1. Which of the following statements are true?

(1) $(\mathbf{a}+\mathbf{b})\times\mathbf{a} = -\mathbf{a}\times\mathbf{b}$

(2) $\mathbf{a}\times(\mathbf{a}\times\mathbf{b}) = \mathbf{0}$

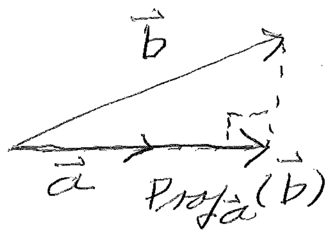
(3) $|\mathrm{proj}_{\mathbf{a}}(\mathbf{b})| \leq |\mathbf{b}|$

(4) $|\mathbf{a}\cdot\mathbf{b}| \leq |\mathbf{a}||\mathbf{b}|$

(1) $(\vec{a}+\vec{b})\times\vec{a} = \vec{a}\times\vec{a} + \vec{b}\times\vec{a}$
$= \vec{0} + \vec{b}\times\vec{a}$
$= -\vec{a}\times\vec{b}$

(2) $\vec{i}\times(\vec{i}\times\vec{j}) = \vec{i}\times\vec{k} = -\vec{j} \neq \vec{0}$

(3)



(4) $|\vec{a}\cdot\vec{b}| = |\vec{a}||\vec{b}||\cos\theta|$
$\leq |\vec{a}||\vec{b}|$

A. (2) and (3) are true. (1) and (4) are false.

B. (3) and (4) are true. (1) and (2) are false.

C. (1) and (4) are true. (2) and (3) are false.

D. (1), (2) and (3) are true. (4) is false.

E. (1), (3) and (4) are true. (2) is false.

2. The graph of

$$x^2 + y^2 + z^2 - 2x + 2ay - 10 = 0$$

is a sphere. If the center is $(1, -3, 0)$, what is the radius r?

$x^2 - 2x + 1 + y^2 + 2a + a^2 + z^2 = 11 + a^2$

$(x-1)^2 + (y+a)^2 + z^2 = 11 + a^2$

$(1, a, 0) = (1, -3, 0) \Rightarrow a = -3$

$r^2 = 11 + a^2 = 11 + 9 = 20$

$r = \sqrt{20} = 2\sqrt{5}$

A. $r = \sqrt{3}$

B. $r = 2\sqrt{5}$

C. $r = \sqrt{10}$

D. $r = 5\sqrt{2}$

E. $r = \sqrt{5}$

3. Find $\text{proj}_{\mathbf{w}}\mathbf{v}$, the vector projection of $\mathbf{v}$ onto $\mathbf{w}$, where

$$\mathbf{v} = \langle 1, 2, -1 \rangle, \quad \mathbf{w} = \langle 2, 0, 4 \rangle.$$

$$\text{Proj}_{\vec{w}}(\vec{v}) = \frac{\vec{v} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \vec{w}$$

$$= \frac{-2}{20} (2, 0, 4)$$

$$= \left(-\frac{1}{5}, 0, -\frac{2}{5}\right)$$

A. $\langle -1/5, 0, -2/5 \rangle$

B. $\langle -1/3, -2/3, 1/3 \rangle$

C. $\langle -2/3, 0, -4/3 \rangle$

D. $\langle -1/5, -2/5, 1/5 \rangle$

E. $\langle 1/3, 0, -1/3 \rangle$

4. Find the area of the triangle whose vertices are

$$P(1, 1, 1), \quad Q(2, -1, 5), \quad R(0, 3, -2).$$

$$\overrightarrow{PQ} = (1, -2, 4)$$

$$\overrightarrow{PR} = (-1, 2, -3)$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (-2, -1, 0)$$

$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{5}$$

$$A = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{\sqrt{5}}{2}$$

A. $3\sqrt{6}$

B. $\dfrac{\sqrt{6}}{2}$

C. $\dfrac{\sqrt{3}}{2}$

D. $\dfrac{\sqrt{5}}{2}$

E. $2\sqrt{5}$

5. A force $\mathbf{F} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ is applied to an object as it moves from the point $P(1, -1, 1)$ to the point $Q(2, 5, -3)$. Find the work done.

