SOLUTION.

MA 161 & 161E EXAM 1 SPRING 2002

1. Solve for x: $x^2 > x + 2$

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

$(x-2>0 \;\&\; x+1>0)$ or $(x-2<0 \;\&\; x+1<0)$

$(x>2$ and $x>-1)$ or $(x<2$ and $x<-1)$

$x > 2$ or $x < -1$.

A. $x > 0$
B. $x < -1$ or $x > 2$
C. $x > -1$
D. $x < 2$
E. $x > 2$

2. Find the radius of the circle whose equation is $x^2 + y^2 + 2x - 4y - 5 = 0$.

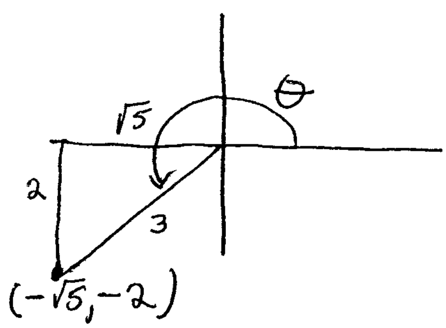
$x^2 + 2x \quad + y^2 - 4y \quad = 5$

$x^2 + 2x + 1 \; + \; y^2 - 4x + 4 \; = 5 + 1 + 4$

$(x+1)^2 + (y-2) = 10$

A. $\sqrt{5}$
B. 5
C. 3
D. $\sqrt{10}$
E. 10

3. If $\sin\theta = -\dfrac{2}{3}$ and $\pi \le \theta \le \dfrac{3\pi}{2}$, then $\tan\theta =$



$\tan\theta = \dfrac{-2}{-\sqrt{5}} = \dfrac{2}{\sqrt{5}}$

A. $\dfrac{3}{2}$
B. $-\dfrac{3}{2}$
C. $-\dfrac{\sqrt{5}}{3}$
D. $-\dfrac{3}{\sqrt{5}}$
E. $\dfrac{2}{\sqrt{5}}$

4. The domain of the function $f(x) = \dfrac{\ln(x+1)}{\sqrt{x+2}}$ is

$x+1>0$ and $x+2>0$.
$x>-1$ and $x>-2$
$\therefore \quad x>-1$

A. $x > -1$
B. $x > -2$
C. $-2 < x < -1$
D. $x > 0$
E. $x > 2$

5. The graph of $y = f(x)$ is compressed horizontally by a factor of 4, then translated 3 units to the left, then reflected about the x-axis. The resulting graph has the equation

compressed horizontally by a factor of 4: $y = f(4x)$.
translated 3 units to the left:
$y = f(4(x+3))$.
reflected about x-axis
$y = -f(4(x+3))$

A. $y = f(4(x+3))$
B. $y = f\left(\frac{1}{4}(x+3)\right)$
C. $y = -f(4(x+3))$
D. $y = -f\left(\frac{1}{4}(x-3)\right)$
E. $y = -f(4(x-3))$

6. A certain radioactive substance has a half-life of 8 days. Initially there are 6 grams of the substance. After 12 days, how many grams of the substance remain?

$m(0) = 6$. $m(8) = 3$.
In t days there will be $t/8$ half-life periods.
$m(t) = 6 \cdot \left(\frac{1}{2}\right)^{t/8}$.
$\therefore$ when $t = 12$,
$m(12) = 6 \cdot \left(\frac{1}{2}\right)^{12/8}$
$= 6 \cdot \left(\frac{1}{2}\right)^{3/2}$

A. $6 \cdot \left(\frac{1}{2}\right)^{96}$
B. $6 \cdot \left(\frac{1}{2}\right)^{12}$
C. $6 \cdot \left(\frac{1}{2}\right)^{1/2}$
D. $\frac{3}{2}$
E. $6 \cdot \left(\frac{1}{2}\right)^{3/2}$

7. If $\ln(x+1) - \ln x = 1$, then

$$\ln(x+1) - \ln(x) = \ln \frac{x+1}{x} = 1.$$

$$\therefore \quad \frac{x+1}{x} = e^1 = e.$$

$$x+1 = ex$$

$$1 = ex - x = (e-1)x$$

$$\therefore \quad x = \frac{1}{e-1}$$

A. $x = \dfrac{1}{e-1}$

B. $x = 1 - \dfrac{1}{e}$

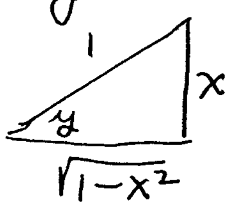
C. $x = 1 + \dfrac{1}{e}$

D. $x = \sqrt{e}$

E. There is no solution

8. Evaluate $\cos(\sin^{-1} x)$.

Let $y = \sin^{-1} x$, then $\sin y = x$.



