Solutions

MA 161 EXAM 3 SPRING 2010

1. A bacteria culture initially contains 200 cells and grows at a rate proportional to its size. After 2 hours, the culture contains 600 cells. How many bacteria are in the culture after 3 hours?

   A. $200\, e^{2\ln 3}$

   B. $200\, e^{3\ln 2}$

   C. $600\, e^{\frac{3}{2}\ln 2}$

   D. $200\, e^{\frac{3}{2}\ln 3}$

   E. $600\, e^{2\ln 3}$

$y = Ce^{kt}$, $C = 200 \Rightarrow y = 200e^{kt}$

$600 = 200e^{k\cdot 2}$

$3 = e^{2k}$

$\ln 3 = 2k$

$k = \frac{\ln 3}{2}$

$\Rightarrow y = 200\, e^{t(\ln 3)/2}$

$y(3) = 200e^{(3\ln 3)/2}$ D

2. A particle is traveling on the ellipse $x^2 + 4y^2 = 8$ (in the first quadrant). When $y = 1$, $\dfrac{dy}{dt} = 1$. Find $\dfrac{dx}{dt}$.

   A. -1

   B. 1

   C. -4

   D. 2

   E. -2

$x^2 + 4y^2 = 8$

$y = 1$: $x^2 + 4 = 8 \Rightarrow x^2 = 4$

$\Rightarrow$ (since $x \geq 0, y \geq 0$) $\underline{x = 2}$

$2x\cdot x' + 4\cdot 2y\cdot y' = 0$.

@ $(2,1)$: $2\cdot 2x' + 8\cdot 1\cdot 1 = 0$

$4x' = -8$

$x' = -2$ E

3. The volume of a sphere ($V = \frac{4}{3}\pi r^3$) is increasing at a rate of 4 $cm^3/\min$. How fast is the radius increasing when the radius is 4 cm?

A. $\frac{1}{16\pi}$ $cm/\min$

B. $\frac{1}{4\pi}$ $cm/\min$

C. $\frac{1}{12\pi}$ $cm/\min$

D. $\frac{1}{24\pi}$ $cm/\min$

E. $\frac{1}{32\pi}$ $cm/\min$

$V' = \frac{4}{3}\pi \cdot 3r^2 \cdot r' = 4\pi r^2 \cdot r'$

$4 = 4\pi r^2 r'$

$r' = \frac{1}{\pi r^2}\Big/_4 = \frac{1}{16\pi}$

A

4. Use linear approximation to compute the approximate value of $\sqrt{24.5}$.

A. 4.90

B. 4.95

C. 4.99

D. 4.80

E. 4.995

$f(x) = x^{1/2}$

$f(25) = 5$

$f(x) \simeq f(a) + f'(a)(x-a)$

Take $x = 24.5$, $a = 5$

$x - a = -.5$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f'(25) = \frac{1}{2 \cdot 5} = \frac{1}{10}$

$\sqrt{24.5} \simeq 5 + \frac{1}{10}(-.5)$

$= 5 - \frac{1}{10} \cdot \frac{1}{2} = 5 - .05$

$= 4.95$ B

5. Compute $\frac{d}{dx}(\cosh(\ln x))$ when $x = 2$.

A. $\frac{5}{8}$

B. $\frac{3}{4}$

C. $\frac{3}{8}$

D. $\frac{7}{8}$

E. $\frac{1}{2}$

$$L = \sinh(\ln x)\cdot\frac{1}{x}\Big|_{x=2}$$
$$= \sinh(\ln 2)\cdot\frac{1}{2}$$
$$= \frac{[e^{\ln 2} - e^{-\ln 2}]}{2}\cdot\frac{1}{2}$$
$$= \frac{(2-\frac{1}{2})}{4} = \frac{(\frac{3}{2})}{4} = \frac{3}{8} \quad C$$

6. Find the absolute minimum of $f(x) = \frac{x}{x^2+2}$ on the interval $[-4, 4]$.

A. $\frac{-1}{3}$

B. $\frac{\sqrt{2}}{4}$

C. $-\frac{1}{4}$

D. $-\frac{2}{9}$

E. $-\frac{\sqrt{2}}{4}$

$$f'(x) = \frac{(x^2+2)\cdot 1 - x(2x)}{(2+x^2)^2}$$
$$= \frac{2-x^2}{(2+x^2)^2}$$

