MA 161 EXAM 3 Fall 2007

Name SOLUTIONS

ten–digit Student ID number

RECITATION Division and Section Numbers

Recitation Instructor

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 12 problems, each worth $8\frac{1}{3}$ points. The maximum score is 100 points.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

1. The most common linear approximation of $\frac{1}{1003}$ that uses the reciprocal function is

Let $f(x) = \frac{1}{x}$, $a = 1000$

Then $f'(x) = -1/x^2$.

$L(x) = f(1000) + f'(1000)(x-1000)$

$= 0.001 - 0.000001\,(x-1000)$

$L(1003) = 0.001 - 0.000001\,(3)$

$= 0.000997$

A. 0.001

B. 0.001003

C. 0.000997009

D. 0.00099

E. 0.000997

2. The ratio $\dfrac{1+\tanh x}{1-\tanh x}$ is identical to

$$\frac{1+\tanh x}{1-\tanh x} = \frac{1+\frac{\sinh x}{\cosh x}}{1-\frac{\sinh x}{\cosh x}} = \frac{\frac{\cosh x + \sinh x}{\cancel{\cosh x}}}{\frac{\cosh x - \sinh x}{\cancel{\cosh x}}}$$

$$= \frac{\frac{e^x+e^{-x}}{2} + \frac{e^x-e^{-x}}{2}}{\frac{e^x+e^{-x}}{2} - \frac{e^x-e^{-x}}{2}} = \frac{\frac{2e^x}{2}}{\frac{2e^{-x}}{2}}$$

$$= \frac{e^x}{e^{-x}} = (e^x)(e^x) = e^{2x}$$

A. sinh

B. cosh

C. e^{2x}

D. e^{-2x}

E. 1

3. If $f(-5) = -1$ and $f'(x) \leq -3$, then the mean value theorem guarantees that

use interval $[-5,-2]$. $f'(x)$ exists $\to$ f continuous.

Mean Value Theorem $\Rightarrow$ $\dfrac{f(-2)-f(-5)}{-2-(-5)} = f'(c)$, where $-5 < c < -2$.

$f'(c) \leq -3 \Rightarrow \dfrac{f(-2)-f(-5)}{-2-(-5)} \leq -3$

$\to \dfrac{f(-2)-f(-5)}{3} \leq -3 \to f(-2)-f(-5) \leq -9$

$\to f(-2)-(-1) \leq -9 \longrightarrow f(-2) \leq -10$

A. $f(-2) \leq -10$

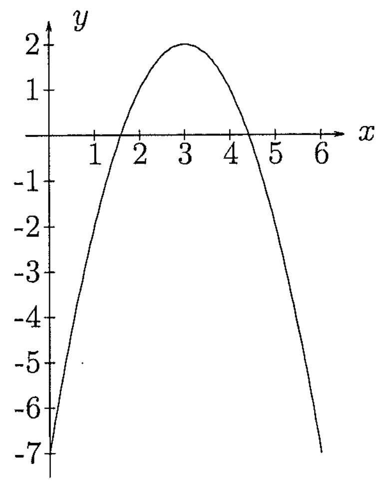
B. $f(-2) \geq -10$

C. $f(-2) \leq -8$

D. $f(-2) \geq -8$

E. None of the above

4. Given the graph of $y = f'(x)$ below,



it follows that

A. f is increasing on $(0, 3)$

B. f is concave down on $(0, 6)$

C. f has a local minimum at $x \approx 4.4$

D. f has an inflection point at $x = 3$

E. None of the above

f' changes from increasing to decreasing at $x=3$, therefore f'' changes sign from positive to negative at $x=3$, therefore $(3, f(3))$ is an inflection point.

