SOLUTIONS

MA 16100 **Exam III** **Fall 2008**

1. What approximate value do you get for $\sqrt{4.1}$ if you use the linear approximation at 4?

A. 2

[✓ B. 2.025]

C. 2.05

D. 2.075

E. 2.1

Let $f(x) = x^{\frac{1}{2}}$, $a = 4$.

Then $f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$

$$L(x) = f(4) + f'(4)(x-4)$$
$$= 2 + \frac{1}{4}(x-4)$$

$$L(4.1) = 2 + \frac{1}{4}(4.1-4)$$
$$= 2 + 0.25(0.1)$$
$$= 2.025$$

2. Evaluate cosh(ln 5).

A. 2.4

B. 2.5

[✓ C. 2.6]

D. 3

E. 5

$$\cosh(\ln 5) = \frac{e^{\ln 5} + e^{-\ln 5}}{2}$$
$$= \frac{5 + (5)^{-1}}{2}$$
$$= \frac{5.2}{2}$$
$$= 2.6$$

3. The maximum value of $x^3 - 3x + 9$ for $-3 \leq x \leq 2$ is

A. 5
B. 7
C. 9
[✓ D. 11]
E. 13

let $f(x) = x^3 - 3x + 9$

Then $f'(x) = 3x^2 - 3$

$f'(x)$ exists for all x,

$f'(x) = 0 \rightarrow x = \pm 1$

$f(-3) = -27 + 9 + 9 = -9$ ← min value of f

$f(-1) = -1 + 3 + 9 = 11$ ← max value of f

$f(1) = 1 - 3 + 9 = 7$

$f(2) = 8 - 6 + 9 = 11$ ← max value of f

4. The minimum value of $x^3 - 3x + 9$ for $-3 \leq x \leq 2$ is

[✓ A. −9]
B. −1
C. 3
D. 5
E. 7

See previous problem

5. Given that $f(3) = 0$ and $f'(x) \geq 3$ for $0 \leq x \leq 3$, the largest $f(0)$ can be is

[✓ A.] -9

B. -3

C. 0

D. 6

E. Cannot be determined.

f differentiable on $[0,3]$ implies Mean Value Theorem hypotheses satisfied by f on $[0,3]$.

Therefore, $\frac{f(3)-f(0)}{3-0} = f'(c)$ where $0 < c < 3$.

$f'(c) \geqslant 3 \rightarrow \frac{f(3)-f(0)}{3-0} \geqslant 3 \rightarrow \frac{0-f(0)}{3} \geqslant 3$

$\rightarrow -f(0) \geqslant 9 \rightarrow f(0) \leq -9$

6. If $f'(x) = x(x-1)^2(x-2)$, then f has

A. 3 local minima.

B. 2 local minima and 1 local maximum.

C. 1 local minimum and 2 local maxima.

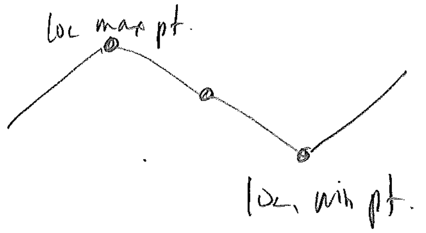
D. 3 local maxima.

[✓ E.] 1 local maximum and 1 local minimum.

$f'(x) = 0 \rightarrow x = 0, 1, 2$

	$-\infty$		0		1		2		∞ x
x		−	0	+		+		+	
$(x-1)^2$		+		+	0	+		+	
x−2		−		−		−	0	+	
$f'(x)$		+		−		−		+	
f		inc.		dec.		dec.		inc	

f



f(0) is a local max.

f(2) is a local min.

