Solutions:

MATH 161 – FALL 2009 – THIRD EXAM – NOVEMBER 2009

TEST NUMBER 01

STUDENT NAME ______________________________

STUDENT ID ______________________________

LECTURE TIME ______________________________

RECITATION INSTRUCTOR ______________________________

RECITATION TIME ______________________________

INSTRUCTIONS

1. Fill in all the information requested above and the version number of the test on your scantron sheet.

2. This booklet contains 14 problems, each worth 7 points. There are two free points. The maximum score is 100 points.

3. For each problem mark your answer on the scantron sheet and also circle it is this booklet.

4. Work only on the pages of this booklet.

5. Books, notes, calculators are not to be used on this test.

6. At the end turn in your exam and scantron sheet to your recitation instructor.

(1) The position function of a particle after t seconds is given by $s = 42t^2 - t^3$. After how many seconds is the acceleration equal to zero?

(a) 1 sec.

(b) 5 sec.

(c) 7 sec.

→ (d) 14 sec.

(e) 28 sec.

$v(t) = s'(t) = 84t - 3t^2$

$a(t) = v'(t) = 84 - 6t = 6(14-t)$

$a(t) = 0 \Leftrightarrow t = 14$

(2) A material has a half–life of 12 hours. If initially there are 4 grams of the material, how much is present after 8 hours?

(a) $2^{2/3}$

(b) $2^{3/4}$

→ (c) $2^{4/3}$

(d) $2^{3/2}$

(e) $8/3$

$P(t) = P_0 e^{ct}$

$P_0 = 4 \Rightarrow P(t) = 4e^{ct}$

$P(12) = \frac{P_0}{2} = 2$

$\Rightarrow 2 = 4e^{12c}$

$\Rightarrow \frac{1}{2} = e^{12c}$

$\Leftrightarrow \ln(\frac{1}{2}) = 12c$

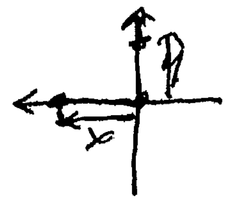
$c = \frac{1}{12}\ln(\frac{1}{2})$.

So $P(t) = 4e^{\frac{t}{12}\ln(\frac{1}{2})}$

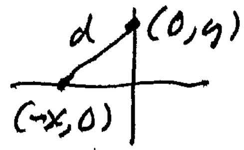
$P(8) = 4e^{\frac{2}{3}\ln(\frac{1}{2})} = 4e^{\ln(\frac{1}{2})^{2/3}} = 4\cdot(\frac{1}{2})^{2/3}$

$= 2^{2-2/3} = \underline{2^{4/3}}$

(3) Two people start from the same point. One walks east at 4 mi./hr. and the other walks north at 2 mi./hr. How fast is the distance between them changing after 10 ~~minutes?~~ hours ?



Let x = distance from start of eastward person
y = dist. from start of north bound person



$d = \sqrt{(0-(-x))^2 + (y-0)^2}$
$= \sqrt{x^2+y^2}$

(a) $\sqrt{20}/2$ mi./hr.

(b) $\sqrt{20}$ mi./hr.

(c) $2\sqrt{20}$ mi./hr.

(d) $6\sqrt{20}$ mi./hr.

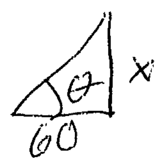
(e) $10\sqrt{20}$ mi./hr.

$d'(t) = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot (2x \cdot \underbrace{x'(t)}_{=2} + 2y \cdot \underbrace{y'(t)}_{4})$

After 10 min: $x = 20$, $y = 40$.

So $d'(10) = \frac{1}{2\sqrt{2000}} \cdot 2(200) = \frac{200}{10\sqrt{20}} = \sqrt{20}$

(4) A balloon is rising vertically from a point on the ground that is 60 feet from a ground–level observer. If the balloon is rising at a rate of 24 feet/sec., how fast is the angle of elevation between the observer and the balloon increasing when this angle is $\frac{\pi}{3}$?



