SOLUTION

1. If $f(x) = (x^3 + 2x - 1)^2$, then $f''(1) =$

$f'(x) = 2(x^3+2x-1)(3x^2+2)$

$f''(x) = 2\left[(x^3+2x-1)6x + (3x^2+2)^2\right]$

$f''(1) = 2[2 \cdot 6 + 5^2] = 74$

A. 4

B. 20

C. 50

D. 74

E. 100

2. If $F(x) = \sin(g(x))$, then $F''(x) =$

$F'(x) = \cos(g(x)) \cdot g'(x)$

$F''(x) = \cos(g(x)) \cdot g''(x) + (-\sin(g(x)) \cdot (g'(x))^2$

A. $\cos(g(x))g'(x) + \sin(g(x))$

B. $-\sin(g(x))g'(x) + \cos(g(x))$

C. $-\sin(g(x))g'(x) + (g'(x))^2$

D. $-\sin(g(x))g''(x) + \cos(g(x))(g'(x))^2$

E. $-\sin(g(x))(g'(x))^2 + \cos(g(x))g''(x)$

3. If $f(x) = \ln\left(\dfrac{x}{1+x}\right)$, then $f'(2) =$

$f(x) = \ln x - \ln(1+x)$

$f'(x) = \dfrac{1}{x} - \dfrac{1}{1+x}$

$f'(2) = \dfrac{1}{2} - \dfrac{1}{3} = \dfrac{1}{6}$

A. $\dfrac{3}{2}$

B. $-\dfrac{3}{8}$

C. $\dfrac{1}{6}$

D. $-\dfrac{1}{6}$

E. $\dfrac{5}{6}$

4. Use logarithmic differentiation to find $\frac{d}{dx}\left(x^{\sin x}\right)$

$y = x^{\sin x}$

$\ln y = \sin x \ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + \cos x \ln x$

$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + \cos x \ln x\right]$

A. $x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$

B. $x^{\sin x}\left(\ln(\sin x) + \frac{\cos x}{\sin x}\right)$

C. $x^{\sin x} \cos x$

D. $\cos x \ln x + \frac{\sin x}{x}$

E. $\ln(\sin x) + \frac{\cos x}{\sin x}$

5. If 40% of a certain radioactive substance decays in 50 days, what is the half-life of the substance?

$y = Ce^{kt}$

0.6 of the substance remains after 50 days

$0.6 = e^{50k} \longrightarrow k = \frac{1}{50}\ln 0.6$

$\therefore \frac{1}{2} = e^{(\ln 0.6)\frac{1}{50}t} = (0.6)^{\frac{1}{50}t}$

$\ln\frac{1}{2} = \frac{1}{50}t \ln 0.6$

$\therefore t = \frac{50 \ln\frac{1}{2}}{\ln(0.6)}$

A. $50\,\frac{\ln 0.5}{\ln 0.4}$

B. $50\,\frac{\ln 0.5}{\ln 0.6}$

C. $50\,\frac{\ln 0.6}{\ln 0.5}$

D. $50\,\frac{\ln 0.4}{\ln 0.5}$

E. $50\,\frac{\ln 0.4}{\ln 0.6}$

6. Use differentials (or, equivalently, a linear approximation) to estimate $\sqrt[3]{8.1}$.

$f(x) = \sqrt[3]{x}$ $\quad f'(x) = \frac{1}{3x^{2/3}}$

$\sqrt[3]{8.1} = \sqrt[3]{8} + \frac{1}{3\cdot 8^{2/3}}\cdot 0.1$

$= 2 + \frac{1}{12}\cdot\frac{1}{10}$

$= 2 + \frac{1}{120}$

A. $2\frac{1}{120}$

B. $2\frac{1}{12}$

C. $2\frac{1}{10}$

D. $2\frac{1}{100}$

E. $2\frac{1}{1000}$

7. The difference between the absolute maximum and the absolute minimum of $f(x) = \dfrac{x}{x^2+1}$ on $[0,2]$ is

$$f'(x) = \frac{(x^2+1)\cdot 1 - x\cdot 2x}{(x+1)^2} = \frac{1-x^2}{(x+1)^2}.$$

$f'(x) = 0 \to x = \pm 1 \quad (-1 \notin [0,2])$.

$x=0$, $f(0)=0$, MIN $\quad x=1$, $f(1)=\frac{1}{2}$, MAX $\quad x=2$, $f(2)=\frac{2}{5}$

$f(1) - f(0) = \frac{1}{2}$

A. $\frac{1}{10}$

B. 1

C. $\frac{1}{2}$

D. $\frac{9}{10}$

E. $\frac{2}{5}$

8. How many critical numbers does the function $G(x) = \sqrt[3]{x^2 - x}$ have?

$G(x) = (x^2-x)^{1/3}, \quad G'(x) = \dfrac{2x-1}{3(x^2-x)^{2/3}}.$

Critical Nos.

