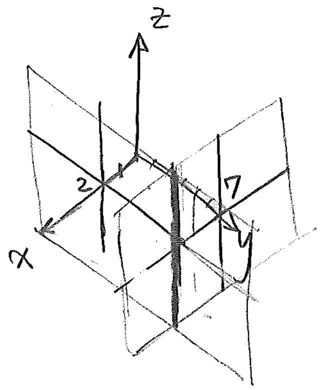
1. What does the pair of equations $x = 2, \quad y = 7$ represent in $\mathbb{R}^3$?



Intersection of 2 non-parallel planes is a line.

A. a point.
B. a line.
C. a plane.
D. a cone.
E. two planes.

2. Find the radius of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y - 6z = 7.$$

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) + (z^2 - 6z + 9) = 7 + 1 + 4 + 9$$

$$\rightarrow (x-1)^2 + (y+2)^2 + (z-3)^2 = 21$$

$$\rightarrow \text{radius} = \sqrt{21}$$

A. 1
B. $\sqrt{5}$
C. $\sqrt{11}$
D. $\sqrt{21}$
E. $\sqrt{23}$

3. Let $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 2\mathbf{j}$. Find $|\mathbf{a} - \mathbf{b}|$.

$$\vec{b} - \vec{a} = (4-2)\vec{i} + (2-5)\vec{j} + (0-(-1))\vec{k}$$
$$= 2\vec{i} - 3\vec{j} + \vec{k}$$
$$|\vec{b} - \vec{a}| = \sqrt{2^2 + (-3)^2 + (1)^2} = \sqrt{14}$$

A. $\sqrt{10}$
B. $\sqrt{14}$
C. $\sqrt{17}$
D. $\sqrt{20}$
E. $\sqrt{30}$

4. Find a unit vector with direction opposite that of $< 2, 4, -4 >$.

$$|\langle 2,4,-4\rangle| = \sqrt{2^2+4^2+(-4)^2} = \sqrt{36} = 6$$

$$-\frac{1}{6}\langle 2,4,-4\rangle = \left\langle -\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right\rangle = \left\langle -\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right\rangle$$

A. $\langle 2, 4, -4\rangle$

B. $\left\langle \frac{2}{\sqrt{10}}, \frac{4}{\sqrt{10}}, \frac{-4}{\sqrt{10}} \right\rangle$

C. $\left\langle \frac{-2}{\sqrt{10}}, \frac{-4}{\sqrt{10}}, \frac{4}{\sqrt{10}} \right\rangle$

D. $\left\langle \frac{1}{3}, \frac{2}{3}, \frac{-2}{3} \right\rangle$

E. $\left\langle \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3} \right\rangle$

5. Let $\mathbf{a} =< 1, 2, 3 >$ and $\mathbf{b} =< 2, -1, 1 >$. Find $\mathbf{a} \times \mathbf{b}$.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = (2-(-3))\vec{i} - (1-6)\vec{j} + (-1-4)\vec{k}$$

$$= 5\vec{i} + 5\vec{j} - 5\vec{k}$$

$$= \langle 5, 5, -5\rangle$$

A. $< -5, -5, 5 >$

B. $< 1, 1, -1 >$

C. $< 5, 5, -5 >$

D. $< -1, -1, 1 >$

E. $< -1, -7, 3 >$

6. Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{j} + \mathbf{k}$. Find $\mathrm{proj}_{\mathbf{a}}\mathbf{b}$,

$$\mathrm{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|a|^2}\right)\vec{a}$$

$$= \left(\frac{1\cdot 0 + 2\cdot 1 + 1\cdot 1}{1^2+2^2+1^2}\right)\vec{a} = \frac{3}{6}\vec{a}$$

$$= \frac{1}{2}\vec{a} = \frac{1}{2}\vec{i} + \vec{j} + \frac{1}{2}\vec{k}$$

A. $\frac{3}{2}\mathbf{j} + \frac{3}{2}\mathbf{k}$

B. $\frac{1}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$

C. $\frac{1}{2}\mathbf{i} + \mathbf{j} + \frac{1}{2}\mathbf{k}$

D. $\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$

E. $\frac{1}{4}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{4}\mathbf{k}$

$$(|b|\cos\theta)\left(\frac{a}{|a|}\right) = |b|\left(\frac{a\cdot b}{|a||b|}\right)\left(\frac{a}{|a|}\right) = \left(\frac{a\cdot b}{|a||a|}\right)a$$