$$\therefore \cos(\sin^{-1} x) = \cos y = \sqrt{1-x^2}$$

A. x

B. $\sqrt{1-x^2}$

C. $\dfrac{1}{x}$

D. $\dfrac{1}{\sqrt{1-x^2}}$

E. $x\sqrt{1-x^2}$

9. The displacement (in feet) of a certain particle moving in a straight line is given by $s = t^2 + t$, where t is measured in seconds. Find the average velocity for the time period beginning at $t = 1$ and lasting for 2 seconds.

$$\text{Average Velocity} = \frac{s(3) - s(1)}{2} = \frac{12 - 2}{2} = 5$$

A. 6 ft/s

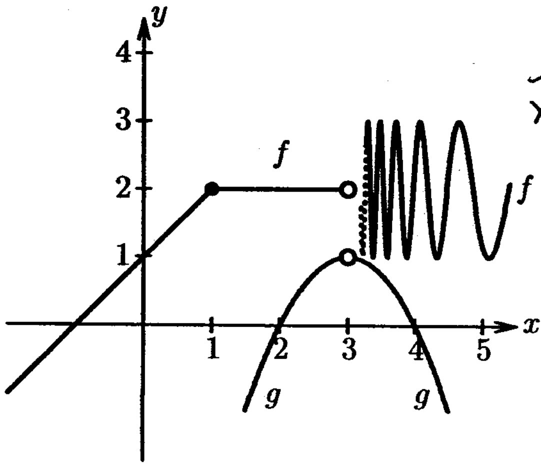
B. 3 ft/s

C. 7 ft/s

D. 4 ft/s

E. 5 ft/s

10. The graphs of the functions f and g are given below. Then $\lim_{x\to 3^-} (g \circ f(x)) =$



$\lim_{x\to 3^-} f(x) = 2.$

$\therefore \lim_{x\to 3^-} g(f(x)) =$ (B) 0

$g(2) = 0$

A. Does not exist

B. 0

C. 1

D. $-\infty$

E. ∞

11. Given that $\lim_{x\to\pi} f(x) = 2$ and $\lim_{x\to\pi} g(x) = 0$, then $\lim_{x\to\pi^-} \frac{\sqrt{3f(x)}}{(x-\pi)(g(x))^2} =$

If $x < \pi$, $x - \pi < 0$. $\therefore$ as

$x \to \pi^-$, $\sqrt{3f(x)} \to \sqrt{6} > 0$

and $(x-\pi)(g(x))^2 \to 0$ Through

negative numbers so

$$\frac{\sqrt{3f(x)}}{(x-\pi)(g(x))^2} \to -\infty.$$

A. 0

B. $\sqrt{3\sqrt{2}}$

C. ∞

D. $-\infty$

E. Cannot be determined

12. $\lim_{h\to 0} \frac{(3+h)^{-1} - 3^{-1}}{h} =$

$$\frac{(3+h)^{-1} - 3^{-1}}{h} = \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

$$= \frac{\frac{3-(3+h)}{3(3+h)}}{h} = \frac{-h}{3h(3+h)}$$

$$= -\frac{1}{3(3+h)} \to -\frac{1}{9} \text{ as } h \to 0$$

A. 0

B. $\frac{1}{9}$

C. $-\frac{1}{9}$

D. $-\frac{1}{3}$

E. Does not exist

13. Find an interval that contains a root of the equation $x^3 + x - 3 = 0$

A. $(-2, -1)$
B. $(-1, 0)$
C. $(0, 1)$
D. $(1, 2)$
E. $(2, 3)$

Let $f(x) = x^3 + x - 3$

$f(1) = -1$

$f(2) = 9$

$\therefore$ By the Intermediate Value Thm, $x^3 + x - 3 = 0$ has a root on the interval $(1, 2)$

14. Suppose $f(x) = \begin{cases} 10 - ax, & \text{if } x > a \\ 20 - 7x, & \text{if } x \leq a \end{cases}$ where a is a constant. Find all values of a for which f is continuous.

A. $a = 2, 5$
B. $a = 2, -5$
C. $a = -2, 5$
D. $a = \frac{5}{3}$
E. $a = 5$

f is continuous for all values of $x \neq a$. $\therefore$ f will be continuous if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$.

$\lim_{x \to a^-} f(x) = \lim_{x \to a^-} (10 - ax) = 10 - a^2$

$\lim_{x \to a^+} f(x) = \lim_{x \to a^+} (20 - 7x) = 20 - 7a$.

$\therefore \quad 10 - a^2 = 20 - 7a$.

$a^2 - 7a + 10 = 0$

$(a-2)(a-5) = 0$

$a = 2, 5$