5. $\lim_{x \to 0} (1 - 2x)^{1/x}$ $= 1^0$

A. $= 0$

B. $= e^{-2}$

C. $= -2$

D. $= 1$

E. does not exist

$$\lim_{x\to 0} (1-2x)^{1/x} = \lim_{x \to 0} \left(e^{\ln(1-2x)}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(e^{\frac{\ln(1-2x)}{x}}\right) = e^{\lim_{x\to 0} \frac{\ln(1-2x)}{x}}$$

Now $\lim_{x\to 0} \frac{\ln(1-2x)}{x} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\frac{-2}{1-2x}}{1} = \frac{\frac{-2}{1}}{1} = -2$

Therefore $\lim_{x\to 0} (1-2x)^{1/x} = e^{-2}$

6. Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

$\rightarrow y^2 = 4 - 4x^2$

$D = \sqrt{(x-1)^2 + (y-0)^2}$ where (x, y) is on ellipse.

$\rightarrow D = \sqrt{(x-1)^2 + (4-4x^2)} = \sqrt{-3x^2 - 2x + 5}$

$\frac{dD}{dx} = \frac{-6x-2}{2\sqrt{-3x^2-2x+5}} = 0 \rightarrow x = -\frac{1}{3}$

$\rightarrow y = \pm\sqrt{4 - 4\left(-\frac{1}{3}\right)^2} = \pm\sqrt{\frac{32}{9}} = \pm \frac{4\sqrt{2}}{3}$

A. $\left(-\frac{1}{3}, -\frac{4\sqrt{2}}{3}\right)$ and $\left(\frac{1}{3}, \frac{4\sqrt{2}}{3}\right)$

B. $\left(-\frac{4\sqrt{2}}{3}, -\frac{1}{3},\right)$ and $\left(\frac{1}{3}, \frac{4\sqrt{2}}{3}\right)$

C. $\left(-\frac{1}{3}, -\frac{4\sqrt{2}}{3}\right)$ and $\left(-\frac{1}{3}, \frac{4\sqrt{2}}{3}\right)$

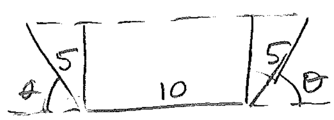
D. $\left(\frac{4\sqrt{2}}{3}, -\frac{1}{3},\right)$ and $\left(\frac{1}{3}, \frac{4\sqrt{2}}{3}\right)$

E. $\left(-\frac{1}{3}, -\frac{4\sqrt{2}}{3}\right)$ and $\left(\frac{1}{3}, -\frac{4\sqrt{2}}{3}\right)$

Farthest points are $\left(-\frac{1}{3}, \frac{4\sqrt{2}}{3}\right)$ and $\left(-\frac{1}{3}, -\frac{4\sqrt{2}}{3}\right)$

7. A rain gutter is to be constructed from a metal sheet of width 20 cm by bending up one fourth of the sheet on each side through an angle θ. The cosine of the angle θ that will result in the gutter capable of carrying the maximum amount of water is $\cos\theta =$

Cross section of gutter



A. $\frac{\sqrt{3}}{2}$

B. $\frac{-\sqrt{3}+1}{2}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}-1}{2}$

E. $\frac{\sqrt{3}+1}{2}$

Cross-sectional area $= A = 2\left(\frac{1}{2}\right)(5\cos\theta)(5\sin\theta) + (10)(5\sin\theta)$
(= 2 triangles + rectangle)

$A(\theta) = 25\cos\theta\sin\theta + 50\sin\theta, \quad 0 \le \theta \le \pi/2$

$A'(\theta) = 25(-\sin^2\theta + \cos^2\theta) + 50\cos\theta$

$= 25(-(1-\cos^2\theta) + \cos^2\theta) + 50\cos\theta$

$= 25(2\cos^2\theta + 2\cos\theta - 1)$

$A'(\theta) = 0 \rightarrow \cos\theta = \frac{-2 \pm \sqrt{4 - 4(2)(-1)}}{4} = \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4}$

$\rightarrow \cos\theta = \frac{-1 \pm \sqrt{3}}{2}$, $\quad 0 \le \frac{-1+\sqrt{3}}{2} \le \pi/2$

8. Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation to the root of the equation $x^4 - x - 1 = 0$. Then, $x_2 =$

let $f(x) = x^4 - x - 1$.