3

7. Let $\mathbf{a} = <4,2,3>$ and $\mathbf{b} = <-2,1,2>$. Find $\mathbf{a} \cdot \mathbf{b}$.

$$\vec{a} \cdot \vec{b} = (4)(-2) + (2)(1) + (3)(2)$$
$$= -8 + 2 + 6$$
$$= 0$$

A. 11

B. $<2,3,5>$

C. 0

D. 8

E. $\frac{3}{2}$

8. Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Find $\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{2 - 4 + 1}{\sqrt{1+4+1}\,\sqrt{4+4+1}}$$
$$= \frac{-1}{\sqrt{6}\sqrt{9}} = \frac{-1}{3\sqrt{6}}$$

A. $\frac{-1}{2\sqrt{5}}$

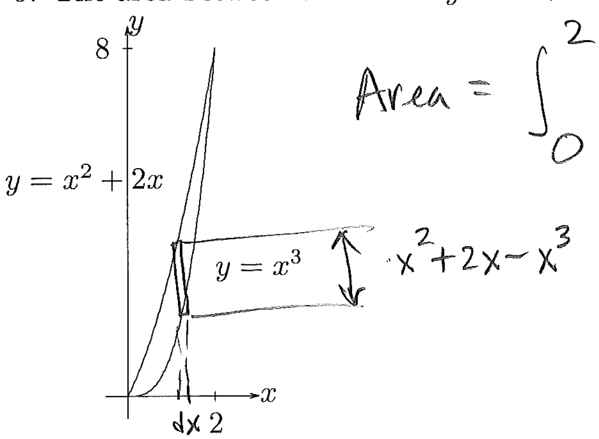
B. $\frac{-7}{3\sqrt{6}}$

C. $\frac{\sqrt{53}}{3\sqrt{6}}$

D. $\frac{7}{3\sqrt{6}}$

E. $\frac{-1}{3\sqrt{6}}$

9. The area between the curves $y = x^2 + 2x$ and $y = x^3$ and between $x = 0$ and $x = 2$ is



$$\text{Area} = \int_0^2 (x^2 + 2x - x^3)\,dx$$
$$= \left(\frac{1}{3}x^3 + x^2 - \frac{x^4}{4}\right)\Big|_0^2$$
$$= \left(\frac{8}{3} + 4 - \frac{16}{4}\right) - (0)$$
$$= \frac{8}{3}$$

A. $\frac{2}{3}$

B. $\frac{4}{3}$

C. $\frac{5}{3}$

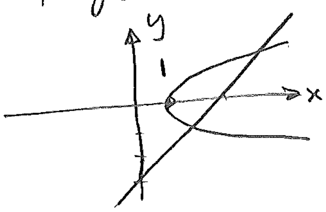
D. $\frac{8}{3}$

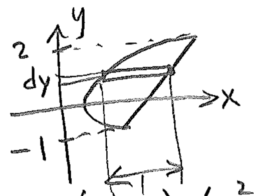
E. $\frac{10}{3}$

Typical vertical rectangular slice has area $= ((x^2+2x) - (x^3))\,dx$

10. The area between the curves $y^2 = x - 1$ and $y = x - 3$ is

Rough sketch





$(y+3)-(y^2+1)$

(A.) $\int_{-1}^{2} \left((y+3) - (y^2+1)\right)\, dy$

B. $\int_{-1}^{2} \left((y^2+1) - (y+3)\right)\, dy$

C. $\int_{1}^{5} \left(\sqrt{x-1} - (x-3)\right)\, dx$

D. $\int_{1}^{5} \left((x-3) - \sqrt{x-1}\right)\, dx$

E. $\int_{-1}^{2} \left((x-3) - (x-1)\right)\, dy$

intersection of curves interms of y:

