1. The equation $x^2 - 2x + y^2 + 4y + z^2 = 0$ represents a sphere with

   A. center $(-1, 2, 0)$ and radius 5

   B. center $(1, -2, 0)$ and radius $\sqrt{5}$

   C. center $(1, -2, 0)$ and radius 5

   D. center $(-1, 2, 0)$ and radius $\sqrt{5}$

   E. This is not an equation of a sphere

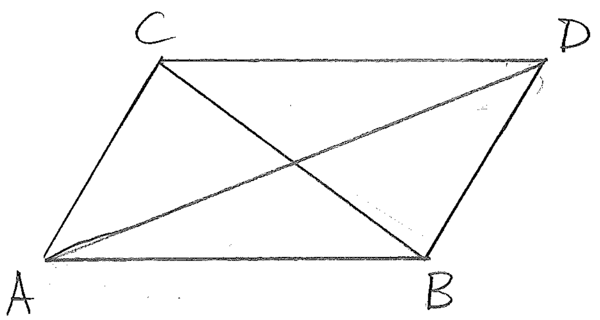
$(x^2 - 2x + 1) + (y^2 + 4y + 4) + z^2 = 0 + 1 + 4$ complete the square

$\to (x-1)^2 + (y+2) + (z-0)^2 = 5$

Center : $(1, -2, 0)$

radius : $\sqrt{5}$

2. In the parallelogram below, $\overrightarrow{AB} - \overrightarrow{BD}$ equals



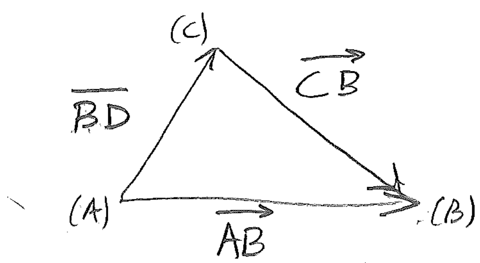
A. $\overrightarrow{CB}$

B. $\overrightarrow{AD}$

C. $\overrightarrow{BC}$

D. $\overrightarrow{DA}$

E. $\overrightarrow{CA}$



$\overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{BD}$

3. Let $\vec{\mathbf{u}}$ be a unit vector, and let $\vec{\mathbf{u}} \cdot \vec{\mathbf{a}} = 5$. If the angle between $\vec{\mathbf{u}}$ and $\vec{\mathbf{a}}$ is $\pi/4$, find $|\vec{\mathbf{a}}|$.

A. $|\vec{\mathbf{a}}| = 5$

B. $|\vec{\mathbf{a}}| = 5/\sqrt{2}$

C. $|\vec{\mathbf{a}}| = \sqrt{2}/5$

D. $|\vec{\mathbf{a}}| = 5\sqrt{2}$

E. $|\vec{\mathbf{a}}| = \frac{1}{5\sqrt{2}}$

$$\cos 45^{\circ} = \frac{\vec{u} \cdot \vec{a}}{|\vec{u}||\vec{a}|}$$

$$\rightarrow \frac{1}{\sqrt{2}} = \frac{5}{(1)(|\vec{a}|)}$$

$$\rightarrow |\vec{a}| = 5\sqrt{2}$$

4. Which of the following expressions are meaningful for vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$?

(a) $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

(b) $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})$ — not meaningful

(c) $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

(d) $(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}) \times \vec{\mathbf{c}}$ — not meaningful

(e) $(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}) \times (\vec{\mathbf{c}} \cdot \vec{\mathbf{d}})$ — not meaningful

(f) $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot (\vec{\mathbf{c}} \times \vec{\mathbf{d}})$

A. (a), (c), and (e) only

B. (b) and (c) only

C. (a), (c), and (f) only

D. (c) and (f) only

E. all of them

(b) and (d) not meaningful since a vector can't be crossed with a number

(e) not meaningful because 2 numbers can't be crossed.

