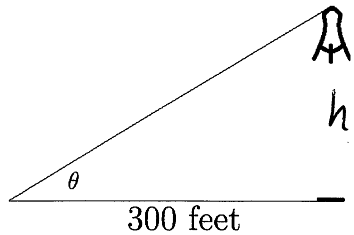
EXAM 3, SPRING 2016

1. An observer is stationed 300 feet from a rocket launch pad. The rocket rises vertically off the launch pad. A few seconds after takeoff, the rocket is 300 feet in the air and rising at 100 feet/sec. How fast is the angle of elevation, θ, changing at that instant?



$$\theta = \tan^{-1}\left(\frac{h}{300}\right)$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{h}{300}\right)^2}\left(\frac{1}{300}\right)\frac{dh}{dt}$$

AT INSTANT:

$$\frac{1}{1+\left(\frac{300}{300}\right)^2}\left(\frac{1}{300}\right)(100)$$

A. $\boxed{\frac{1}{6}\text{ radian/sec}}$

B. 2 radian/sec

C. $\frac{1}{2}$ radian/sec

D. $\frac{1}{4}$ radian/sec

E. $\frac{2}{3}$ radian/sec

2. Use a linear approximation (or differentials) to estimate the value of $\sqrt{24.8}$

A. 5.20

B. $\boxed{4.98}$

C. 4.95

D. 4.92

E. 4.90

$$\sqrt{25} + \frac{1}{2\sqrt{25}}(24.8-25)$$

3. Find the minimum value of $f(x) = x^3 - x$ on the closed interval $[-1, 1]$.

Hint: Find the actual value of f and NOT the x-value at which that minimum occurs.

A. 0

B. $-\dfrac{1}{\sqrt{3}}$

C. $-\dfrac{1}{3}$

D. $\boxed{-\dfrac{2}{3\sqrt{3}}}$

E. There is no absolute minimum value.

$f'(x) = 3x^2 - 1$

CRIT NUMBERS: $X = \pm\sqrt{\frac{1}{3}}$

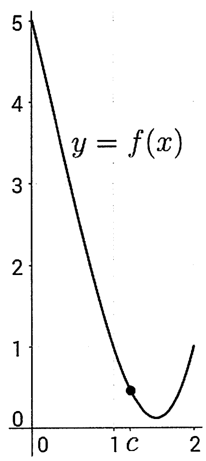
$f(-1) = 0$

$f\left(-\sqrt{\frac{1}{3}}\right) = \frac{2}{3\sqrt{3}}$

$f\left(\sqrt{\frac{1}{3}}\right) = -\frac{2}{3\sqrt{3}} \leftarrow$

$f(1) = 0$

4. The function f is continuous on $[0, 2]$ and differentiable on $(0, 2)$, and consequently the direct application of the mean value theorem guarantees the existence of c, where c is between 0 and 2 and pictured below. Find $f'(c)$.



$$\frac{f(2) - f(0)}{2 - 0} = \frac{1 - 5}{2} = -2$$

A. $\boxed{-2}$

B. 1.2

C. 0.5

D. -4

E. -1

5. Which statement accurately describes the function

$$f(x) = x^4 - 6x^3$$

on the interval $(0, 3)$?

A. f is increasing and its graph is concave up.

B. f is decreasing and its graph is concave up.

C. f is increasing and its graph is concave down.

D. $\boxed{f \text{ is decreasing and its graph is concave down.}}$

E. None of the above.

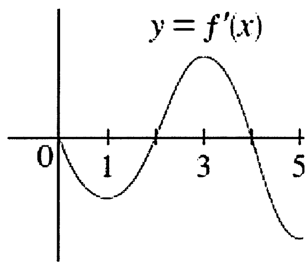
$f'(x) = 4x^3 - 18x^2$
$= 2x^2(2x-9)$

f': $\ominus$ | $\ominus$ | $\oplus$
0, 4.5

$f''(x) = 12x^2 - 36x$
$= 12x(x-3)$

f'': $\oplus$ | $\ominus$ | $\oplus$
0, 3

6. The graph of $y = f'(x)$, the **derivative** of f, is shown below.



Which of the following statements about f are true?

