MATH 161 – FALL 2009 – SECOND EXAM – OCTOBER 20, 2009

TEST NUMBER 01

STUDENT NAME______________________________________

STUDENT ID______________________________________

LECTURE TIME SOLUTIONS

RECITATION INSTRUCTOR________________________________

RECITATION TIME___________________________________

INSTRUCTIONS

1. Fill in all the information requested above and the version number of the test on your scantron sheet.

2. This booklet contains 12 problems, each worth 8 points. There are four free points. The maximum score is 100 points.

3. For each problem mark your answer on the scantron sheet and also circle it is this booklet.

4. Work only on the pages of this booklet.

5. Books, notes, calculators are not to be used on this test.

6. At the end turn in your exam and scantron sheet to your recitation instructor.

1) The graphs shown below are graphs of a function f, its first derivative f', and its second derivative f''. Determine the appropriate labels for the graph of f, f' and f'', in that order.

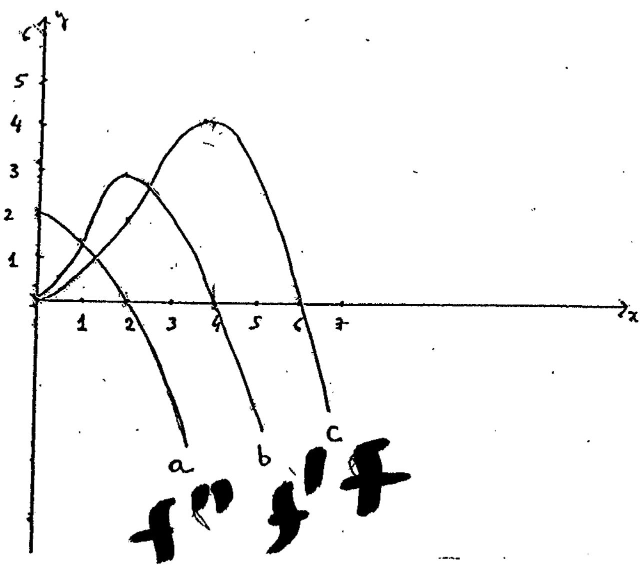
A) a, b, c

B) a, c, b

C) b, c, a

D) b, a,c

E) c, b a



2) The derivative of $2\sqrt{t} - \frac{6}{\sqrt{t}}$ is

A) $\frac{1}{\sqrt{t}} + \frac{3}{t^{3/2}}$

B) $\frac{2}{\sqrt{t}} + \frac{3}{t^{3/4}}$

C) $2t^{3/2} - 6t^{2/3}$

D) $\frac{4}{3}t^{3/2} - 9t^{1/3}$

E) $t^{3/2} + 2t^{2/3}$

$$f(t) = 2\sqrt{t} - \frac{6}{\sqrt{t}} = 2t^{1/2} - 6t^{-1/2}$$

$$f'(t) = 2 \cdot \tfrac{1}{2}\, t^{1/2 - 1} - 6 \cdot \left(-\tfrac{1}{2}\right) t^{-1/2 - 1}$$

$$= t^{-1/2} + 3t^{-3/2}$$

$$= \frac{1}{\sqrt{t}} + \frac{3}{t^{3/2}}$$

Answer A

3) At what values of x does the curve $y = \frac{1}{3}x^3 + \frac{5}{2}x^2 - 14x + 1$ have a horizontal tangent?

A) $x = \frac{-5\pm\sqrt{97}}{2}$

B) $x = \pm\sqrt{\frac{54}{3}}$

C) $x = 7$ and $x = 2$

D) $\boxed{x = 2 \text{ and } x = -7}$

E) $x = 2$ and $x = 3$

We look for the points where $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = x^2 + 5x - 14 = (x+7)(x-2) = 0$$

$$x = 2 \quad \text{and} \quad x = -7$$

4) If $f(x)$ is a differentiable function, compute the derivative of $\frac{4+x^2f(x)}{x^3}$.

A) $\frac{x^3f'(x)+5x^2f(x)+12}{x^4}$

B) $\boxed{\frac{x^3f'(x)-x^2f(x)-12}{x^4}}$

C) $\frac{-x^2f'(x)+(3x^2-2x)f(x)+12}{x^4}$

D) $\frac{2x^5f'(x)-3x^2f(x)-12}{x^4}$

E) $\frac{2f'(x)}{3x^2}$

$$g(x) = \frac{4 + x^2 f(x)}{x^3} = \frac{4}{x^3} + \frac{f(x)}{x}.$$

$$g'(x) = -\frac{12}{x^4} + \frac{xf'(x) - f(x)}{x^2}.$$

$$g'(x) = \frac{x^3f'(x) - x^2f(x) - 12}{x^4}.$$

5) An equation for the tangent line to $y = e^x(x^2+5)$ at $x = 1$ is

A) $y = 2ex + 4e$

B) $y = 3ex + 3e$

C) $y = 4ex + 4e$

D) $y = 8ex - 2e$

E) $y = 10ex - 6e$

When $x=1$, $y = 6e$. The slope of the line is given by $\frac{dy}{dx}$ at $x=1$

$\frac{dy}{dx} = e^x(x^2+5) + 2xe^x$ when $x=1$

$\frac{dy}{dx} = 8e$. So the line is given

by $y - 6e = 8e(x-1)$ $\boxed{y = 8ex - 2e}$

6) If $y = (\tan x)(\sec x)$, then $\frac{dy}{dx}$ at $\pi/3$ is

A) $8\sqrt{3}$

B) $\frac{19}{6\sqrt{3}}$

C) 14

D) $\frac{8}{3}$

E) 11

$y = \tan x \sec x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sec x = \sec x . \tan x$