$\frac{dx}{dt} = 24$, $\tan\theta = \frac{x}{60}$

so $\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{60}\frac{dx}{dt} = \frac{24}{60}$

When $\theta = \pi/3$, $\sec\theta = 2$

so $4\frac{d\theta}{dt} = \frac{24}{60}$

$\frac{d\theta}{dt} = \frac{6}{60} = \frac{1}{10}$

(a) 1/10 radians/sec.

(b) 1/15 radians/sec.

(c) 3/10 radians/sec.

(d) $4\sqrt{3}/15$ radians/sec.

(e) 8/5 radians/sec.

(5) Use a linearization to approximate the value of $\sqrt[3]{27.01}$:

(a) $3 + \frac{1}{30}$

(b) $3 + \frac{1}{90}$

(c) $4 + \frac{1}{900}$

(d) $3 + \frac{1}{270}$

(e) $3 + \frac{1}{2700}$

$f(x) = x^{1/3}$. $f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

$f(x) \simeq f(a) + f'(a)(x-a)$, Take $a=27$,

$x=27.01 \Rightarrow \sqrt[3]{27.01} \simeq \sqrt[3]{27} + \frac{1}{3 \cdot 27^{2/3}} \cdot (.01)$

$= 3 + \frac{1}{3 \cdot 9} \cdot \frac{1}{100}$

$= 3 + \frac{1}{2700}$

(6) Let $f(x) = x^3 - 3x^2 + 3$. Find all values of x where f has a local maximum.

(a) $x = 0,\ x = 2$

(b) $x = 1$

(c) $x = 2$

(d) $x = 0$

(e) $x = 1, 2$

$f'(x) = 3x^2 - 6x = 2x(x-3)$

$= 0$ @ $x = 0$, $x = 3$.

$f'>0$, $f'<0$, $f'>0$

0 3

f: ↑ ↓ ↑

0 3

local max @ $x = 0$

(7) Find all open intervals in $[0, 2\pi]$ where the function $f(t) = \sin t + \cos t$ is decreasing.

(a) $(\frac{\pi}{4}, \frac{3\pi}{4})$

(b) $(\frac{\pi}{4}, \frac{5\pi}{4})$

(c) $(\frac{\pi}{2}, \frac{3\pi}{2})$

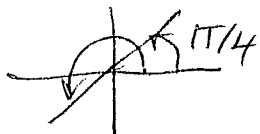
(d) $(0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, 2\pi)$

(e) $(0, \frac{\pi}{4}) \cup \frac{(5\pi}{4}, 2\pi)$

$f'(t) = \cos t - \sin t = 0$

$\Leftrightarrow \cos t = \sin t$

$\Leftrightarrow \tan t = 1$



$\Rightarrow t = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

$f'>0$, $f'<0$, $f'>0$ on 0, $\frac{\pi}{4}$, $\frac{5\pi}{4}$, 2π

f: $\uparrow$ $\downarrow$ $\uparrow$ on 0, $\frac{\pi}{4}$, $\frac{5\pi}{4}$, 2π

Decr on $(\frac{\pi}{4}, \frac{5\pi}{4})$

(8) If $f(x)$ is continuous on $[5, 7]$ and differentiable on $(5, 7)$ and its derivative satisfies $3 \geq f'(x) > 2$ for every x in the interval $(5, 7)$, we can conclude that $f(7) - f(5)$ is in the following interval:

(a) $(4, 6)$

(b) $(3, 7)$

(c) $(4, 6]$

(d) $[3, 7]$

(e) $(0, 1]$

$f(7) - f(5) = f'(c)(7-5)$

$= 2f'(c), \quad 5 < c < 7$

$2 < f'(c) \leq 3$

$\Rightarrow 4 < \underbrace{2f'(c)}_{= f(7) - f(5)} \leq 6$

$(4, 6]$

(9) The graph of the first derivative of a function f is sketched below. We can conclude that f is concave upward in the following intervals