I. The graph of f is concave up on the interval $(2, 4)$.
II. $f(x)$ has a local minimum at $x = 2$.
III. $(1, f(1))$ is an inflection point for f.

A. None of these statements are true.

B. I and III

C. $\boxed{\text{II and III}}$

D. I and II

E. I, II, and III

7. Find the limit.

$$\lim_{x\to 0} \frac{\tan x - x}{x^3} \qquad \frac{0}{0}$$

A. $-\infty$

B. $\boxed{\frac{1}{3}}$

C. 1

D. $-\frac{1}{6}$

E. 0

$$= \lim_{x\to 0} \frac{\sec^2 x - 1}{3x^2} \qquad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{2\sec x(\sec x \tan x)}{6x} = \lim_{x\to 0} \frac{\tan x}{3x\cos^2 x} \qquad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\sec^2 x}{3\cos^2 x + (-6x\cos x \sin x)} = \frac{1}{3+0} = \frac{1}{3}$$

8. Find the x-coordinate of the inflection point of the function $f(x) = \dfrac{1}{\ln x}$ on the interval $0 < x < 1$.

A. $x = \frac{1}{2}$

B. $\boxed{x = \frac{1}{e^2}}$

C. $x = \frac{1}{\sqrt{e}}$

D. $x = \frac{1}{e}$

E. $x = \ln 2$

$$f'(x) = -(\ln x)^{-2}\left(\frac{1}{x}\right) = \frac{-1}{(\ln x)^2 x}$$

$$f''(x) = \frac{2(\ln x)\left(\frac{1}{x}\right)x + (\ln x)^2}{(\ln x)^4 x^2}$$

$$2\ln x + (\ln x)^2 = 0$$

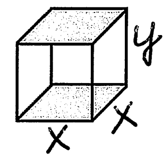
$$\ln x(2 + \ln x) = 0$$

$$\ln x = -2$$

$$x = e^{-2}$$

9. A six-sided box is to have four clear plastic sides, a wooden square top, and a wooden square bottom. The volume of the box must be 24 ft^3. Plastic costs \$1 per ft^2 and wood costs \$3 per ft^2. Find the dimensions of the box which minimize cost.

VOLUME $= X^2 y = 24 \rightarrow y = \frac{24}{X^2}$



MIN COST $= 1(4xy) + 3(2x^2) = 4x\left(\frac{24}{x^2}\right) + 6x^2$

$C(x) = \frac{96}{x} + 6x^2$

$C'(x) = -\frac{96}{x^2} + 12x = \frac{-96 + 12x^3}{x^2}$

$-96 + 12x^3 = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2$

A. $\boxed{2 \text{ ft} \times 2 \text{ ft} \times 6 \text{ ft}}$

B. $\sqrt{6}$ ft $\times$ $\sqrt{6}$ ft $\times$ 4 ft

C. $\sqrt[3]{4}$ ft $\times$ $\sqrt[3]{4}$ ft $\times$ $6\sqrt[3]{4}$ ft

D. $\sqrt[3]{3}$ ft $\times$ $\sqrt[3]{3}$ ft $\times$ $8\sqrt[3]{3}$ ft

E. $2\sqrt[3]{2}$ ft $\times$ $2\sqrt[3]{2}$ ft $\times$ $3\sqrt[3]{2}$ ft

10. A rectangle is formed with one corner at $(0, 0)$ and the opposite corner on the graph of $y = -\ln x$, where $0 < x < 1$. What is the largest possible area of such a rectangle?