$f' = 0$ at $\pm\sqrt{2}$. endpts: ± 4

f': $-$ $+$ $-$; -4, $-\sqrt{2}$, $\sqrt{2}$, 4

f: $\downarrow$ $\uparrow$ $\downarrow$; -4, $-\sqrt{2}$, $\sqrt{2}$, 4

local min: at $x = -\sqrt{2}$. $f(-\sqrt{2}) = -\frac{\sqrt{2}}{4}$

$f(-4) = \frac{-4}{18} = \frac{-2}{9}$, $f(4) = \frac{4}{18} = \frac{2}{9}$

$-\frac{\sqrt{2}}{4} < \frac{-2}{9} \Rightarrow$ abs. min $= -\sqrt{2}/4$ E

$= x^2(3x^2 - 4x - 12)$

7. Find the absolute minimum of $f(x) = 3x^4 - 4x^3 - 12x^2$ on the interval $[-2, 2]$.

   A. 16

   B. 0

   C. -32

   D. -16

   E. -24

$$f'(x) = 12x^3 - 12x^2 - 24x$$
$$= 12x(x^2 - x - 2)$$
$$= 12x(x-2)(x+1)$$

Crit pts: $x = 0, 2, -1$

f': $-$ $+$ $-$ (-2, -1, 0, 2)

f: $\downarrow$ $\uparrow$ $\downarrow$ (-2, -1, 0, 2)

local min @ -1 + test endpts.

$f(-2) = 4(12 + 8 - 12) = 32$

$f(-1) = 3 + 4 - 12 = -5$

abs min $\rightarrow$ $f(2) = 4(12 - 8 - 12) = -32$

C

8. Assume f is continuous in $[1, 4]$ and differentiable in $(1, 4)$. If $f(1) = -2$ and $3 \le f'(x) \le 5$, how small can $f(4)$ be?

   A. $f(4) \ge 5$

   B. $f(4) \ge 9$

   C. $f(4) \ge 6$

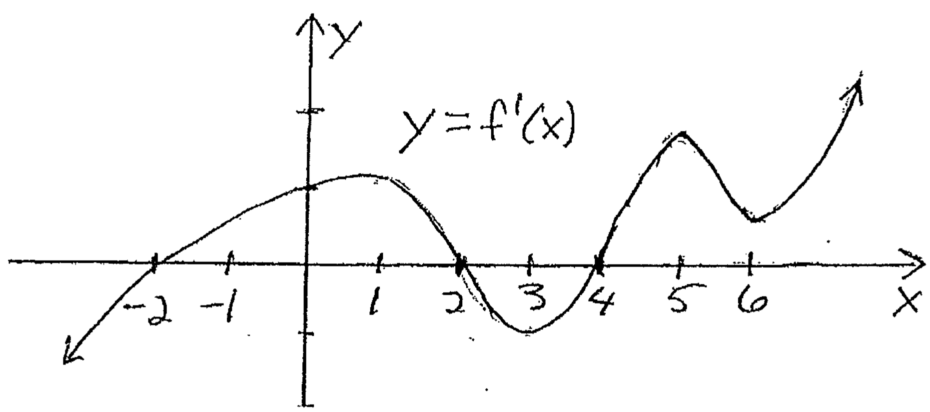
   D. $f(4) \ge 7$

   E. $f(4) \ge 11$

$$f(4) - f(1) = f'(c)(4-1)$$
$$= 3f'(c)$$
$$\ge 3 \cdot 3 = 9$$
$$f(4) \ge f(1) + 9 = -2 + 9 = 7$$

D

9. Assume f is a differentiable function whose derivative, $f'(x)$, has the graph given by:



Which of the following describes all intervals on which f is increasing?