$y = x-3 \rightarrow x = y+3 \rightarrow y^2 = (y+3)-1$

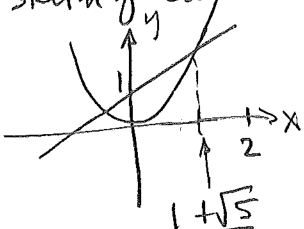
$\rightarrow y^2 - y - 2 = 0 \rightarrow (y-2)(y+1) = 0$

$\rightarrow y = -1, 2$

Typical horizontal rectangular slice has area $= \left((y+3)-(y^2+1)\right)dy$

11. What is the distance between the points (x, x^2) and $(x, x+1)$ for $x > 2$?

Sketch of curves



intersection of curves:

$x^2 = x+1$

$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$. Note: $\frac{1+\sqrt{5}}{2} < 2 \rightarrow y = x^2$ is above $y = x+1$ for $x \geq 2$.

(A.) $x^2 - x - 1$

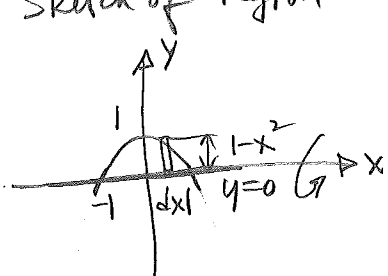
B. $x + 1 - x^2$

C. Cannot be determined.

12. The region bounded by $y = 1 - x^2$ and $y = 0$ is rotated about the x-axis. Find the volume of the solid generated.

By Disks:

Sketch of region



$$V = \int_{-1}^{1} \pi \left(1 - x^2\right)^2 dx$$

$$= \int_{-1}^{1} \pi \left(1 - 2x^2 + x^4\right) dx$$

$$= \pi \left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\Big|_{-1}^{1}$$

$$= \pi\left(\left(1 - \frac{2}{3} + \frac{1}{5}\right) - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)\right) = \frac{16}{15}\pi$$

A. $\frac{3}{5}\pi$

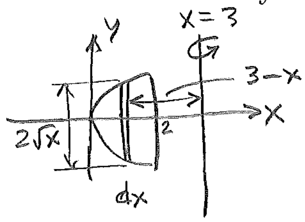
B. $\frac{7}{15}\pi$

C. $\frac{11}{15}\pi$

(D.) $\frac{16}{15}\pi$

E. $\frac{21}{15}\pi$

13. The region bounded by $x = y^2$ and $x = 2$ is rotated about the line $x = 3$. Using the method of cylindrical shells, the volume of the solid generated is



$$V = \int_0^2 2\pi (3-x)\, 2\sqrt{x}\, dx$$

$$= \int_0^2 2\pi \left(6x^{1/2} - 2x^{3/2}\right) dx$$

A. $\int_0^2 2\pi \left(3x^{1/2} - x^{3/2}\right) dx$

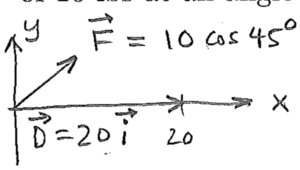
B. $\int_0^2 2\pi \left(6x^{1/2} - 2x^{3/2}\right) dx$

C. $\int_0^2 2\pi (3 - x)\ dx$

D. $\int_0^2 2\pi \left(6x - 2x^2\right) dx$

E. $\int_0^2 2\pi \left(3x - x^2\right) dx$

14. A person slides a block of ice 20 feet along a horizontal floor by pulling with a force of 10 lbs at an angle of 45° to the floor. How much work is done by the person?



$$\vec{F} = 10\cos 45^\circ\, \vec{i} + 10 \sin 45^\circ\, \vec{j} = \frac{10}{\sqrt{2}}\vec{i} + \frac{10}{\sqrt{2}}\vec{j}$$

$$\text{Work} = \vec{F} \cdot \vec{D} = \left(\frac{10}{\sqrt{2}}\right)(20) + \left(\frac{10}{\sqrt{2}}\right)(0)$$

$$= \frac{200}{\sqrt{2}} \text{ ft-lbs}$$

A. $\frac{200}{\sqrt{2}}$ ft-lbs

B. 200 ft-lbs

C. 100 ft-lbs

D. $\frac{400}{\sqrt{3}}$ ft-lbs

E. $\frac{400}{\sqrt{2}}$ ft-lbs

15. A water trough with triangular cross–section (see figure) is 2 feet high, 4 feet wide at the top and 10 feet long, and is full of water (62.5 lbs/ft^3). Find the work done pumping all the water to the top of the tank.

