. $f^{-1}(x) = \log_3\left(\frac{2x}{\pi + 1}\right)$

Solve $x = \frac{3^y}{2+\pi 3^y}$ for y.

$$2x + \pi x 3^y = 3^y$$
$$2x = 3^y - \pi x 3^y$$
$$2x = (1 - \pi x) 3^y$$
$$\frac{2x}{1 - \pi x} = 3^y$$
$$y = \log_3\left(\frac{2x}{1 - \pi x}\right)$$
$$y = \log_3(2x) - \log_3(1 - \pi x)$$

6. Select all true statements about the following function:

$$f(x) = \begin{cases} |x| & \text{if } x < 2 \\ 0 & \text{if } x \geq 2 \end{cases}$$



I. $f(x)$ is differentiable at $x = 0$.

II. $f(x)$ is continuous at $x = 2$.

III. $\lim_{x \to 0} f(x)$ exists.

A. II and III

B. I and II

C. I only

D. $\boxed{\text{III only}}$

E. None of the statements are true

7. Find values for the constants a and b that make $f(x)$ continuous for all values of x.

$$f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x < 1 \\ ax+b & \text{if } 1 \le x \le 3 \\ x^2 - 1 & \text{if } x > 3 \end{cases}$$

A. $\boxed{a = 3,\ b = -1}$

B. $a = 2,\ b = 8$

C. $a = 1,\ b = 1$

D. $a = 2,\ b = 2$

E. No values of a and b will make the function f continuous everywhere

Need: $f(1) = \lim_{x\to 1^-} f(x)$

$a + b = \lim_{x\to 1^-} \frac{x^2-1}{x-1} = \lim_{x\to 1^-} x+1 = 2$

Need: $f(3) = \lim_{x\to 3^+} f(x)$

$a(3) + b = \lim_{x\to 3^+} x^2 - 1 = 3^2 - 1$

$a+b=2$
$3a+b=8$ $\rightarrow$ $a=3$, $b=-1$

8. Evaluate the following limit:

$$\lim_{x\to\infty} (\sqrt{16x^2+x} - 4x)$$

A. ∞

B. $\boxed{\frac{1}{8}}$

C. 0

D. 4

E. $\frac{1}{4}$

$= \lim_{x\to\infty} \frac{(\sqrt{16x^2+x} - 4x)(\sqrt{16x^2+x} + 4x)}{(\sqrt{16x^2+x} + 4x)}$

$= \lim_{x\to\infty} \frac{16x^2 + x - 16x^2}{\sqrt{16x^2+x} + 4x}$

$= \lim_{x\to\infty} \frac{x}{\sqrt{16x^2+x} + 4x}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{16+1/x} + 4} = \frac{1}{\sqrt{16+0} + 4} = \frac{1}{4+4} = \frac{1}{8}$

9. How many vertical and horizontal asymptotes does the following function have?

$$f(x) = \frac{3x^2 + x - 1}{x^2 + x - 6}$$

A. 2 horizontal and 2 vertical asymptotes

B. 2 horizontal and 1 vertical asymptotes

C. $\boxed{\text{1 horizontal and 2 vertical asymptotes}}$

D. 1 horizontal asymptote

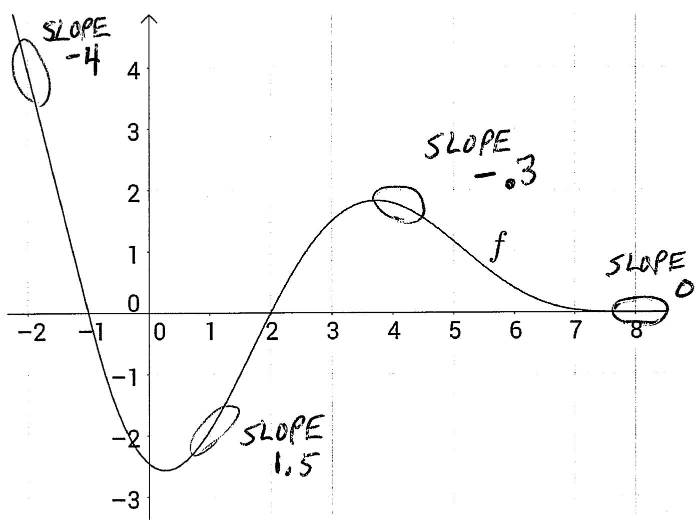
E. 2 horizontal asymptotes

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 3$

$\lim_{x\to 2} |f(x)| = \infty$

$\lim_{x\to -3} |f(x)| = \infty$

10. The graph of $y = f(x)$ is given below. Choose the correct ordering for the values of f'.



A. $f'(-2) < f'(1) < f'(4) < f'(8)$

B. $f'(1) < f'(8) < f'(4) < f'(-2)$

C. $f'(-2) < f'(4) < f'(1) < f'(8)$

D. $f'(1) < f'(8) < f'(-2) < f'(4)$

E. $\boxed{f'(-2) < f'(4) < f'(8) < f'(1)}$

11. Find the equation of the tangent line to $f(x) = \frac{1}{x}$ at $x = 2$.

A. $y = -\frac{x}{4} + \frac{1}{2}$

B. $y = -\frac{x}{2} + \frac{1}{2}$

C. $y = -\frac{x}{2} + 1$

D. $\boxed{y = -\frac{x}{4} + 1}$

E. $y = \frac{x}{2} - \frac{1}{2}$

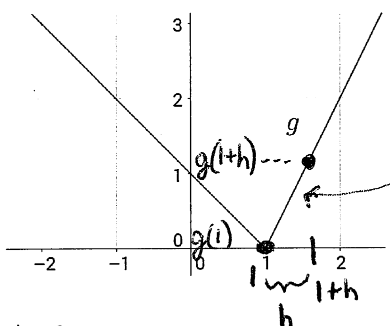
$$f'(2) = \lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} = \lim_{x \to 2} \frac{\frac{2-x}{2x}}{x-2}$$

$$= \lim_{x \to 2} \frac{-1}{2x} = -\frac{1}{4}$$

$$\frac{y - f(2)}{x - 2} = -\frac{1}{4} \rightarrow y - \frac{1}{2} = -\frac{1}{4}(x-2)$$

$$\rightarrow y = -\frac{x}{4} + 1$$

12. Given the graph of $y = g(x)$ below, find the one-sided limit, $\lim_{h \to 0^+} \frac{g(1+h) - g(1)}{h}$.



SLOPE of SECANT LINE:

$$\frac{g(1+h) - g(1)}{h}$$

A. 0

B. 1

C. $\boxed{2}$

D. −1

E. Does not exist

"right-hand slope" at $x = 1$.