SOLUTION

1. The curve defined by the equation $y = \dfrac{2 - x - x^2}{x^2 + 4}$ has

A. no asymptotes

B. exactly one horizontal asymptote and one vertical asymptote

C. exactly one horizontal asymptote and no vertical asymptotes

D. exactly one vertical asymptote and no horizontal asymptotes

E. exactly two horizontal asymptotes and no vertical asymptote

$x^2 + 4 \neq 0$ so there are no vertical asymptotes.

$$\lim_{x \to \pm\infty} \frac{2 - x - x^2}{x^2 + 4} = \lim_{x \to \pm\infty} \frac{\frac{2}{x^2} - \frac{1}{x} - 1}{1 + \frac{4}{x^2}}$$

$$= -1.$$

$\therefore$ $y = -1$ is the only horizontal asymptote.

2. Find an equation of the line tangent to the curve with the equation $y = \dfrac{1}{\sqrt{x}}$ at the point where $x = 3$.

A. $x + 6\sqrt{3}y = 9$

B. $x - 3y = 3 - \sqrt{3}$

C. $\sqrt{3}x + 9y = 6\sqrt{3}$

D. $x + 3y = 3 + \sqrt{3}$

E. $x - 6\sqrt{3}y = -3$

when $x = 3$, $y = \frac{1}{\sqrt{3}}$

$y = \sqrt{x} = x^{-1/2}$

$\frac{dy}{dx} = -\frac{1}{2} x^{-3/2} = -\frac{1}{2x\sqrt{x}}$

$\left.\frac{dy}{dx}\right|_{x=3} = -\frac{1}{6\sqrt{3}}$.

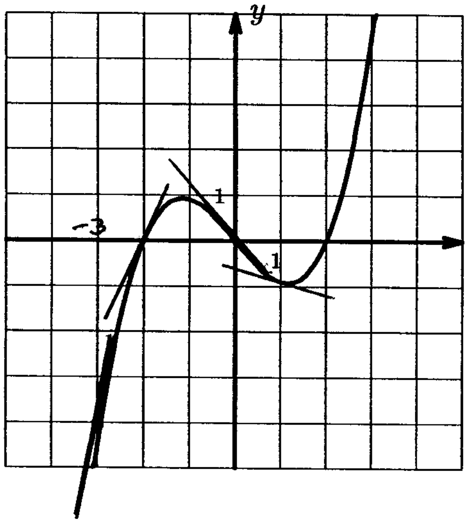
equation of the tangent line at $(3, \frac{1}{\sqrt{3}})$ is

$y - \frac{1}{\sqrt{3}} = -\frac{1}{6\sqrt{3}}(x-3)$

$6\sqrt{3}y - 6 = -x + 3$

$x + 6\sqrt{3}y = 9$

3. For the function u whose graph is given, arrange the following numbers in decreasing order: $u'(-3)$, $u'(-2)$, 0, $u'(0)$, $u'(1)$.



From the graph!

$u'(-3) \approx 5$

$u'(-2) \approx 2$

$u'(0) \approx -1$

$u'(1) \approx -\frac{2}{5}$

A. $u'(-3), u'(-2), u'(0), u'(1), 0$

B. $u'(-2), u'(0), 0, u'(1), u'(-3)$

C. $0, u'(1), u'(0), u'(-2), u'(-3)$

D. $u'(-3), u'(-2), 0, u'(1), u'(0)$

E. $0, u'(1), u'(0), u'(-2), u'(-3)$

$\therefore u'(-3) > u'(-2) > 0 > u'(1) > u'(0)$

4. $\lim\limits_{x \to 1} \dfrac{x^6 - 1}{x - 1}$ is the derivative of some function f at some number a. find f and a.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

$\therefore f(x) = x^6$ and $a = 1$

A. $f(x) = x^6, \ a = 1$

B. $f(x) = x^6, \ a = 0$

C. $f(x) = 6x^5, \ a = 1$

D. $f(x) = 6x^5, \ a = 0$

E. The limit doesn't exist

5. A ball is thrown upward from the ground with an initial velocity of 160 ft/sec. Its height above the ground, in feet, t seconds later is given by $s(t) = 160t - 16t^2$. Its velocity when $t = 2$ is

If $s(t) = 160t - 16t^2 =$ height

velocity $= s'(t) = 160 - 32t$.

$s'(2) = 160 - 64 = 96$

A. 64 ft/sec

B. 96 ft/sec

C. 32 ft/sec

D. 106 ft/sec

E. 128 ft/sec

6. The graph of $f(x) = 2x - e^x$ has a horizontal tangent when $x =$

There is a horizontal tangent when $f'(x) = 0$.