$\overrightarrow{PQ} = (1, 6, -4)$

$\vec{F} = (2, -1, -3)$

$W = \vec{F} \cdot \overrightarrow{PQ} = 2 - 6 + 12 = 8$

A. 6

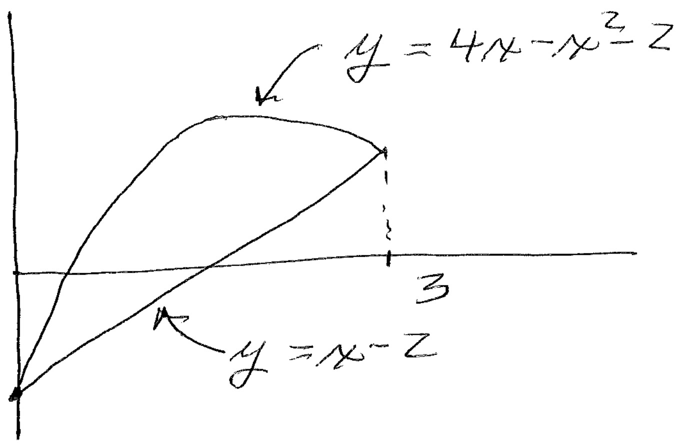
B. 8

C. 4

D. -6

E. -4

6. Find the area of the region bounded by the curves $y = x - 2$ and $y = 4x - x^2 - 2$.



A. $\frac{5}{3}$

B. $\frac{3}{4}$

C. $\frac{9}{2}$

D. $\frac{4}{3}$

E. $\frac{2}{7}$

PTS. OF INTERSECTION

$x - 2 = 4x - x^2 - 2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, 3$

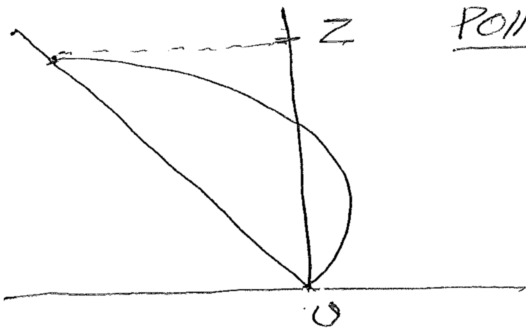
$$A = \int_0^3 \left((4x - x^2 - 2) - (x - 2) \right) dx$$

$$= \int_0^3 (3x - x^2)\, dx = \left. \frac{3x^2}{2} - \frac{x^3}{3} \right]_0^3$$

$$= \frac{3^3}{2} - \frac{3^3}{3} = 3^3\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{3^3}{6} = \frac{9}{2}$$

7. Find the area of the region bounded by the curves $x = -y$ and $x = y - y^2$.



POINTS OF INTERSECTION

$-y = y - y^2$

$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0, 2$

A. $\frac{4}{3}$

B. $\frac{3}{4}$

C. $\frac{2}{3}$

D. $\frac{5}{2}$

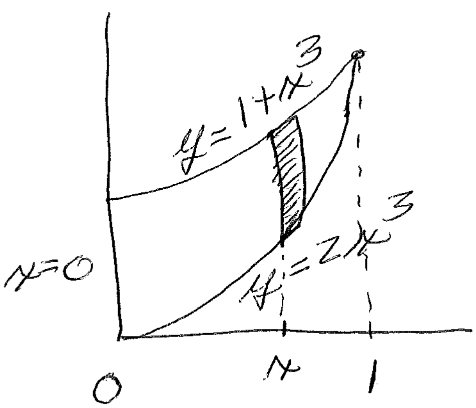
E. $\frac{8}{3}$

$$A = \int_0^2 \left((y-y^2) - (-y)\right) dy = \int_0^2 (2y - y^2)\, dy$$

$$= y^2 - \frac{y^3}{3}\Big]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$

8. Find the volume of the solid obtained by rotating about the x–axis the region bounded by the curves

$$y = 2x^3, \qquad y = 1 + x^3, \qquad x = 0.$$



$dV = \pi(1+x^3)^2 dx - \pi(2x^3)^2 dx$

$dV = \pi(1 + 2x^3 - 3x^6) dx$

A. $\frac{\pi}{15}$

B. $\frac{7\pi}{5}$

C. $\frac{16\pi}{15}$

D. $\frac{46\pi}{25}$

E. $\frac{15\pi}{14}$

$$V = \int_0^1 \pi(1 + 2x^3 - 3x^6)\, dx$$

$$= \pi\left[x + \frac{x^4}{2} - \frac{3x^7}{7}\right]_0^1 = \pi\left(1 + \frac{1}{2} - \frac{3}{7}\right) = \frac{15}{14}\pi$$

9. Use the method of cylindrical shells to set up an integral for the volume of the solid obtained by rotating the region bounded by $y = x^2 - x$ and $y = x + 3$ about the axis $x = 3$.