$G'(x) = 0 \to 2x-1=0 \to x = 1/2$

$G'(x)$ DNE $\to x^2 - x = 0 \to x = 0, 1.$

$\therefore$ Three Critical Nos

A. 0

B. 1

C. 2

D. 3

E. 4

9. Classify the local extrema of the function $f(x) = x^4(x-2)^3$.

$$f'(x) = x^4\cdot 3(x-2)^2 + 4x^3(x-2)^3$$
$$= x^3(x-2)^2[3x + 4(x-2)]$$
$$= x^3(x-2)^2(7x-8).$$

Critical Nos are $0, \frac{7}{8}, 2$.

$f'(-1)>0 \quad f'(1)<0 \quad f'(\frac{3}{2})>0 \quad f'(3)>0.$

$\therefore$ $x=0$ is a local Max.

$x = \frac{7}{8}$ is a local Min.

$x=2$ is neither.

A. Exactly two local maximum and one local minimum

B. Exactly one local maximum and one local minimum

C. Exactly one local maximum and two local minimum

D. Exactly two local maximum and two local minimum

E. None

10. Over what intervals is the function $f(x) = x^4 - 8x^3 + 200$ concave up?

$f'(x) = 4x^3 - 24x^2$

$f''(x) = 12x^2 - 48x$

$= 12x(x-4)$.

$x < 0$ $x = 0$ $0 < x < 4$ $x = 4$ $x > 4$

$f''(x) > 0$ concave up $f''(x) < 0$ con cave down $f''(x) > 0$ concave up.

A. $(0, 4)$

B. $(-\infty, 0)$

C. $(4, \infty)$

D. $(-\infty, 0)$ and $(4, \infty)$

E. $(0, 4)$ and $(4, \infty)$

11. $\lim_{x \to 0} \dfrac{\sin 2x}{e^x - 1} =$

$$\lim_{x \to 0} \frac{\sin 2x}{e^x - 1} \overset{H}{=} \lim_{x \to 0} \frac{2\cos 2x}{e^x} = \frac{2}{1}$$

A. 1

B. ∞

C. 0

D. $-\infty$

E. 2

12. $\lim_{x \to 0^-} (1+x)^{\frac{1}{x}} =$

Let $y = (1+x)^{\frac{1}{x}}$

$\ln y = \dfrac{\ln(1+x)}{x}$

$$\lim_{x \to 0^-} \ln y = \lim_{x \to 0^-} \frac{\ln(1+x)}{x} \overset{H}{=} \lim_{x \to 0^-} \frac{\frac{1}{1+x}}{1}$$

$= 1$

$\therefore \lim_{x \to 0^-} y = e^1 = e$

A. e

B. $\frac{1}{e}$

C. 1

D. 0

E. ∞

13. Suppose f is a function which is continuous on the interval $[-1, 2]$ and differentiable on the interval $(-1, 2)$. Suppose also that $f(-1) = 3$ and $f(2) = 1$. Then there is a c in the interval $(-1, 2)$ such that

By the Mean Value Theorem.
There is a c in $(-1, 2)$ such that

$$f'(c) = \frac{f(2) - f(-1)}{2 - (-1)} = \frac{1-3}{3} = -\frac{2}{3}$$

A. $f(c) = -\frac{2}{3}$

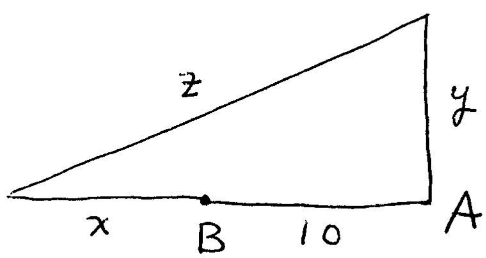
B. $f(c) = \frac{2}{3}$

C. $f'(c) = 0$

D. $f'(c) = \frac{2}{3}$

E. $f'(c) = -\frac{2}{3}$

14. At noon, ship A is 10 miles east of ship B. Ship A is traveling north at 20 mph and ship B is traveling west at 10 mph. How fast is the distance between them changing at 3 pm?



$\frac{dx}{dt} = 10 \quad \frac{dy}{dt} = 20.$

at 3 pm $x = 30, y = 60$

$$z = \sqrt{(x+10)^2 + y^2}$$

$$= \sqrt{(40)^2 + (60)^2} = \sqrt{5200}$$

A. $\frac{1400}{\sqrt{5200}}$ mph

B. $\frac{100}{\sqrt{5200}}$ mph

C. $\frac{90}{\sqrt{4500}}$ mph

D. $\frac{1600}{\sqrt{5200}}$ mph

E. $\frac{1500}{\sqrt{4500}}$ mph

$$z^2 = (x+10)^2 + y^2.$$

$$\cancel{2} z \frac{dz}{dt} = \cancel{2}(x+10)\frac{dx}{dt} + \cancel{2} y \frac{dy}{dt}.$$

$$\sqrt{5200}\,\frac{dz}{dt} = 40 \cdot 10 + 60 \cdot 20$$

$$= 400 + 1200$$

$$= 1600.$$

$$\therefore \frac{dz}{dt} = \frac{1600}{\sqrt{5200}}$$