5. Find $\vec{\mathbf{i}} \times (\vec{\mathbf{j}} \times \vec{\mathbf{i}})$.

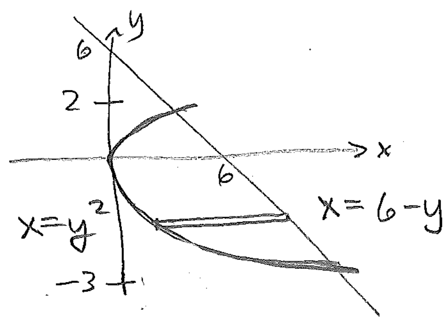
   A. $\vec{\mathbf{j}}$
   B. $-\vec{\mathbf{j}}$
   C. $\vec{\mathbf{0}}$
   D. $\vec{\mathbf{k}}$
   E. $-\vec{\mathbf{k}}$

$\vec{j} \times \vec{i} = -\vec{k}$

$\vec{i} \times (-\vec{k}) = -(\vec{i} \times \vec{k})$
$= -(-\vec{j})$
$= \vec{j}$

6. Find the area of the region enclosed by the curves $x = y^2$ and $x + y = 6$.

   A. 20
   B. 25
   C. 125/6
   D. 127/6
   E. 151/6



intersections : (substitute y^2 for x in 2nd equ.)

$y^2 + y = 6$
$\rightarrow y^2 + y - 6 = 0$
$\rightarrow (y+3)(y-2) = 0$
$\rightarrow y = 2, -3$

$$\text{Area} = \int_{-3}^{2} ((6-y)-(y^2))\,dy$$
$$= \left(6y - \tfrac{1}{2}y^2 - \tfrac{1}{3}y^3\right)\Big|_{-3}^{2}$$
$$= \left(12 - 2 - \tfrac{8}{3}\right) - \left(-18 - \tfrac{9}{2} + 9\right)$$
$$= \frac{125}{6}$$

7. If the region bounded by the curve $y = 1 - x^2$ and the x–axis is rotated about the line $x = -1$, then the solid generated will have volume

SHELLS :

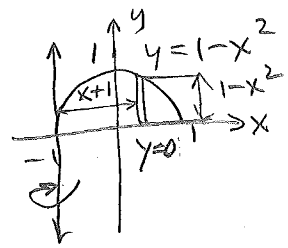
A. $\int_{-1}^{1} \pi(1-x^2)^2 dx$

B. $\int_{-1}^{1} 2\pi(1-x^2)^2 dx$

C. $\int_{-1}^{1} \pi(x+1)(1-x^2) dx$

D. $\int_{-1}^{1} 2\pi(x+1)(1-x^2) dx$

E. $\int_{0}^{1} \pi(1-y) dy$



$$\text{Volume} = \int_{-1}^{1} 2\pi(x+1)(1-x^2)\,dx$$

$$= \int_{-1}^{1} 2\pi(x - x^3 + 1 - x^2)\,dx$$

$$= 2\pi\left(\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4 + x - \tfrac{1}{3}x^3\right)\Big|_{-1}^{1}$$

$$= 2\pi\left(\left(\cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{4}} + 1 - \tfrac{1}{3}\right) - \left(\cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{4}} - 1 + \tfrac{1}{3}\right)\right)$$

$$= 2\pi\left(2 - \tfrac{2}{3}\right) = 2\pi\left(\tfrac{4}{3}\right) = \tfrac{8\pi}{3}$$

(oops! I got carried away !)

8. If the region in the first quadrant bounded by $y = x^2, y = 2$ and $x = 0$ is rotated about the x–axis, then the resulting solid will have volume