Then $f'(x) = 4x^3 - 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = x_1 - \frac{x_1^4 - x_1 - 1}{4x_1^3 - 1}$$

$$x_1 = 1 \rightarrow x_2 = 1 - \frac{-1}{3} = \frac{4}{3}$$

A. $\frac{7}{12}$

B. $\frac{4}{3}$

C. $\frac{1}{\sqrt[3]{4}}$

D. $\sqrt[3]{4}$

E. $\frac{2}{3}$

9. Given $f''(x) = \sin\theta + \cos\theta$, $f(0) = -1$, $f'(0) = 4$, it follows that $f(\pi/4) =$

$f''(x) = \sin\theta + \cos\theta$

$\rightarrow f'(x) = -\cos\theta + \sin\theta + C$

$f'(0) = 4 = -\cos 0 + \sin 0 + C = -1 + 0 + C$

$\rightarrow C = 5$

$\rightarrow f'(x) = -\cos\theta + \sin\theta + 5$

$\rightarrow f(x) = -\sin\theta - \cos\theta + 5\theta + C_1$

$f(0) = -1 = -\sin 0 - \cos 0 + 5(0) + C_1 = -1 + C_1$

$\rightarrow C_1 = 0$

$\rightarrow f(x) = -\sin\theta - \cos\theta + 5\theta$

$$f\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} - \cos\frac{\pi}{4} + 5\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{5\pi}{4} = -\sqrt{2} + \frac{5\pi}{4}$$

A. $\frac{5\pi\sqrt{2}}{4}$

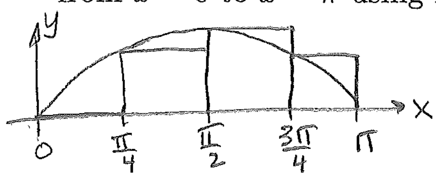
B. $\frac{5\pi}{4} - \frac{\sqrt{2}}{2}$

C. $\frac{5\pi\sqrt{2}}{2}$

D. $\frac{5\pi}{4}$

E. $\frac{5\pi}{4} - \sqrt{2}$

10. The left-endpoint Riemann sum to estimate the area under the graph of $f(x) = \sin x$ from $x = 0$ to $x = \pi$ using four approximating rectangles is



$$\frac{\pi}{4}\left[\sin 0 + \sin\frac{\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{3\pi}{4}\right]$$

$$= \frac{\pi}{4}\left[0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}\right] = \frac{\pi}{4}\left[1+\sqrt{2}\right]$$

A. $\frac{1+\sqrt{2}}{4}$

B. $\frac{3\sqrt{3}}{8}$

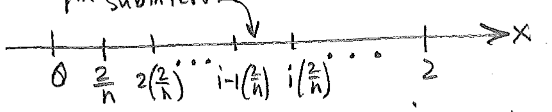
C. 2

D. $\frac{\pi}{2}$

E. $\frac{\pi(1+\sqrt{2})}{4}$

11. The definite integral $\int_0^2 \frac{1}{1+x}\,dx$ is the limit of which Riemann sums?

ith subinterval



f(x) at right endpt of ith subinterval is

$$f\left(\frac{2i}{n}\right) = \frac{1}{1+\frac{2i}{n}} = \frac{1}{\frac{n+2i}{n}} = \frac{n}{n+2i}$$

$$\int_0^2 \frac{1}{1+x}\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\cdot\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} \frac{\cancel{n}}{n+2i}\left(\frac{2}{\cancel{n}}\right)$$

A. $\sum_{i=1}^{n} \frac{2}{n+2i}$

B. $\sum_{i=1}^{n} \frac{2}{n+i}$

C. $\sum_{i=1}^{n} \frac{1}{n+2i}$

D. $\sum_{i=1}^{n} \frac{1}{n+i}$

E. None of the above

12. Given $g(x) = \int_0^{2x} \frac{u^2-2}{u^2+2}\,du$ the value of $g'(1)$ is

$$g'(x) = \left(\frac{(2x)^2-2}{(2x)^2+2}\right)(2)$$

chain rule!

$$g'(1) = \left(\frac{4-2}{4+2}\right)(2) = \left(\frac{2}{6}\right)(2) = \frac{4}{6} = \frac{2}{3}$$

A. 2

B. 3

C. $\frac{1}{3}$

D. $\frac{2}{3}$

E. $\frac{4}{5}$