Name SOLUTIONS

10-digit PUID

RECITATION Section Number and time

Recitation Instructor

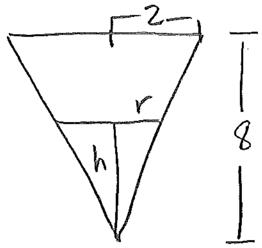
Lecturer

Instructions:

1. Fill in all the information requested above and on the scantron sheet. On the scantron sheet also fill in the little circles for your name, section number and PUID.
2. This booklet contains 12 problems, each worth 8 points (except problems 3,4,7 and 9 are worth 9 points each). The maximum score is 100 points. The test booklet has 7 pages, including this one.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators or any electronic devices are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

1) A tank has the shape of an inverted circular cone with radius 2 m and height 8 m. If water is poured into the tank at a rate of 4 m^3 per minute, find the rate at which the water level is rising (in m per minute) when the water is 4 m deep.

cross section of tank



know: $\frac{dV}{dt} = 4 \frac{m^3}{min}$

want: $\frac{dh}{dt}$ when $h=4$

A) $\frac{4}{\pi}$

B) $\frac{2}{\pi}$

C) $\frac{8}{3\pi}$

D) $\frac{3}{\pi}$

E) $\frac{4}{3\pi}$

$V = \frac{\pi}{3} r^2 h$. $\frac{h}{r} = \frac{8}{2} \rightarrow r = \frac{1}{4} h$

$V(h) = \frac{\pi}{3}\left(\frac{h}{4}\right)^2 h = \frac{\pi}{48} h^3$

$\frac{dV}{dt} = \frac{\pi}{16} h^2 \frac{dh}{dt} \longrightarrow 4 = \frac{\pi}{16} \cdot 16 \cdot \frac{dh}{dt} \longrightarrow \frac{dh}{dt} = \frac{4}{\pi}$

2) Use a linear approximation to compute the approximate value of $\sqrt[3]{8.06}$.

A) 2.04

B) 2.02

C) 2.005

D) 2.01

E) 2.0025

let $f(x) = x^{1/3}$. $a = 8$.

$f'(x) = \frac{1}{3} x^{-2/3}$.

$L(x) = f(8) + f'(8)(x-8)$

$= 2 + \frac{1}{12}(x-8)$

$L(8.06) = 2 + \frac{1}{12}(8.06-8)$

$= 2 + 0.005$

3) If $f(x) = x^3 + x - 1$ on the interval $[0, 2]$, find a number c that satisfies the Mean Value Theorem.

A) $\frac{2}{\sqrt{3}}$

B) $\sqrt{2}$

C) $\sqrt{\frac{5}{3}}$

D) $\frac{\sqrt{3}}{3}$

E) $\frac{4}{\sqrt{3}}$

f satisfies hypotheses of Mean Value Theorem on any interval. $f'(x) = 3x^2 + 1$

$$\frac{f(2) - f(0)}{2 - 0} = f'(c), \quad 0 < c < 2.$$

$$\frac{(2^3 + 2 - 1) - (0^3 + 0 - 1)}{2} = 3c^2 + 1$$

$$5 = 3c^2 + 1$$

$$\frac{4}{3} = c^2$$

$$\frac{2}{\sqrt{3}} = c$$

4) If m_1 is the minimum of $f(x) = x^3 + 3x^2 - 9x$ on $[0, 2]$ and m_2 is the maximum, find $m_1 + m_2$.

A) 7

B) -3

C) 5

D) 2

E) -52

f is continuous. $[0, 2]$ is a closed interval.

$$f'(x) = 3x^2 + 6x - 9$$
$$= 3(x^2 + 2x - 3)$$
$$= 3(x + 3)(x - 1) = 0 \rightarrow x = \cancel{-3}, 1$$

$f(0) = 0^3 + 3(0)^2 - 9(0) = 0$

$f(1) = 1^3 + 3(1)^2 - 9(1) = -5$ ← min (m_1)

$f(2) = 2^3 + 3(2)^2 - 9(2) = 2$ ← max (m_2)

$$m_1 + m_2 = -5 + 2 = -3$$

5) if $f(x) = \sinh(\ln x)$, calculate $f'(2)$.

$$f(x) = \frac{e^{\ln x} - e^{-\ln x}}{2}$$

$$= \frac{x - \frac{1}{x}}{2}$$

$$f'(x) = \frac{1 + \frac{1}{x^2}}{2}$$

$$f'(2) = \frac{1 + \frac{1}{4}}{2} = \frac{\frac{5}{4}}{2} = \frac{5}{8}$$

A) $\frac{3}{8}$

B) $\frac{5}{4}$

C) $\frac{3}{4}$

D) $\frac{5}{8}$

E) $\frac{5}{2}$

6) If $f(\cancel{x})$ t $= t^2 + 4\cos t$ on $(0, 2\pi)$ find the interval(s) where the graph of f is concave upward.

