Name SOLUTIONS

10-digit PUID

RECITATION Section Number and time

Recitation Instructor

Lecturer

Instructions:

1. Fill in all the information requested above and on the scantron sheet. On the scantron sheet also fill in the little circles for your name, section number and PUID.
2. This booklet contains 14 problems, each worth 7 points (except problems 10 and 14 are worth 8 points each). The maximum score is 100 points. The test booklet has 8 pages, including this one.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators or any electronic devices are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

1) The position of a particle moving along a line is $s = 4 + \frac{t}{2} - \ln(t+1)$, for $t \geq 0$. For what t is the velocity equal to 0.

$$v = \frac{ds}{dt} = \frac{1}{2} - \frac{1}{t+1} = \frac{t+1-2}{2(t+1)} = \frac{t-1}{2(t+1)}$$

$$v(t) = 0 \rightarrow t-1=0 \rightarrow t=1$$

A) $t = 2$

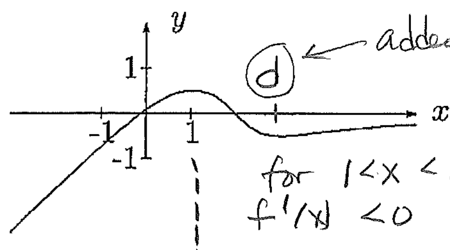
B) $t = \frac{1}{2}$

C) $t = 1$

D) $t = \frac{1}{4}$

E) $t = 4$

2) If the graph of $y = f(x)$ is



added for grading key only

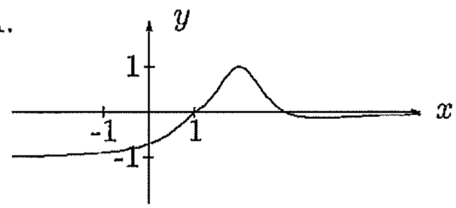
for $x < 1$, $f'(x) > 0$ and $\lim_{x \to -\infty} f'(x) \approx 1$

for $1 < x < d$ $f'(x) < 0$

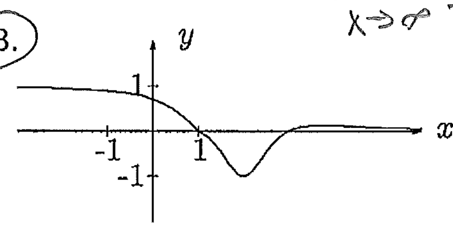
for $x > d$ $f'(x) > 0$ and $\lim_{x \to \infty} f'(x) \approx 0$

which graph represents the graph of $f'(x)$?

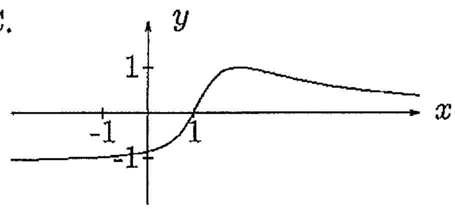
A.



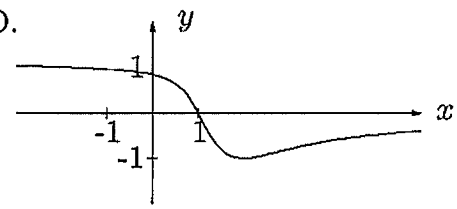
B.



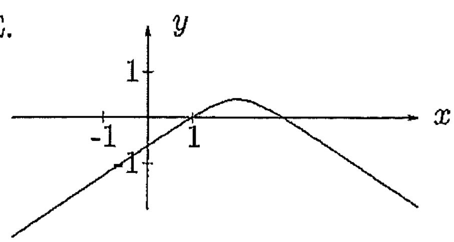
C.



D.



E.



3) For which values of x is the tangent line to the curve $y = \frac{x^3}{3} + \frac{x^2}{2}$ parallel to the line $y = 2x$.

$y = 2x$ has slope 2.

$y = \frac{x^3}{3} + \frac{x^2}{2} \Longrightarrow \frac{dy}{dx} = x^2 + x$

Want $\frac{dy}{dx} = 2$.