7. If $f'(x) = 3(x-1)^{2/3} - x$, the interval(s) where f is concave down is (are)

A. $(-\infty, 9)$ only

B. $(-\infty, 1)$ only

C. $(9, \infty)$ only

[✓ D.] $(-\infty, 1)$ and $(9, \infty)$

E. $(-\infty, 9)$ and $(9, \infty)$

$$\to f''(x) = 2(x-1)^{-1/3} - 1$$

$$f''(x) = \frac{2}{(x-1)^{1/3}} - \frac{(x-1)^{1/3}}{(x-1)^{1/3}} = \frac{2-(x-1)^{1/3}}{(x-1)^{1/3}}$$

$f''(x)$ does not exist for $x = 1$

$f''(x) = 0$ for $(x-1)^{1/3} = 2 \to x-1 = 8 \to x = 9$

	$-\infty$	1		9	$\infty \to x$
$2-(x-1)^{1/3}$	+		+	0	−
$(x-1)^{1/3}$	−	0	+		+
$f''(x)$	−	u	+	0	−
f	concave down		concave up		concave down

8. $\lim_{x\to\infty} \frac{\ln(1+2x)}{\ln(3x)} = \frac{\infty}{\infty}$

A. 2/3

B. 3/2

C. 6

[✓ D.] 1

E. 0

use l'Hôpital's Rule

$$\lim_{x\to\infty} \frac{\ln(1+2x)}{\ln(3x)}$$

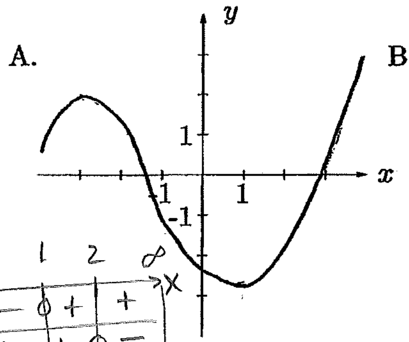
$$\overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{\frac{2}{1+2x}}{\frac{3}{3x}}$$

$$= \lim_{x\to\infty} \frac{\cancel{\frac{1}{x}}\left(\frac{2}{\frac{1}{x}+2}\right)}{\cancel{\frac{1}{x}}\left(\frac{3}{3}\right)}$$

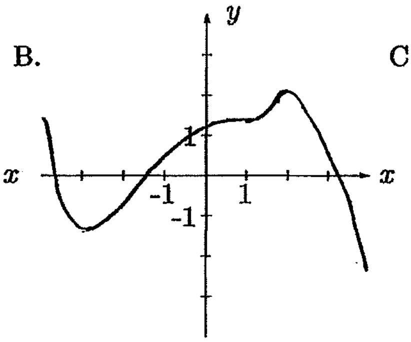
$$= \frac{\frac{2}{0+2}}{\frac{3}{3}} = \frac{1}{1} = 1$$

9. If $f'(x) = (x-1)(2-x)(x+3)$, then the graph of f can look like which one of the following graphs?

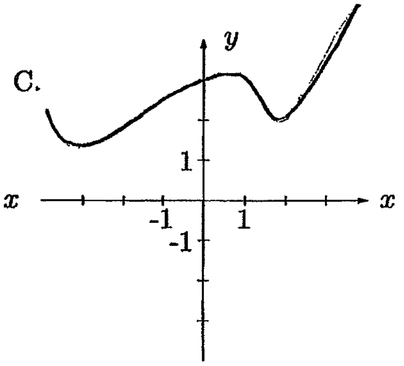
A.



B.



C.

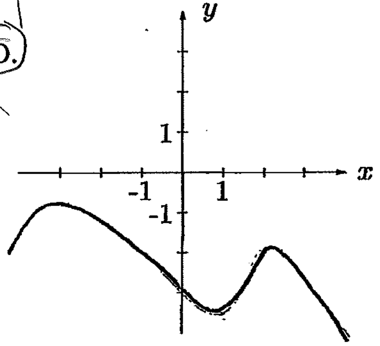


−∞ −3 1 2 ∞ →X

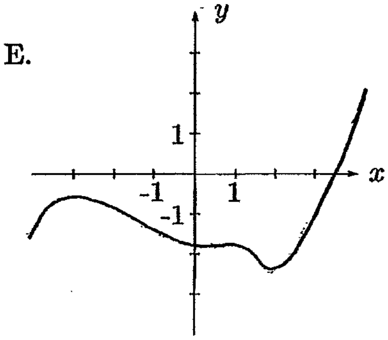
X−1	—	—	0 +	+
2−X	+	+	+ 0	—
X+3	— 0	+	+	+
f'(x)	+	—	+	—

f

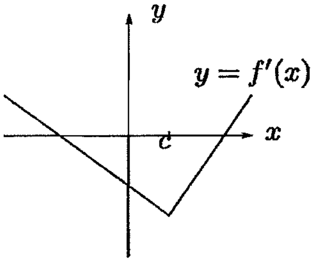
[✓ D.]