$f(x) = 2x - e^x$

$f'(x) = 2 - e^x$

If $2 - e^x = 0$, $e^x = 2$

which implies $x = \ln 2$

A. $\frac{e}{2}$

B. $\frac{\ln 2}{2}$

C. e^2

D. $e \ln 2$

E. $\ln 2$

7. If $f(x) = |x-2|$, then $f'(2) =$

$$|x-2| = \begin{cases} x-2 & \text{if } x \geq 2 \\ -x+2 & \text{if } x < 2. \end{cases}$$

$$\therefore \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2^+} \frac{x-2-0}{x-2} = 1$$

$$\lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2^-} \frac{-x+2-0}{x-2} = -1$$

A. 0

B. -1

C. 1

D. ∞

E. Does not exist

8. Let $f(x) = (x^2 - 3x + 1)\left(\frac{1}{x} - \sqrt{x}\right)$. Then $f'(1) =$

$$f'(x) = (x^2 - 3x + 1)\left(-\frac{1}{x^2} - \frac{1}{2\sqrt{x}}\right) + \left(\frac{1}{x} - \sqrt{x}\right)(2x-3)$$

$$f'(1) = (-1)\left(-\frac{3}{2}\right) + 0 \cdot (-1)$$

$$= \frac{3}{2}$$

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. $-\frac{3}{2}$

E. $-\frac{5}{2}$

9. If $g(t) = \dfrac{t^2}{t + \frac{3}{t}}$ then $g'(1) =$

$$g'(t) = \frac{\left(t + \frac{3}{t}\right)(2t) - t^2\left(1 - \frac{3}{t^2}\right)}{\left(t + \frac{3}{t}\right)^2}$$

$$g'(1) = \frac{4 \cdot 2 - (-2)}{16} = \frac{5}{8}$$

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 2

D. $\frac{5}{8}$

E. $\frac{3}{8}$

10. If $f(x) = \dfrac{x^3}{g(x)}$ and $f(2) = 4$, $f'(2) = 3$ and $g(2) = 2$, then $g'(2) =$

$$f'(x) = \frac{g(x) \cdot 3x^2 - x^3 \cdot g'(x)}{(g(x))^2}$$

$\therefore$ if $x = 2$.

$$f'(2) = \frac{g(2) \cdot 12 - 8 \cdot g'(2)}{g(2)^2}$$

$$3 = \frac{2 \cdot 12 - 8(g'(2))}{4}$$

$$3 = 6 - 2g'(2)$$

$$\therefore \; g'(2) = \frac{3}{2}$$

A. $\frac{3}{2}$

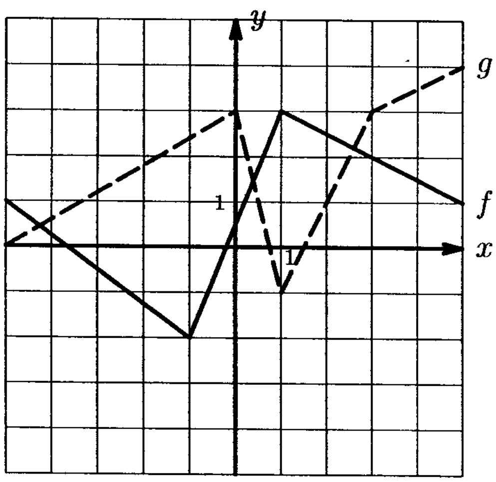
B. 1

C. -1

D. 4

E. -4

11. The graphs of f and g are shown below. Let $u(x) = f(x)g(x)$. Find $u'(2)$.



$f(2) = \frac{5}{2}$

$g(2) = 1$

$f'(2) = \dfrac{1-3}{5-1} = -\dfrac{1}{2}$

$g'(2) = \dfrac{1-(-1)}{2-1} = 2$

A. 3

B. $\frac{3}{2}$

C. $-\frac{3}{2}$

D. $-\frac{9}{2}$

E. $\frac{9}{2}$

$$u'(2) = f(2)\,g'(2) + g(2) \cdot f'(2)$$

$$u'(2) = \frac{5}{2} \cdot 2 + 1 \cdot \left(-\frac{1}{2}\right)$$

$$= 4\frac{1}{2} = \frac{9}{2}$$

12. If $y = \tan^{-1}(3x^2)$, find $\dfrac{dy}{dx}$ when $x = -1$.

$$\frac{dy}{dx} = \frac{1}{1+9x^4} \cdot 6x$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = \frac{1}{10} \cdot (-6) = -\frac{3}{5}$$

A. $\frac{5}{3}$

B. $-\frac{3}{5}$

C. $\frac{3}{5}$

D. $\frac{1}{10}$

E. $-\frac{1}{10}$

13. Find $\dfrac{dy}{dx}$ if $4\cos y \sin x = 2$.

$$\frac{d}{dx}(4\cos y \sin x) = \frac{d}{dx}(2) = 0$$

$$4\left(\cos y \cdot \cos x + (-\sin y \cdot \sin x)\frac{dy}{dx}\right) = 0$$

$$\therefore \frac{dy}{dx} = \frac{\cos y \cos x}{\sin y \sin x}$$

A. $\dfrac{-\sin y + \cos x}{\sin x + \cos y}$

B. $\dfrac{\cos y \sin x}{\cos x \sin y}$

C. $\dfrac{\cos y \cos x}{\sin y \sin x}$

D. $\dfrac{\cos x \sin y}{\cos y \sin x}$

E. $\dfrac{\cos^2 x}{\sin^2 y}$

14. Let $y^2 = 5x^4 - x^2$. Find the slope of the tangent line at the point $(1, -2)$.

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(5x^4 - x^2)$$

$$2y\frac{dy}{dx} = 20x^3 - 2x$$

$$\frac{dy}{dx} = \frac{10x^3 - x}{y}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=-2}} = \frac{10-1}{-2} = -\frac{9}{2}$$

A. 9

B. -9

C. $\frac{9}{2}$

D. $-\frac{9}{2}$

E. $-\frac{9}{4}$