1. The equation

$$x^2 + y^2 + z^2 + 2x - 4y + 6z = C$$

is a sphere of radius 2. What is C?

A. $C = -12$

B. $C = 4$

C. $C = 14$

D. $C = 2$

E. $\boxed{C = -10}$

$$x^2+2x \underline{+1} + y^2-4y \underline{+4} + z^2+6z \underline{+9} = C+1+4+9$$

$$(x+1)^2 + (y-2)^2 + (z+3)^2 = C + 14$$

$$C+14 = 2^2$$

$$C = -10$$

2. $\vec{v} = \langle 2, 3, 6\rangle$ and $\vec{w} = \langle 1, -2, 3\rangle$. Find $\text{proj}_{\vec{v}}\vec{w}$, the vector projection of $\vec{w}$ onto $\vec{v}$.

A. $\langle 3, 0, -1\rangle$

B. $\left\langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right\rangle$

C. $\langle 2, -4, 6\rangle$

D. $\boxed{\left\langle \frac{4}{7}, \frac{6}{7}, \frac{12}{7}\right\rangle}$

E. $\langle 1, -2, 3\rangle$

$$\vec{w}\cdot\vec{v} = 2-6+18 = 14$$

$$|\vec{v}| = \sqrt{2^2+3^2+6^2} = 7$$

$$\vec{w}\cdot\frac{\vec{v}}{|\vec{v}|}\left(\frac{\vec{v}}{|\vec{v}|}\right) = \frac{2}{7}\vec{v}$$

3. Which of the following statements are true?
(i) $2\vec{\imath}$ and $\vec{\jmath}+\vec{k}$ are orthogonal vectors
(ii) If $\vec{v}$ is a vector in three dimensions, $\vec{v}\times\vec{v}=\vec{0}$
(iii) If $\vec{w}$ is a vector, $\vec{w}\cdot\vec{w}=|\vec{w}|$

A. $\boxed{\text{(i) and (ii) only}}$

B. (i) and (iii) only

C. Only one of the three statements is true

D. (ii) and (iii) only

E. (i), (ii), and (iii) are all true

4. Suppose $|\vec{a}|=4$ and $\vec{b}=\vec{\imath}+4\vec{\jmath}+8\vec{k}$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$. What is $|\vec{a}\times\vec{b}|$?

A. 36

B. 26

C. $9\sqrt{3}$

D. 9

E. $\boxed{18}$

$|\vec{b}| = \sqrt{1^2+4^2+8^2} = 9$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta = (4)(9)(\frac{1}{2})$

5. The area between the curves $y = x^2$ and $x + y = 2$ is given by

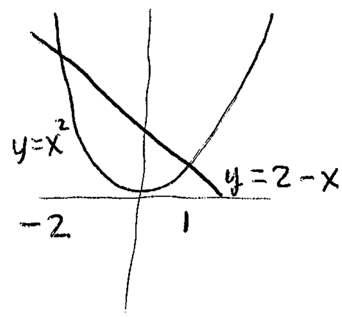
A. $\boxed{\int_{-2}^{1} (2 - x - x^2)\, dx}$

B. $\int_{0}^{2} (\sqrt{y} + y - 2)\, dy$

C. $\int_{-2}^{1} (x^2 + x - 2)\, dx$

D. $\int_{-1}^{2} (2 - x - x^2)\, dx$

E. $\int_{-1}^{2} (x^2 + x - 2)\, dx$



$x^2 = 2 - x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

6. Find the area bounded by the curves $y = 2 - x^2$ and $y = |x|$

A. $\frac{10}{3}$

B. $\frac{7}{6}$

C. $\frac{13}{3}$

D. $\boxed{\frac{7}{3}}$

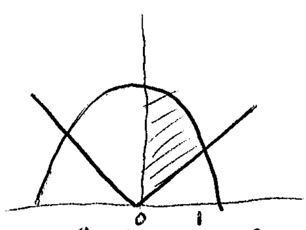
E. $\frac{13}{6}$

If $x \geq 0$, $2 - x^2 = x$

$0 = x^2 + x - 2$

$0 = (x-1)(x+2)$

$x = 1$



SYMMETRIC

$\downarrow$

$$2\int_0^1 (2 - x^2) - x \;dx$$

$$= 2\left[2x - \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2\right]_0^1$$

$$= 2\left(2 - \tfrac{1}{3} - \tfrac{1}{2}\right) = 4 - \tfrac{2}{3} - 1$$

7. Use cylindrical shells to find a formula for the volume of the following solid: the region bounded by $y = x^2$ and $y = 1$ is revolved about the line $y = 3$.

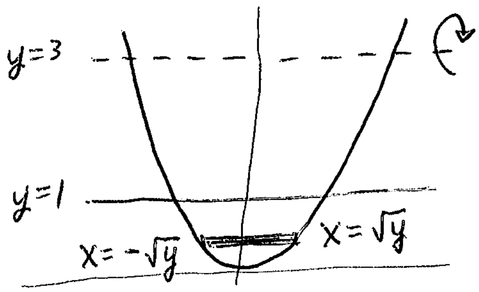
A. $\int_{-1}^{1} 2\pi(1-y)y^{1/2}dy$

B. $\int_{0}^{1} 2\pi(3-y)y^{1/2}dy$

C. $\boxed{\int_{0}^{1} 4\pi(3-y)y^{1/2}\,dy}$

D. $\int_{0}^{1} 2\pi(3-x)x^2dx$

E. $\int_{-1}^{1} \pi(1-x^2)\,dx$



$$\int_0^1 2\pi\,(3-y)\left[\sqrt{y}-(-\sqrt{y})\right]dy$$

8. It requires a force of 12 N to hold a spring 0.1 m longer than its natural length. How much work (in N·m) is required to stretch it from its natural length into this position?

