MA 16100, EXAM 2, FALL 2015

1. If $y = x^2 e^{\sin x}$, find $\dfrac{dy}{dx}$.

$$\frac{d}{dx}(x^2)e^{\sin x} + x^2 \frac{d}{dx}\left(e^{\sin x}\right)$$

A. $2xe^{\sin x}$

B. $2xe^{\sin x} + x^2 e^{\sin x}$

C. $2xe^{\sin x} \cos x$

D. $\boxed{2xe^{\sin x} + x^2 e^{\sin x} \cos x}$

E. $2xe^{\sin x} + x^2 e^{\cos x}$

2. Find the limit.

$$\lim_{x \to 0} \frac{\sin(4x)\sin(3x)}{x^2} = \lim_{x \to 0} \frac{\sin 4x}{4x} \cdot \frac{\sin 3x}{3x} \cdot 12$$

$$= 1 \cdot 1 \cdot 12 = 12$$

A. 0

B. $\boxed{12}$

C. $\dfrac{1}{12}$

D. $\dfrac{3}{4}$

E. Does not exist.

3. If $y = \cos^{-1}(2x)$, find $(\cot(y))^2$.

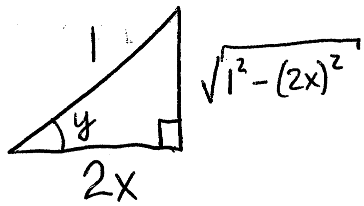
A. $\dfrac{\sqrt{1-x^2}}{x}$

B. $\dfrac{1-4x^2}{x^2}$

C. $\boxed{\dfrac{4x^2}{1-4x^2}}$

D. $\dfrac{1-x^2}{2x}$

E. $\dfrac{1-x^2}{x^2}$



$\cot y = \dfrac{\text{adj}}{\text{opp}} = \dfrac{2x}{\sqrt{1-4x^2}}$

4. Compute $f''(e)$, if $f(x) = \ln(\ln x)$.

A. $\dfrac{-1}{e}$

B. $\dfrac{e^2}{3+e^2}$

C. $\dfrac{-2e-1}{e^2}$

D. $\dfrac{1}{e^2}$

E. $\boxed{\dfrac{-2}{e^2}}$

$f'(x) = \dfrac{1}{x \ln x}$

$f''(x) = \dfrac{-\ln x - 1}{(x \ln x)^2}$

5. Using logarithmic differentiation, the derivative of $y = x^{(e^x)}$ is:

A. $x^{(e^x)}(e^x + \ln x)$

B. $e^x\left(x + x^{(e^x)}\right)$

C. $e^x(1/x + \ln x)$

D. $x^{(e^x - 1)}e^x$

E. $\boxed{x^{(e^x)}e^x(1/x + \ln x)}$

$\ln y = e^x \ln x$

$\frac{1}{y} y' = e^x \ln x + e^x\left(\frac{1}{x}\right)$

$y' = y\left(e^x\right)\left(\ln x + \frac{1}{x}\right)$

6. At what point on the curve $y = 1 + 5e^x - 4x$ is the tangent line parallel to the line $x - y = 5$?

A. $(\ln \frac{3}{5}, 4 - 4\ln \frac{3}{5})$

B. $\boxed{(0, 6)}$

C. $(1, 5e - 3)$

D. $(\ln \frac{9}{5}, 10 - 4\ln \frac{9}{5})$

E. $(5, 5e^5 - 19)$

$y' = 5e^x - 4 = 1$

$\rightarrow x = 0$

7. Suppose $f(x) = \tanh(1 - \tan x)$. Find $f'(\frac{\pi}{4})$.

A. $\frac{1}{2}$

B. -1

C. $\boxed{-2}$

D. $-\frac{1}{4}$

E. $\frac{1}{4}$

$$f'(x) = \operatorname{sech}^2(1 - \tan x) \cdot (-\sec^2 x)$$

$$f'\left(\tfrac{\pi}{4}\right) = \left(\frac{2}{e^{1-\tan \pi/4} + e^{-1+\tan \pi/4}}\right)^2 \cdot \frac{-1}{\left(\cos \frac{\pi}{4}\right)^2}$$

$$= \left(\frac{2}{1+1}\right)^2 \left(\frac{-1}{\left(\frac{1}{\sqrt{2}}\right)^2}\right)$$