$$f'(t) = 2t - 4\sin t$$

$$f''(t) = 2 - 4\cos t$$

$$f''(t) = 0 \rightarrow \cos t = \frac{1}{2}$$

$$\rightarrow t = \frac{\pi}{3}, \frac{5\pi}{3}$$

	0		$\frac{\pi}{3}$		$\frac{5\pi}{3}$		2π t
$2-4\cos t$		$-$	0	$+$	0	$-$	

concave upward

A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$

B) $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

C) $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

D) $\left(\frac{\pi}{6}, \frac{11\pi}{6}\right)$

E) $\left(0, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)$

7) Let $f(x) = 2x^3 - 3x^2$. f has

A) 1 local max and 2 points of inflection

B) 1 local max and 1 point of inflection

C) 1 local min and 2 points of inflection

D) 1 local min and 1 point of inflection

E) 1 local min, 1 local max and 1 point of inflection

$f'(x) = 6x^2 - 6x = 6x(x-1)$

$f'(x) = 0 \rightarrow x = 0, 1$

$-\infty$		0		1		∞ x
$6x$	$-$	ϕ	$+$		$+$	
$x-1$	$-$		$-$	ϕ	$+$	
$f'(x)$	$+$		$-$		$+$	
f	inc		dec.		inc	

f has 1 local max

f has 1 local min

$f''(x) = 12x - 6$,

$f''(x) = 0 \rightarrow x = \frac{1}{2}$

$-\infty$		$1/2$		∞ x
$12x-6$	$-$	ϕ	$+$	
f	concave down		concave up	

inflection point at $(\frac{1}{2}, f(\frac{1}{2}))$

8) Find the least distance between the hyperbola $x^2 - y^2 = 1$ and the point $(4, 0)$.

$\rightarrow y^2 = 1 - x^2$

A) 2

B) 4

C) $\sqrt{7}$

D) $\sqrt{8}$

E) 4

$$D = \sqrt{(x-4)^2 + (y-0)^2}$$

$$= \sqrt{(x-4)^2 + (1-x^2)}$$

$$= \sqrt{2x^2 - 8x + 15}$$

$$\frac{dD}{dx} = \frac{4x - 8}{2\sqrt{2x^2 - 8x + 15}} = 0 \rightarrow x = 2$$

$$D(2) = \sqrt{2(2)^2 - 8(2) + 15} = \sqrt{7}$$

9) $\lim_{x\to 0} \dfrac{\sin x - x}{\tan x - x} =$

$$\overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\cos x - 1}{\sec^2 x - 1}$$

$$\overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{-\sin x}{\cancel{2\sec x \sec x \tan x}\ 2\sec^2 x \tan x}$$

$$\overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{-\cos x}{2(\sec^2 x \sec^2 x + 2\sec x \sec x \tan x \tan x)}$$

$$= \frac{-1}{2(1+0)} = -\frac{1}{2}$$

A) $-\frac{1}{2}$

B) -1

C) 0

D) $\frac{1}{2}$

E) 1

10) $\lim_{x\to 0^+} (1-3x)^{1/5x} =$ 1^0

$$= \lim_{x\to 0^+} e^{\ln(1-3x)^{1/5x}}$$

see below

$$= e^{\boxed{\lim_{x\to 0^+} \ln(1-3x)^{1/5x}}} = e^{-3/5}$$

A) 1

B) e^{-15}

C) $e^{-3/5}$

D) $e^{-5/3}$

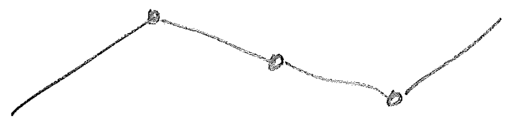
E) $e^{-1/15}$

$$\lim_{x\to 0^+} \ln(1-3x)^{\frac{1}{5x}} = \lim_{x\to 0^+} \frac{\ln(1-3x)}{5x} \overset{\frac{0}{0}}{=} \lim_{x\to 0^+} \frac{\frac{-3}{1-3x}}{5} = -\frac{3}{5}$$

11) Let $f'(x) = (x+1)(x-1)^2(x-2)$. f has

	$-\infty$ to -1	-1 to 1	1 to 2	2 to ∞
$x+1$	$-$	0 $+$	$+$	$+$
$(x-1)^2$	$+$	$+$ 0	$+$	$+$
$x-2$	$-$	$-$	$-$ 0	$+$
$f'(x)$	$+$	$-$	$-$	$+$
	inc	dec	dec	inc

f



1 local max, 1 local min

A) no local maxima and 2 local minima

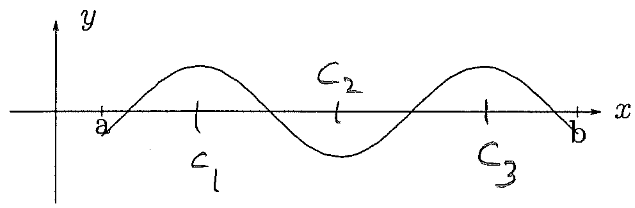
B) 2 local maxima and no local minima

C) 1 local maximum and 2 local minima

D) 2 local maxima and 1 local minimum

E) 1 local maximum and 1 local minimum

12) The graph of f' is given below, $a \le x \le b$.



A) f has exactly 2 points of inflection and exactly 4 local extrema.

B) f has exactly 2 points of inflection and exactly 3 local extrema.

C) f has exactly 4 points of inflection and exactly 3 local extrema.

D) f has exactly 3 points of inflection and exactly 4 local extrema.

E) f has exactly 3 points of inflection and exactly 5 local extrema.

4 local extrema (f' changes sign at 4 x-intercepts)

3 pts. of inflection (at c_1, c_2 and c_3 f' changes between increasing and decreasing)