A. $(-2, 2) \cup (4, \infty)$.

B. $(-2, 2) \cup (4, 6)$.

C. $(-2, 1) \cup (3, 5)$.

D. $(-\infty, 1) \cup (6, \infty)$.

E. $(-\infty, 1) \cup (3, 5) \cup (6, \infty)$.

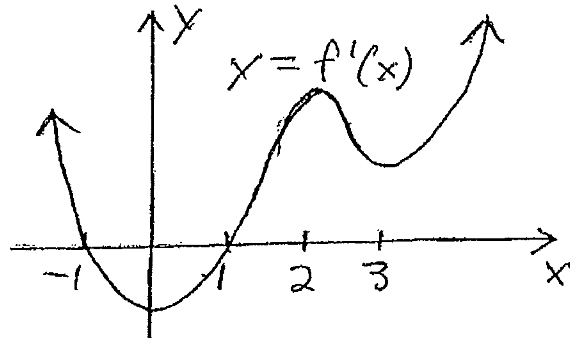
f': $-$ | $+$ | $-$ | $+$ (at -2, 2, 4)

f: $\downarrow$ | $\uparrow$ | $\downarrow$ | $\uparrow$ (at -2, 2, 4)

f incr on $(-2, 2) \cup (4, \infty)$

A

10. For the function f whose derivative, $f'(x)$, has the graph given by:



find all values of x at which the graph of f has an inflection point.

A. $x = -1, 2$, and 3

B. $x = -1$ and 1

C. $x = 0, 2$, and 3

D. $x = 1.5$ and 2.5

E. $x = -1, 0, 2$, and 3

f': ↓ 0 ↑ 2 ↓ 3 ↑

So f'': − 0 + 2 − 3 +

f: ∩ 0 ∪ 2 ∩ 3 ∪

infl. pt at $x = 0, 2, 3$.

C

11. If $f(x) = 2x^3 - 15x^2 - 36x + 1$, find all values of x at which f has a local maximum.

A. $x = -6$

B. $x = -1$

C. $x = 1$

D. $x = 6$

E. $x = 7$

$f'(x) = 6x^2 - 30x - 36 = 6(x^2 - 5x - 6)$
$= 6(x-6)(x+1)$

$f' = 0$ @ $\boxed{x = 6, -1}$. crit. pts

$f''(x) = 12x - 30 = 6(2x - 5)$.

$f''(6) = 6(12 - 5) > 0$: ∪ local min

$f''(-1) = 6(-2-5) < 0$: ∩ local max

local max at $x = -1$ B

12. Assume $f(t) = 4\sin t + t^2$ for $-\frac{\pi}{2} < t < \frac{3\pi}{2}$. Find all intervals on which f is concave down.

A. $(-\frac{\pi}{2}, \frac{\pi}{3}) \cup (\frac{4\pi}{3}, \frac{3\pi}{2})$

B. $(-\frac{\pi}{2}, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, \frac{3\pi}{2})$

C. $(\frac{\pi}{6}, \frac{5\pi}{6}) \cup (\frac{7\pi}{6}, \frac{3\pi}{2})$

D. $(\frac{\pi}{3}, \frac{4\pi}{3})$

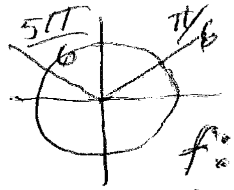
E. $(\frac{\pi}{6}, \frac{5\pi}{6})$

$f'(t) = 4\cos t + 2t$

$f''(t) = -4\sin t + 2$

$f'' = 0$ @ $4\sin t = 2$

$\sin t = \frac{1}{2}$



in $(-\frac{\pi}{2}, \frac{3\pi}{2})$ get $t = \frac{\pi}{6}, \frac{5\pi}{6}$

f'': ∪ on $(-\frac{\pi}{2}, \frac{\pi}{6})$, ∩ on $(\frac{\pi}{6}, \frac{5\pi}{6})$, ∪ on $(\frac{5\pi}{6}, \frac{3\pi}{2})$

$f''(0) = 2 > 0$: cc up in $(-\frac{\pi}{2}, \frac{\pi}{6})$

$\boxed{f''(\frac{\pi}{2}) = -4 + 2 < 0 \text{: cc down in } (\frac{\pi}{6}, \frac{5\pi}{6})}$

$f''(\pi) = 4 + 2 > 0$: cc up in $(\frac{5\pi}{6}, \frac{3\pi}{2})$

Ans. E

13. Evaluate $\lim_{x\to\infty} \frac{\ln(1+x^2)}{\ln x}$.

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. 4

type $\frac{\infty}{\infty}$ indeterminate

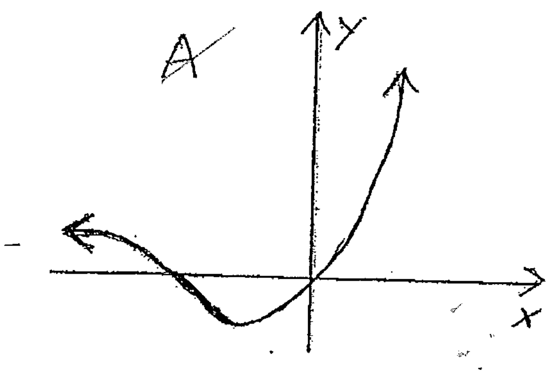
LH $= \lim_{x\to\infty} \frac{\frac{1}{1+x^2}\cdot 2x}{\frac{1}{x}}$