E.



10. The graph of f' is given below. Only one of the following is true. Which one?



A. f has a local min at $x = c$.

B. f is not differentiable at $x = c$.

[✓ C.] f has an inflection point at $x = c$.

D. f is increasing for all x such that $x > c$.

E. $f(c) < 0$.

11. Find the x–coordinate of the point on the line $3x - 2y = 2$ that is closest to the point $(2, 1)$.

$\to y = \frac{2-3x}{-2} = \frac{3x-2}{2}$

let $D = \sqrt{(x-2)^2 + (y-1)^2}$, (x,y) on line.

$\to D = \sqrt{(x-2)^2 + \left(\frac{3x-2}{2} - 1\right)^2}$

$= \sqrt{x^2 - 4x + 4 + \left(\frac{9x^2-12x+4}{4} - (3x-2) + 1\right)}$

$= \sqrt{\frac{13}{4}x^2 - 10x + 8}$

$\frac{dD}{dx} = \frac{\frac{13}{2}x - 10}{2\sqrt{\frac{13}{4}x^2 - 10x + 8}} = 0 \to x = \frac{20}{13}$

[✓ A.] $\frac{20}{13}$

B. $\frac{10}{13}$

C. $\frac{8}{13}$

D. $\frac{20}{17}$

E. $\frac{10}{17}$

12. Suppose at the point $(2, -3)$ on the curve $y = f(x)$, the tangent line has slope 4. If Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 2$, find the second approximation x_2.

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2 - \frac{-3}{4}$

$= 2 + \frac{3}{4}$

$= \frac{11}{4}$

A. $x_2 = -\frac{11}{4}$

B. $x_2 = -\frac{4}{11}$

C. $x_2 = \frac{4}{11}$

[✓ D.] $x_2 = \frac{11}{4}$

E. $x_2 = \frac{3}{2}$

13. Find the most general antiderivative of the function $g(x) = \cos(2x) - 3\sin(x)$.

A. $2\sin(2x) + \frac{1}{3}\cos(3x) + C$

[✓ B.] $\frac{1}{2}\sin(2x) + 3\cos(x) + C$

C. $\frac{1}{2}\sin(2x) - 3\cos(x) + C$

D. $-2\sin(2x) + \frac{1}{3}\cos(3x) + C$

E. $2\sin(2x) - \frac{1}{3}\cos(3x) + C$

$\int \cos(2x)\,dx = *$

let $u = 2x \rightarrow du = 2\,dx \rightarrow \frac{1}{2}du = dx$

$* = \int \cos(u)\left(\frac{1}{2}du\right) = \frac{1}{2}\int \cos u\,du$

$= \frac{1}{2}\sin u + C = \frac{1}{2}\sin(2x) + C$

$\int (\cos 2x - 3\sin(x))\,dx = \frac{1}{2}\sin 2x + 3\cos x + C$

14. If $f''(x) = x^{1/3}$, $f'(8) = 10$, and $f(1) = 0$, then $f(0) =$

A. $-\frac{9}{28}$

B. $\frac{9}{28}$

C. $\frac{45}{28}$

D. $\frac{8}{28}$

[✓ E.] $\frac{47}{28}$

$\frac{9}{28} - \frac{56}{28} = -\frac{47}{28}$

$f''(x) = x^{1/3}$

$\rightarrow f'(x) = \frac{3}{4}x^{4/3} + c$

$f'(8) = 10 = \frac{3}{4}(8^{4/3}) + c$

$\rightarrow 10 = \frac{3}{4}(16) + c$

$\rightarrow 10 = 12 + c$

$\rightarrow c = -2$

$\rightarrow f'(x) = \frac{3}{4}x^{\frac{4}{3}} - 2$

$\rightarrow f(x) = \frac{3}{4}\left(\frac{3}{7}x^{\frac{7}{3}}\right) - 2x + C_1 = \frac{9}{28}x^{7/3} - 2x + C_1$

$f(1) = 0 = \frac{9}{28} - 2 + C_1 \rightarrow C = \frac{47}{28}$

$\rightarrow f(x) = \frac{9}{28}x^{7/3} - 2x + \frac{47}{28} \rightarrow f(0) = \frac{47}{28}$