A. $\boxed{0.6}$

B. 1.2

C. 12

D. 0.006

E. 6

$f = kx$

$12 = k(0.1)$

$120 = k$

$$\text{Work} = \int_0^{0.1} 120x\,dx = 60x^2\Big|_0^{0.1} = 60(0.01) - 0$$

9. What is the average value of the function $f(x) = \sin^3 x$ over the interval $0 \le x \le \pi$?

A. $\frac{1}{8\pi}$

B. $\boxed{\frac{4}{3\pi}}$

C. $\frac{\sqrt{2}}{4\pi}$

D. $\frac{8}{3\pi}$

E. $\frac{1}{2\pi}$

$$\frac{1}{\pi - 0}\int_0^{\pi} \sin^3 x \, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin^2 x \sin x \, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} (1-\cos^2 x)\sin x \, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} (\sin x - \cos^2 x \sin x) \, dx$$

$$= -\frac{1}{\pi}\cos x + \frac{1}{\pi}\left(\frac{1}{3}\cos^3 x\right)\Big|_0^{\pi}$$

$$= -\frac{1}{\pi}(-1-1) + \frac{1}{\pi}\left(-\frac{1}{3} - \frac{1}{3}\right)$$

10. Compute $\int_0^{\pi/4} \tan x \sec^3 x \, dx$ $= \int_0^{\pi/4} \sec^2 x \sec x \tan x \, dx$

A. 1

B. $\frac{3}{4}$

C. $2\sqrt{2} - 1$

D. $\frac{\sqrt{2}}{3}$

E. $\boxed{\frac{2\sqrt{2} - 1}{3}}$

$$= \frac{1}{3}\sec^3 x \Big|_0^{\pi/4}$$

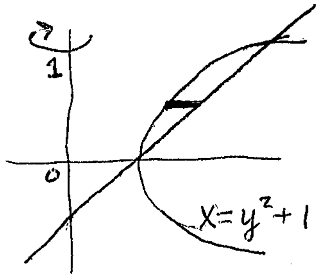
$$= \frac{1}{3}(\sqrt{2})^3 - \frac{1}{3}(1)^3$$

11. Find the volume of the solid that results from rotating the area between the curves $y^2 = x-1$ and $y = x-1$ around the y-axis.

$x = y+1$

$y+1 = y^2+1$

$0 = y^2 - y \rightarrow y=0 \text{ or } y=1$



A. $\dfrac{3\pi}{7}$

B. $\boxed{\dfrac{7\pi}{15}}$

C. $\dfrac{\pi}{6}$

D. $\dfrac{\pi}{3}$

E. $\dfrac{\pi}{10}$

$$\int_0^1 \pi (y+1)^2 - \pi (y^2+1)^2 \, dy$$

$$= \pi \int_0^1 (y^2 + 2y + 1 - y^4 - 2y^2 - 1) \, dy$$

$$= \pi \int_0^1 (2y - y^4 - y^2) \, dy$$

$$= \pi \left[y^2 - \tfrac{1}{5}y^5 - \tfrac{1}{3}y^3 \right]_0^1$$

$$= \pi \left(1 - \tfrac{1}{5} - \tfrac{1}{3}\right)$$

$$= \pi \left(\frac{15-3-5}{15}\right)$$

12. Find $\displaystyle\int \frac{\cos(\frac{x}{4})}{e^x}\, dx$

$$= 4e^{-x}\sin\left(\tfrac{x}{4}\right) + \int 4\sin\left(\tfrac{x}{4}\right) e^{-x}\, dx$$

$$\boxed{\begin{array}{ll} u = e^{-x} & v = 4\sin(\frac{x}{4}) \\ du = -e^{-x}dx & dv = \cos(\frac{x}{4})dx \end{array}}$$

$$\boxed{\begin{array}{ll} u = e^{-x} & v = 16\cos(\frac{x}{4}) \\ du = -e^{-x}dx & dv = 4\sin(\frac{x}{4})dx \end{array}}$$

A. $\dfrac{4\cos(\frac{x}{4}) - 16\sin(\frac{x}{4})}{15e^x} + C$

B. $\boxed{\dfrac{4\sin(\frac{x}{4}) - 16\cos(\frac{x}{4})}{17e^x} + C}$

C. $\dfrac{4\sin(\frac{x}{4}) - 16\cos(\frac{x}{4})}{15e^x} + C$

D. $\dfrac{16\sin(\frac{x}{4}) - 4\cos(\frac{x}{4})}{15e^x} + C$

E. $\dfrac{16\sin(\frac{x}{4}) - 4\cos(\frac{x}{4})}{17e^x} + C$

$$= 4e^{-x}\sin\left(\tfrac{x}{4}\right) + \left(-16e^{-x}\cos\left(\tfrac{x}{4}\right)\right) - \int \left(-16\cos\left(\tfrac{x}{4}\right)\right)\left(-e^{-x}\right) dx$$

$$= \frac{4\sin(\frac{x}{4})}{e^x} - \frac{16\cos(\frac{x}{4})}{e^x} - 16\int \frac{\cos(\frac{x}{4})}{e^x}\, dx$$

$$I = \frac{4\sin(\frac{x}{4}) - 16\cos(\frac{x}{4})}{e^x} - 16I$$