MAX AREA $= xy$

$A(x) = x(-\ln x)$

$A'(x) = -\ln x - x\left(\frac{1}{x}\right)$

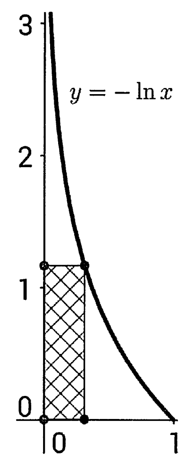
$-\ln x - 1 = 0$

$\ln x = -1$

$x = e^{-1}$

$y = -\ln(e^{-1}) = 1$

$xy = (e^{-1})(1)$



A. $\frac{\sqrt{e}}{2}$

B. e

C. $\boxed{\frac{1}{e}}$

D. $\frac{\ln 2}{2}$

E. There is no maximum.

11. Suppose f is a differentiable function with $f''(x) > 0$ for all real numbers x. Assume that $f(1) = 3$ and $f(5) = 3$. Which one of these statements must be true?

A. $f'(x)$ is decreasing at $x = 3$.

B. $f(x) \geq 0$ for all real numbers x.

C. f has an inflection point.

D. $f'(3) > 0$.

E. $\boxed{f \text{ has a local minimum.}}$

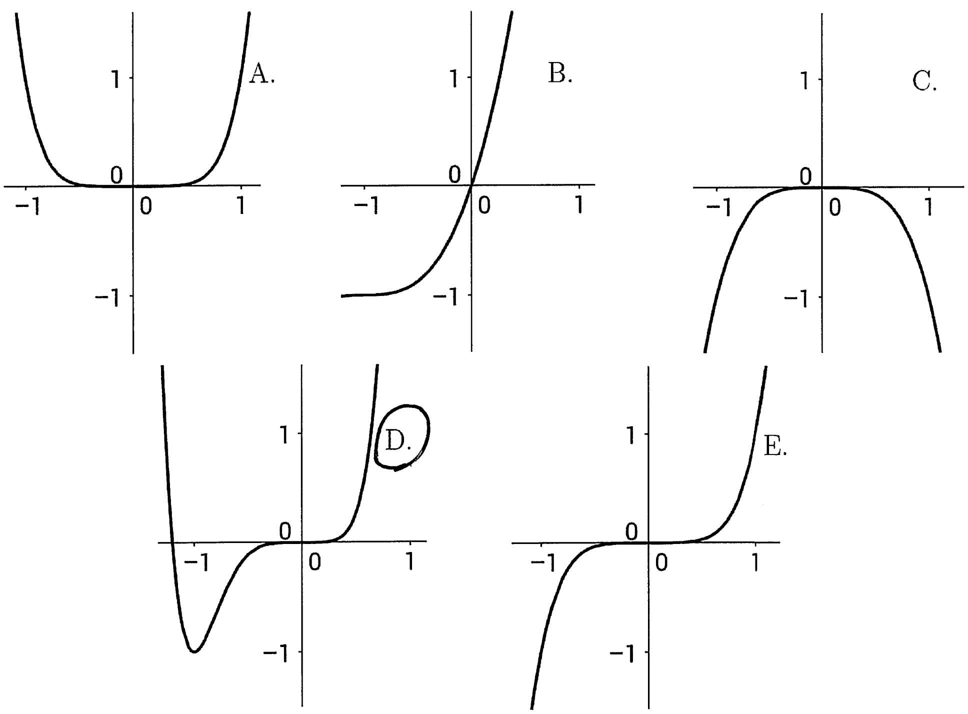
ROLLES:

$f'(c) = 0$ for some $1 < c < 5$

SECOND DERIVATIVE TEST:

$f''(c) > 0 \Rightarrow$ LOCAL MIN at c

12. Which of these curves is the graph of $y = 5x^6 + 6x^5$?



$y' = 30x^4(x+1)$

$y'' = 30x^3(5x+4)$

y is DECR: $(-\infty, -1)$ INCR: $(-1, 0) \cup (0, \infty)$

y is CONCAVE DOWN: $(-\frac{4}{5}, 0)$ UP: $(-\infty, -\frac{4}{5}) \cup (0, \infty)$