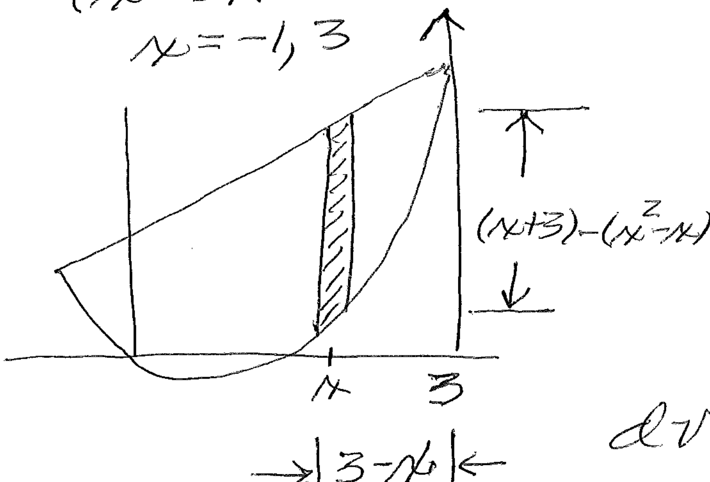
POINTS OF INTERSECTION

$x^2 - x = x + 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, 3$



A. $2\pi \int_{-3}^{1} x(2x - x^2 + 3)\, dx$

B. $2\pi \int_{-3}^{1} (3 - x)(x - x^2 + 3)\, dx$

C. $2\pi \int_{-1}^{3} (3 - x)(2x - x^2 + 3)\, dx$

D. $2\pi \int_{-1}^{3} (3 - x)(x^2 - 2x + 3)\, dx$

E. $2\pi \int_{-1}^{3} x(x^2 - x + 3)\, dx$

$dV = 2\pi(3-x)(2x - x^2 + 3)\,dx$

10. Suppose a force of 10 lbs is required to hold a spring 4 feet beyond its natural length. How much work is required to stretch it 6 feet beyond its natural length.

$F(x) = kx$

$10 = k(4)$

$k = \frac{5}{2}$

$F(x) = \frac{5}{2}x$

$W = \int_0^6 \frac{5}{2}x\,dx = \frac{5x^2}{4}\Big]_0^6$

$= 45$ FT-LB

A. 25 ft-lb

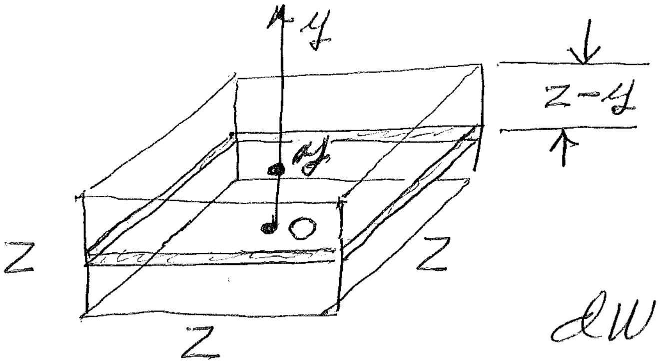
B. 60 ft-lb

C. $\frac{45}{8}$ ft-lb

D. 45 ft-lb

E. $\frac{25}{3}$ ft-lb

11. A tank in the shape of a cube with sides 2 meters long is full of a liquid having density $1\,\mathrm{kg}/\mathrm{m}^3$. Find the work required to empty the tank by pumping all of the liquid to the top of the tank. (Use $g = 10\,\mathrm{m}/\mathrm{s}^2$.)



$dV = 4dy$

$dF = (1)(10)dV$

$dF = 40dy$

$dW = (2-y)dF$

$dW = (2-y)40dy$

A. 200 J

B. 150 J

C. 80 J

D. 160 J

E. 180 J

$$W = \int_0^2 40(2-y)dy = 40\left[2y - \frac{y^2}{2}\right]_0^2 = 80\text{ J}$$

12. Compute $\displaystyle\int_0^{\pi/2} x\cos x\,dx$.

A. $\frac{2\pi}{3}$

B. $\frac{\pi}{4} - \frac{1}{3}$

C. $\frac{\pi}{5} - \frac{2}{3}$

D. $\frac{\pi}{2} - 1$

E. $\frac{\pi}{4}$

$\int x\cos x\,dx$

$u = x \qquad du = dx$

$dv = \cos x\,dx \qquad v = \sin x$

$= x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + C$

$$\int_0^{\pi/2} x\cos x\,dx = x\sin x + \cos x\Big]_0^{\pi/2} = \frac{\pi}{2} - 1$$