$\frac{dy}{dx} = \sec^3 x + \tan^2 x . \sec x$

when $x = \pi/3$ $\cos \frac{5\pi}{3} = \frac{1}{2}$, $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

So $\tan \frac{\pi}{6} = \sqrt{3}$ and ~~sec~~ $\sec \frac{\pi}{6} = 2$

$\frac{dy}{dx} = 8 + 2 \cdot 3 = 14$

7) Evaluate the limit

$$\lim_{t\to 0}\frac{\tan 4t}{t}. = \lim_{t\to 0} \frac{\sin 4t}{t} \cdot \frac{1}{\cos t}$$

$\boxed{\text{A) } 4}$

B) 1

C) 2

D) 0

E) The limit does not exist

$$= \lim_{t\to 0} 4 \cdot \frac{\sin 4t}{4t} \cdot \frac{\cancel{\cos}\,1}{\cos t}$$

when $t \to 0$ $\lim_{t\to 0} \frac{\sin 4t}{4t} = 1$

$\lim_{t\to 0} \frac{1}{\cos t} = 1$. So $\boxed{\lim_{t\to 0} \frac{\tan 4t}{t} = 4}$

8) The derivative of $f(x) = 2^{(x^2+3x+2)}$ at $x = 0$ is equal to

A) $24 \ln 2$

B) $10 \ln 2$

C) 10

D) $13e$

$\boxed{\text{E) } 12 \ln 2}$

$\ln f(x) = (x^2 + 3x + 2.) \cdot \ln 2$

So, differentiating this inequality we get

$$\frac{f'(x)}{f(x)} = (2x+3) \ln 2$$

Therefore $f'(x) = f(x) \cdot (2x+3) \ln 2$

when $x = 0$ $f(0) = 2^2 = 4$

$$\boxed{f'(0) = 12 \ln 2}$$

9) Suppose that C is the curve defined by $2y^2 - xy^3 - x + 2 = 0$. Find an equation of the tangent line of C at the point $(2, 1)$.

A) $y = 2x - 3$

B) $y = x/2$

C) $y = -2x + 5$

D) $\boxed{y = 3 - x}$

E) $y = \frac{1}{2}x + 2$

$$2y^2(x) - x\,y^3(x) - x + 2 = 0$$

So

$$4\,y(x)\,y'(x) - y^3(x) - 3x\,y^2(x)\,y'(x) - 1 = 0$$

when $x = 2$, $y = 1$ we obtain

$$4y'(2) - 1 - 6\,y'(2) - 1 = 0$$

$$-2y'(2) - 2 = 0 \qquad \boxed{y'(2) = -1}$$

this is the slope of the line.

Equation of the line:

$y - 1 = -1\,(x - 2)$

$\boxed{y = -x + 3}$

10) Let $f(x)$ be differentiable at 2 and let g be differentiable at 1. Suppose that $f(2) = 4$, $f'(2) = -1$, $g(1) = 2$ and $g'(1) = -3$. Let $h(x) = (f[(g(x))])^2$. Compute $h'(1)$.

A) $h'(1) = 36$

B) $\boxed{h'(1) = 24}$

C) $h'(1) = -12$

D) $h'(1) = 12$

E) $h'(1) = -48$.

$$h(x) = (f(g(x)))^2$$

$$h'(x) = 2\,f(g(x)) \cdot f'(g(x)) \cdot g'(x)$$

Since $g(1) = 2$ and $g'(1) = -3$

$$h'(1) = 2 \cdot f(2) \cdot f'(2) \cdot (-3)$$

$$h'(1) = 2 \cdot (4) \cdot (-1) \cdot (-3) = 24.$$

11) Let $f(x) = \arctan(x^2 + 1)$ Then $f'(1)$ is equal to

$\frac{d}{dy} \arctan y = \frac{1}{1+y^2}$

A) π

B) 2/3

C) 2/5

D) 1/4

E) 3

$f'(x) = \frac{1}{1+(x^2+1)^2} \cdot 2x$

when $x=1$ $f'(1) = \frac{2}{1+4} = 2/5$

12) Find the derivative of $f(x) = x^{(\ln x)}$ at e.

A) $f'(e) = 3e$

B) $f'(1) = 2e + 1$

C) $f'(e) = 1/4$

D) $f'(e) = -1/3$

E) $f'(e) = 2$

$\ln f(x) = \ln x \cdot \ln x = (\ln x)^2$

$\frac{f'(x)}{f(x)} = 2 \frac{\ln x}{x}$

when $x = e$ $f(e) = e$

$\frac{f'(e)}{e} = \frac{2}{e}$; $\boxed{f'(e) = 2}$