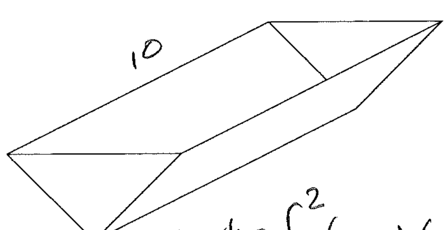
A. (62.5)(30) ft-lbs

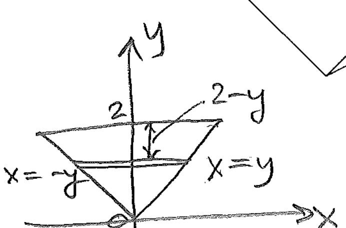
B. (62.5)(25) ft-lbs

C. $(62.5)\left(\frac{40}{3}\right)$ ft-lbs

D. $(62.5)\left(\frac{70}{3}\right)$ ft-lbs

E. $(62.5)\left(\frac{80}{3}\right)$ ft-lbs





$$\text{Work} = \int_0^2 (2-y)(62.5)(10)(2y)\, dy = 62.5 \int_0^2 \left(40y - 20y^2\right) dy = *$$

$$* = 62.5\left(20y^2 - \frac{20}{3}y^3\right)\Big|_0^2 = 62.5\left(80 - \frac{160}{3}\right) = 62.5\left(\frac{80}{3}\right)$$

16. $\int_0^{\pi/6} x\sin x\;dx =$

A. $\frac{1}{2}-\frac{\sqrt{3}}{12}\pi$

B. $\frac{1}{2}-\frac{\sqrt{3}}{2}\pi$

C. $\frac{\sqrt{3}}{2}-\frac{\pi}{12}$

D. $\frac{\sqrt{3}}{2}+\frac{\pi}{12}$

E. $\frac{1}{2}+\frac{\sqrt{3}}{6}\pi$

L.I.A.T.E. let $u = x$, $dv = \sin x\,dx$

then $du = dx$, $v = -\cos x$

$$\int_0^{\pi/6} x\sin x\,dx = x(-\cos x)\Big|_0^{\pi/6} - \int_0^{\pi/6} -\cos x\,dx$$

$$= \left(-x\cos x + \sin x\right)\Big|_0^{\pi/6}$$

$$= \left(-\frac{\pi}{6}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\right) - (0+0)$$

$$= -\frac{\sqrt{3}}{12}\pi + \frac{1}{2}$$

17. $\int_0^{\pi/2} \sin^3 x\cos^2 x\;dx =$

A. $\frac{4}{15}$

B. $\frac{2}{15}$

C. $\frac{-2}{15}$

D. $\frac{-4}{15}$

E. $\frac{1}{3}$

$$= \int_0^{\pi/2} \sin^2 x\cos^2 x\,\sin x\,dx$$

$$= \int_0^{\pi/2} (1-\cos^2 x)\cos^2 x\,\sin x\,dx$$

$$= \int_0^{\pi/2} (\cos^2 x - \cos^4 x)\sin x\,dx$$

$$\left[\begin{array}{l} \text{let } u = \cos x, \text{ then } du = -\sin x\,dx \\ \qquad\qquad\qquad\qquad \text{and } -du = \sin x\,dx \\ u(0) = \cos 0 = 1,\quad u(\pi/2) = \cos \pi/2 = 0 \end{array}\right]$$

$$= \int_1^0 (u^2-u^4)(-du) = -\int_1^0 (u^2-u^4)\,du$$

$$= -\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)\Big|_1^0 = -(0) - \left(-\left(\frac{1}{3}-\frac{1}{5}\right)\right) = \frac{2}{15}$$