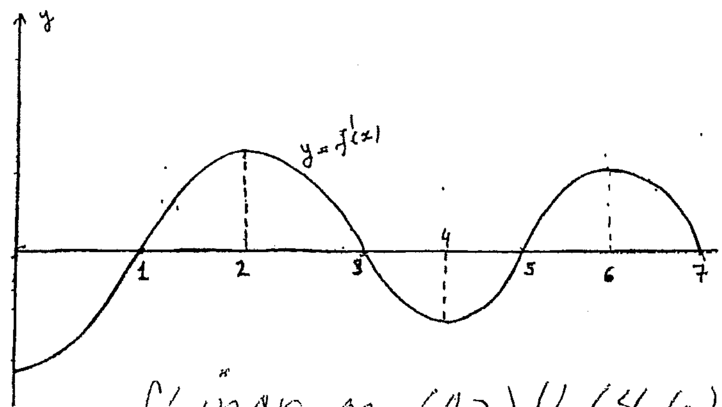
(a) $(0, 3)$ and $(5, 7)$

(b) $(2, 4)$ and $(6, 7)$

(c) $(1, 3)$ and $(5, 7)$

(d) $(2, 4)$ and $(6, 7)$

(e) $(0, 2)$ and $(4, 6)$



f' incr. on $(0,2) \cup (4,6)$
so $f'' > 0$ on $(0,2) \cup (4,6)$

(10) If f is a function such that the graph of $f'(x)$ is as sketched below, we can conclude that the following are local minimum values of f.

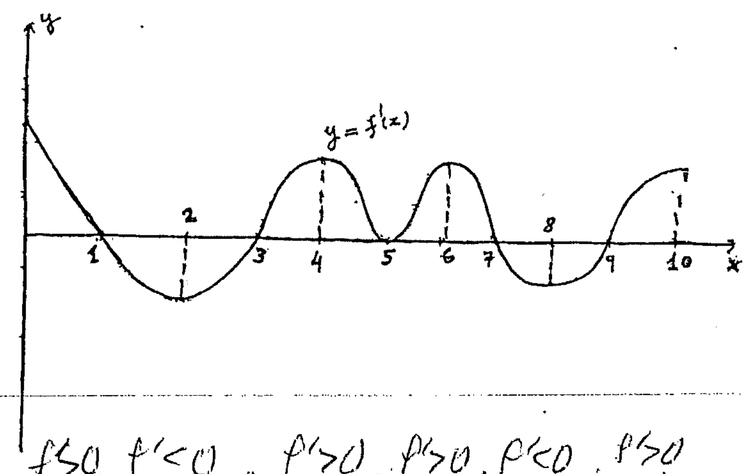
(a) $f(2)$ and $f(8)$

(b) $f(1)$ and $f(5)$

(c) $f(1)$, $f(3)$, $f(5)$ and $f(7)$

(d) $f(3)$ and $f(9)$

(e) $f(1)$ and f(3)



$f'>0$, $f'<0$, $f'>0$, $f'>0$, $f'<0$, $f'>0$

0 1 3 5 7 9 10

f: ↑ ↓ ↑ ↓ ↑

3 7 9

Local min at 3 & 9.

$f(3)$ & $f(9)$ are local min

(11) The limit $\lim_{x\to 0} \frac{\sin x - x}{x^3}$ is equal to

indet $\frac{0}{0}$

L-Hosp.

$= \lim_{x\to 0} \left(\frac{\cos x - 1}{3x^2}\right)$ indet, $\frac{0}{0}$

$= \lim_{x\to 0} -\frac{\sin x}{6x} = \frac{-1}{6}\cdot 1$

→ (a) $-1/6$

(b) $1/3$

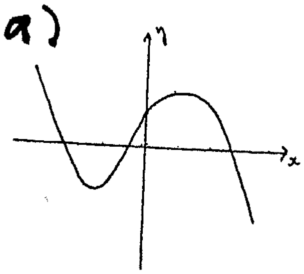
(c) 1

(d) $1/4$

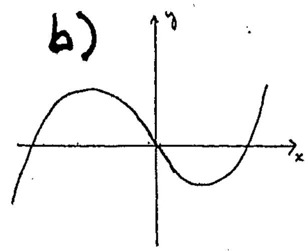
(e) $-1/3$

(12) The graph of $f(x) = 2x^3 + 3x^2 - 12x + 1$ looks most like which of the following?