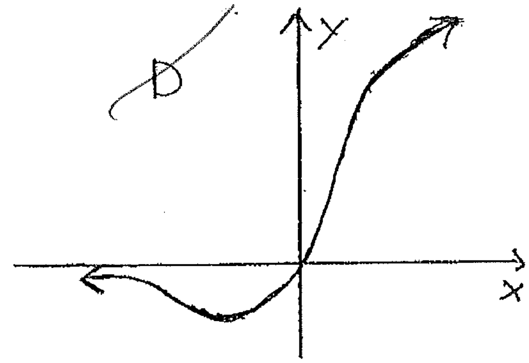
simplify $= \lim_{x\to\infty} \frac{2x}{1+x^2}\cdot\frac{x}{1} = \lim_{x\to\infty} \frac{2x^2}{x^2+1}$

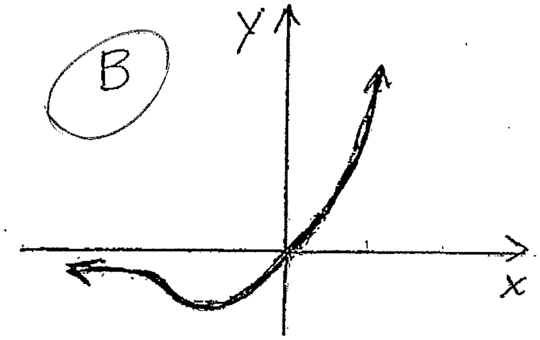
LH $= \lim_{x\to\infty} \frac{4x}{2x} = 2$

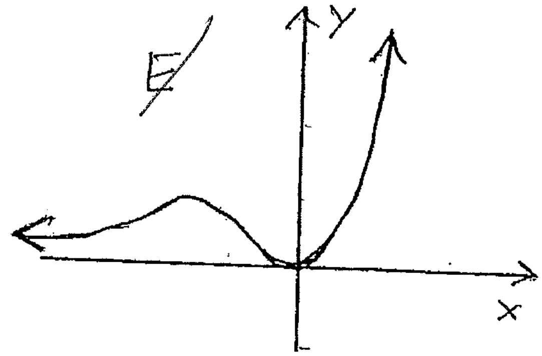
D

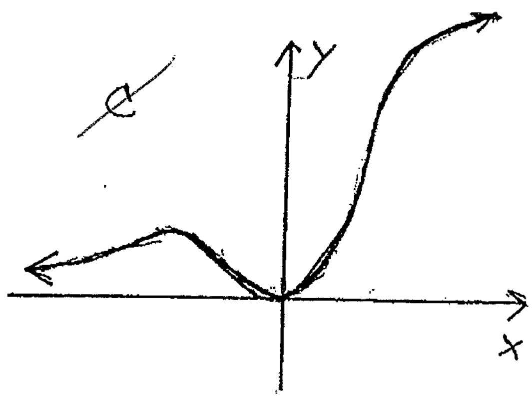
14. The graph of $y = xe^x$ looks most like:











intercepts: $f(0) = \boxed{0 = y}$ intercept

$0 = xe^x \Rightarrow \boxed{x=0}$: x intercept

$\boxed{f > 0 \text{ on } (0,\infty)}, f < 0 \text{ on } (-\infty, 0)$

$f'(x) = x \cdot e^x + e^x = (x+1)e^x$

$= 0 \text{ @ } x = -1$

f': $-$ | $+$ at -1

f: $\downarrow$ | $\uparrow$ at -1 So ans. is B or D

$f''(x) = x \cdot e^x + e^x + e^x = (x+2)e^x = 0 \text{ @ } x = -2$

f'': $-$ | $+$ at -2 f: $\cap$ | $\cup$ at -2 so ans is B