Therefore $x^2 + x = 2 \longrightarrow x^2 + x - 2 = 0 \longrightarrow (x+2)(x-1) = 0$

$\longrightarrow x = -2, 1$

A) $x = -2, 1$

B) $x = 2, -1$

C) $x = 3, 1$

D) $x = 1, -3$

E) $x = -2, -1$

4) Compute $y'(-1)$ if $y(x) = \frac{x^2 + 3x}{2x - 1}$.

$$y'(x) = \frac{(2x+3)(2x-1) - (x^2+3x)(2)}{(2x-1)^2}$$

$$\longrightarrow y'(-1) = \frac{(1)(-3) - (-2)(2)}{9}$$

$$= \frac{1}{9}$$

A) $\frac{1}{9}$

B) $-\frac{1}{3}$

C) $\frac{1}{3}$

D) -1

E) $\frac{2}{9}$

5) If $y(x) = 2x^{3/2}\ln x$, compute $y'(4)$.

$$y'(x) = 2\left(\tfrac{3}{2}x^{1/2}\right)(\ln x) + 2x^{3/2}\left(\tfrac{1}{x}\right)$$
$$= 3x^{1/2}\ln x + 2x^{1/2}$$

$$y'(4) = 3(2)(\ln 2) + 2(2)$$
$$= 6\ln 2 + 4$$

A) $4 + 3\ln 4$

B) $8 + 8\ln 4$

C) $8 + 3\ln 4$

D) $4 + 6\ln 4$

E) $8 + 6\ln 4$

6) Compute the slope of the tangent line of the curve $y = \dfrac{\tan x}{1+\sec x}$ at $\left(\dfrac{\pi}{3}, \dfrac{\sqrt{3}}{3}\right)$.

$$y'(x) = \frac{(\sec^2 x)(1+\sec x) - (\tan x)(\sec x \tan x)}{(1+\sec x)^2}$$

$$y'\left(\tfrac{\pi}{3}\right) = \frac{(4)(3) - (\sqrt{3})(2)(\sqrt{3})}{(3)^2}$$

$$= \frac{12 - 6}{9}$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

A) $\frac{4}{9}$

B) $\frac{2}{9}$

C) $\frac{1}{3}$

D) $\frac{2}{3}$

E) $\frac{4}{3}$

7) If the position of a particle on a line at time t is $\frac{t+1}{t^2+1}$, what is the average velocity during the interval $0 \le t \le 2$.

Let $s(t) = \frac{t+1}{t^2+1}$

$$\frac{s(2)-s(0)}{2-0} = \frac{\frac{3}{5}-1}{2} = \frac{-\frac{2}{5}}{2} = -\frac{2}{10} = -\frac{1}{5}$$

A) -1

B) 1

C) $\frac{1}{10}$

D) $\frac{3}{10}$

E) $\frac{-1}{5}$

8) If $f(x) = \log_4 x^3$, $f'(x) =$

$$f'(x) = \frac{3x^2}{x^3} \cdot \frac{1}{\ln 4} = \frac{3}{x \ln 4}$$

A) $\frac{1}{x^3 \ln 4}$

B) $\frac{1}{x^3} \ln 4$

C) $3x^2 \ln 4$

D) $\frac{3}{x \ln 4}$

E) $\frac{3x^2}{\ln 4}$

9) A bacteria population triples every 2 hours. How long does it take a population of 200 bacteria to grow to 1500?

$P(t) = P(0)e^{kt}$

triples every 2 hours

$\longrightarrow 3\cancel{P(0)} = \cancel{P(0)}e^{2k} \longrightarrow \ln 3 = 2k$

$\longrightarrow k = \frac{1}{2}\ln 3$

$P(0) = 200 \longrightarrow P(t) = 200\, e^{\left(\frac{1}{2}\ln 3\right)t}$

Solve for t: $1500 = 200\, e^{\left(\frac{1}{2}\ln 3\right)t}$

$\longrightarrow \frac{15}{2} = e^{\left(\frac{1}{2}\ln 3\right)t}$

$\longrightarrow \ln\frac{15}{2} = \left(\frac{1}{2}\ln 3\right)t$

$\longrightarrow t = 2\,\frac{\ln(15/2)}{\ln(3)}$

A) $2\left(\frac{\ln 3}{\ln 30}\right)$

B) $2\left(\frac{\ln 3}{\ln 15}\right)$

C) $2\left(\frac{\ln 2}{\ln 15}\right)$

D) $2\left(\frac{\ln 15}{\ln \frac{3}{2}}\right)$

E) $2\left(\frac{\ln \frac{15}{2}}{\ln 3}\right)$

10) If y is a differentiable function of x and $xy - (x+y)^2 + \sqrt{y} + 19 = 0$, find $\frac{dy}{dx}$ at the point $(1, 4)$.