8. Suppose $f(1) = 4$ and $f'(1) = 3$. If

$$g(x) = \sqrt{f(x)}$$

then $g'(1)$ equals

A. $\boxed{\frac{3}{4}}$

B. $\frac{3}{2}$

C. $\frac{1}{2\sqrt{3}}$

D. $\frac{2}{3}$

E. $\frac{1}{4}$

$$g'(x) = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$$

$$g'(1) = \frac{1}{2\sqrt{4}} \cdot 3$$

9. Find the slope of the tangent line to the curve

$$\sin(x+y) = 4x - 4y$$

at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$$\left[\cos(x+y)\right](1+y') = 4 - 4y'$$

$$y' \cos(x+y) + 4y' = 4 - \cos(x+y)$$

$$y' = \frac{4-\cos(x+y)}{4+\cos(x+y)}$$

$$y'\Big|_{\left(\frac{\pi}{2}, \frac{\pi}{2}\right)} = \frac{4-\cos(\pi)}{4+\cos(\pi)}$$

A. $\frac{4}{3}$

B. $\boxed{\frac{5}{3}}$

C. $\frac{5}{4}$

D. $\frac{3}{5}$

E. 1

10. Let $h(x) = \dfrac{2g(x)}{1+f(x)}$. Calculate $h'(2)$, if $f(2) = -3, g(2) = 5, f'(2) = 2$, and $g'(2) = 6$.

$$h'(x) = \frac{(1+f(x))(2g'(x)) - 2g(x)\,f'(x)}{(1+f(x))^2}$$

$$h'(2) = \frac{(1+(-3))(2)(6) - 2(5)(2)}{(1+(-3))^2}$$

A. 6

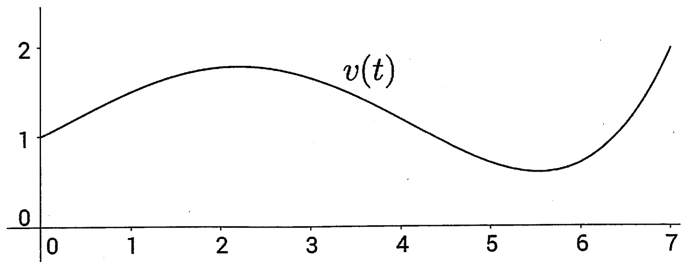
B. 4

C. $\frac{44}{9}$

D. 22

E. $\boxed{-11}$

11. A particle's velocity graph, $v(t)$, is pictured below. Which of the following are true?



I. The particle is speeding up when $3 < t < 5$.

II. The acceleration is positive when $0 < t < 2$.

III. The particle is moving in a positive direction when $0 < t < 7$.

A. I and II

B. II only

C. $\boxed{\text{II and III}}$

D. I and III

E. I only

12. Newton's Law of Cooling states that the rate of cooling of an object is proportional to the termperature difference between the object and its surroundings. Suppose a roast turkey has a temperature of 180°C in an oven. At 12:00pm the turkey is removed from the oven and placed in a room where the temperature is 20°C. At 1:00pm the turkey has cooled to 140°C. What is the temperature of the turkey at 2:00pm?

A. $\boxed{110°C}$

B. 80°C

C. 120°C

D. 90°C

E. 100°C

$T_s = 20$

$Y_0 = 180 - 20 = 160$

$y(1) = 140 - 20 = 120 = 160\, e^{k(1)}$

$y(2) = 160\, e^{k(2)} = 160\left(\frac{3}{4}\right)^2 = 90$

$T(2) = y(2) + T_s = 90 + 20 = 110$