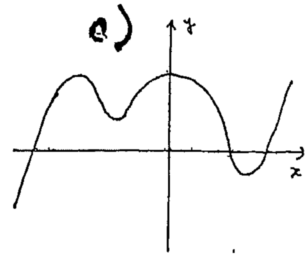
a)



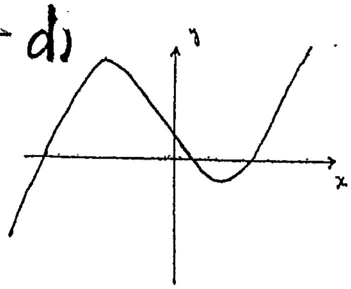
b)



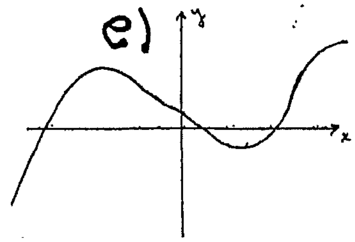
c)



→ d)



e)



$f(0) = 1$ $\boxed{\text{So b) is \underline{not} the answer.}}$

$f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x+2)(x-1)$

$$\underset{-2}{f' > 0 \,|\, f' < 0} \,\underset{1}{|}\, f' > 0$$

$$f: \uparrow \underset{-2}{|} \downarrow \underset{1}{|} \uparrow$$

f inc on $(-\infty, -2) \cup (1, \infty)$

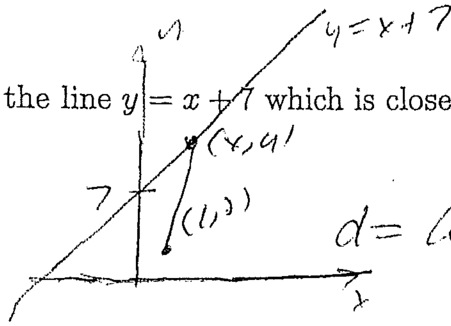
dec on $(-2, 1)$

so $\boxed{\text{a) \& c) are \underline{not} the answer}}$

$f''(x) = 12x + 6$ $\quad f'' < 0 \underset{-1/2}{|} f'' > 0 \Rightarrow f: \cap \underset{-1/2}{|} \cup$

so $\boxed{\text{e) is \underline{not} the answer.}}$ <u>most like d).</u>

(13) The point on the line $y = x + 7$ which is closest to $(1, 2)$ is:



(a) $(1, 8)$

(b) $(-2, 5)$

(c) $(-1, 6)$

(d) $(0, 7)$

(e) $(3, 10)$

$d = \text{dist}\,((x,y),(1,2))$

$= \sqrt{(x-1)^2 + (y-2)^2}$

minimize $\text{(dist)}^2 = (x-1)^2 + (y-2)^2$

" $D(x) = (x-1)^2 + (x+5)^2$

$D'(x) = 2(x-1) + 2(x+5)$

$= 2(2x+4) = 0$ @ $x = -2$

$\Rightarrow y = (-2) + 7 = 5$

$(-2, 5)$

(14) The maximum and minimum values of $f(x) = x^2 + 4x - 3$ on the interval $[-3, 3]$ are respectively

(a) 18 and -5

(b) 18 and -6

(c) 18 and -7

(d) 24 and -6

(e) 24 and -7

$f'(x) = 2x + 4 = 0$ @ $x = -2$

f ↓ ↑

-3 -2 3

$f' < 0$ on $(-3, -2)$

$f' > 0$ on $(-2, 3)$

local min @ -2:
$f(-2) = 4 - 8 - 3$
$= -7$

no local max

endpts: $f(-3) = 9 - 12 - 3 = 9 - 15 = -6$
$f(3) = 9 + 12 - 3 = 18$

Abs. Max $= 18$, min $= -7$