Differentiate with respect to $x \Rightarrow$

$(1)(y) + (x)\left(\frac{dy}{dx}\right) - 2(x+y)\left(1+\frac{dy}{dx}\right) + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

$(x,y) = (1,4) \Rightarrow$

$4 + \frac{dy}{dx} - 2(5)\left(1+\frac{dy}{dx}\right) + \frac{1}{4}\frac{dy}{dx} = 0$

$\longrightarrow 4 + \frac{dy}{dx} - 10 - 10\frac{dy}{dx} + \frac{1}{4}\frac{dy}{dx} = 0$

$\longrightarrow -\frac{35}{4}\frac{dy}{dx} = 6 \Longrightarrow \frac{dy}{dx} = \frac{6}{-\frac{35}{4}} = -\frac{24}{35}$

A) $-\frac{8}{3}$

B) $-\frac{12}{17}$

C) $-\frac{24}{35}$

D) $-\frac{12}{35}$

E) $-\frac{5}{7}$

11) If $y = \tan^{-1}(\sin x)$, then $\dfrac{dy}{dx} =$

$$\frac{dy}{dx} = \frac{1}{1+\sin^2 x}(\cos x)$$

A) $\cos x + \cot x \csc x$

B) $\dfrac{\cos x}{1+\sin^2 x}$

C) $\dfrac{\cos x}{\sqrt{1-\sin^2 x}}$

D) $\dfrac{1}{1+\sin^2 x}$

E) $\dfrac{x}{\sqrt{1-x^2}}$

12) If $f(x) = (\cos 2x)^3$, then $f'(\pi/3) =$

$$f'(x) = 3(\cos 2x)^2(-\sin 2x)(2)$$

$$f'\left(\frac{\pi}{3}\right) = 3\left(-\frac{1}{2}\right)^2\left(-\frac{\sqrt{3}}{2}\right)(2)$$

$$= -\frac{6\sqrt{3}}{8}$$

$$= -\frac{3\sqrt{3}}{4}$$

A) $-\dfrac{3\sqrt{3}}{4}$

B) $3\sqrt{3}$

C) $-3\sqrt{3}$

D) 3

E) $\dfrac{3\sqrt{3}}{2}$

13) If $f(x) = x^{\sin x}$, then $f'(x) =$

$$\ln f(x) = \ln x^{\sin x} = (\sin x)(\ln x)$$

$$\rightarrow \frac{f'(x)}{f(x)} = (\cos x)(\ln x) + (\sin x)\left(\frac{1}{x}\right)$$

$$\rightarrow f'(x) = f(x)\left[(\cos x)(\ln x) + (\sin x)\left(\frac{1}{x}\right)\right]$$

$$= x^{\sin x}\left[\cos x \ln x + \frac{\sin x}{x}\right]$$

A) $f'(x) = x^{\sin x}\left(\cos x + \frac{1}{x}\right)$

B) $f'(x) = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$

C) $f'(x) = x^{\sin x}(\cos x)\left(\frac{1}{x}\right)$

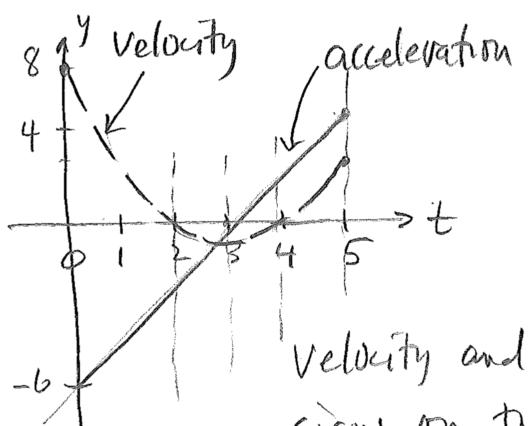
D) $f'(x) = x^{\sin x - 1}(\sin x)$

E) $f'(x) = x^{\sin x}(\cos x)$

14) A particle has position s at time t given by $s(t) = \frac{1}{3}t^3 - 3t^2 + 8t + 161$. On the time interval $[0, 5]$, when is the particle slowing up?

$s'(t) = t^2 - 6t + 8$ = velocity

$s''(t) = 2t - 6$ = acceleration



Velocity and acceleration have opposite signs on the intervals $(0,2)$ and $(3,4)$

A) $(0, 2) \cup (4, 5)$

B) $(2, 3) \cup (4, 5)$

C) $(0, 2) \cup (3, 4)$

D) $(2, 4)$

E) $(3, 5)$