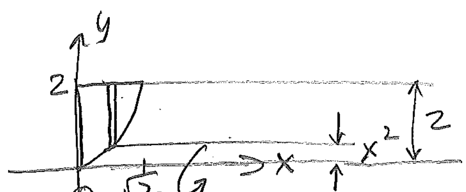
A. $\int_{0}^{\sqrt{2}} \pi(4-x^4) dx$

B. $\int_{0}^{2} \pi(4-x^4) dx$

C. $\int_{0}^{\sqrt{2}} \pi(2-x^2)^2 dx$

D. $\int_{0}^{2} \pi(2-x^2)^2 dx$

E. $\int_{0}^{2} 2\pi x(2-x^2) dx$

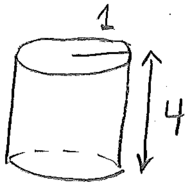


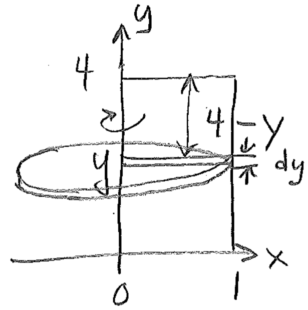
Washer method :

$$\text{Volume} = \int_{0}^{\sqrt{2}} \pi\left(2^2 - (x^2)^2\right) dx$$

$$= \int_{0}^{\sqrt{2}} \pi(4 - x^4)\, dx$$

9. A cyclindrical tank, 2 ft. in diameter and 4 ft. tall, is full of water. How much work is done in pumping the water to the top of the tank? (Assume the water weighs 62.5 lb/ft^3.)





A. 200π ft-lb

B. 300π ft-lb

C. 400π ft-lb

D. 500π ft-lb

E. 1000π ft-lb

$$\text{Work} = \int_0^4 (4-y)(62.5)(\pi \cdot 1^2)\,dy$$
$$= 62.5\pi \int_0^4 (4-y)\,dy$$
$$= 62.5\pi \left(4y - \tfrac{1}{2}y^2\right)\Big|_0^4 = 62.5\pi(16-8) = 500\pi$$

10. Find the average value of the function $f(x) = x^5$ on the interval $[0, 2]$.

A. $\frac{2^6}{2}$

B. $\frac{2^6}{3}$

C. $\frac{2^6}{4}$

D. $\frac{2^6}{6}$

E. $\frac{2^6}{12}$

$$\text{average value} = \frac{1}{2-0}\int_0^2 x^5\,dx$$
$$= \frac{1}{2} \cdot \frac{x^6}{6}\Big|_0^2$$
$$= \frac{2^6}{12}$$

11. $\int_0^2 xe^x\,dx =$

A. $2(e^2-1)$

B. e^2-1

C. $2e^2-1$

D. e^2+1

E. $2e^2+1$

L.I.A.T.E.

let $u = x$, $dv = e^x dx$

then $du = dx$, $v = e^x$

$$\int_0^2 xe^x dx = xe^x\Big|_0^2 - \int_0^2 e^x dx$$

$$= (xe^x - e^x)\Big|_0^2$$

$$= (2e^2 - e^2) - (0-1)$$

$$= e^2 + 1$$

12. $\int_0^{\pi/4} \sec x \tan^3 x\,dx =$

A. $\frac{1}{4}$

B. $2^{\frac{1}{2}} - \frac{1}{2}$

C. $2^{\frac{3}{2}} - \frac{1}{3}$

D. $2^{\frac{3}{2}} - 2^{\frac{1}{2}} + \frac{1}{3}$

E. $\frac{2^{\frac{3}{2}}}{3} - 2^{\frac{1}{2}} + \frac{2}{3}$

$$\int_0^{\pi/4} \tan^2 x(\sec x \tan x)\,dx$$

$$= \int_0^{\pi/4} (\sec^2 x - 1)(\sec x \tan x)\,dx$$

$$\left[\begin{array}{l} \text{let } u = \sec x, \text{ then } du = \sec x \tan x\,dx \\ u(0) = \sec 0 = 1, \quad u(\pi/4) = \sec \pi/4 = \sqrt{2} \end{array}\right]$$

$$= \int_1^{\sqrt{2}} (u^2 - 1)\,du$$

$$= \left(\tfrac{1}{3}u^3 - u\right)\Big|_1^{\sqrt{2}}$$

$$= \left(\tfrac{1}{3}\cdot 2\sqrt{2} - \sqrt{2}\right) - \left(\tfrac{1}{3} - 1\right)$$

$$= \frac{2^{3/2}}{3} - 2^{1/2} + \frac{2}{3}$$