MA 162 **Exam 2** **Spring 2008**

Name SOLUTION KEY

10-digit PUID

RECITATION Division and Section Numbers

Recitation Instructor

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 17 problems. Problems 11 and 13 are worth 5 points each. The rest of the Problems are worth 6 points each. The maximum score is 100 points.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators or any electronic devices are not to be used on this test.

1. What's an appropriate trig substitution for the integral $\int x^3\sqrt{4-9x^2}\,dx$?

$\sqrt{a^2-(u(x))^2} \Longrightarrow$ Let $u(x) = a\sin\theta$

$\therefore\ 3x = 2\sin\theta$

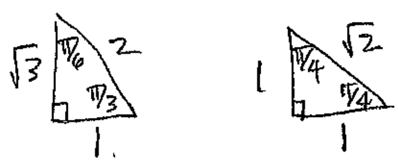
A. $3x = 2\sin(\theta)$
B. $3x = 2\tan(\theta)$
C. $3x = 2\sec(\theta)$
D. $2x = 3\sin(\theta)$
E. $2x = 3\tan(\theta)$

2. Using an appropriate trig substitution, the corresponding θ limits of integration of the integral $\int_{\sqrt{3}}^{3} \frac{x^3}{\sqrt{x^2+9}}\,dx$ are

$\sqrt{x^2+9} \Longrightarrow x = 3\tan\theta \Longrightarrow \theta = \tan^{-1}\left(\frac{x}{3}\right)$

$\theta(\sqrt{3}) = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$

$\theta(3) = \tan^{-1}\left(\frac{3}{3}\right) = \tan^{-1}(1) = \frac{\pi}{4}$



A. $\int_{\pi/4}^{\pi/6}$
B. $\int_{\pi/6}^{\pi/4}$
C. $\int_{\pi/3}^{\pi/4}$
D. $\int_{\pi/6}^{\pi/3}$
E. $\int_{\pi/3}^{\pi/2}$

3. Using an appropriate trig substitution, $\int \frac{\sqrt{x^2-1}}{x}\,dx =$

$\sqrt{x^2-1} \Longrightarrow x = \sec\theta$

Then $\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = \tan\theta$

and $dx = \sec\theta\tan\theta\,d\theta$

$\int \frac{\sqrt{x^2-1}}{x}\,dx = \int \frac{\tan\theta}{\sec\theta}\sec\theta\tan\theta\,d\theta$

$= \int \tan^2\theta\,d\theta$

A. $\int \tan(\theta)\sec(\theta)\,d\theta$
B. $\int \sin(\theta)\cos(\theta)\,d\theta$
C. $\int \sin^2(\theta)\,d\theta$
D. $\int \sec^2(\theta)\,d\theta$
E. $\int \tan^2(\theta)\,d\theta$

4. What's an appropriate trig substitution for the integral $\int \frac{\sqrt{4x - x^2}}{3x}\, dx$?

$$4x - x^2 = -(x^2 - 4x + 4) + 4$$
$$= 4 - (x-2)^2$$
$$\therefore \text{ Let } x - 2 = 2\sin\theta$$

A. $x - 2 = 3\sin(\theta)$

B. $x - 4 = 2\sin(\theta)$

C. $x - 2 = 3\tan(\theta)$

D. $x - 2 = 2\sin(\theta)$

E. $x - 4 = 2\tan(\theta)$

5. The form of the partial fraction decomposition of $\frac{162x}{x^4 - 16}$ is

$$\frac{162x}{x^4-16} = \frac{162x}{(x^2-4)(x^2+4)}$$
$$= \frac{162x}{(x-2)(x+2)(x^2+4)}$$
$$= \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+4}$$

A. $\frac{Ax+B}{x^2-4} + \frac{Cx+D}{(x^2-4)^2}$

B. $\frac{A}{x-4} + \frac{B}{x+4}$

C. $\frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+4}$

D. $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x^2+4}$

E. $\frac{A}{x^2-4} + \frac{B}{x^2+4}$

6. $\int \frac{3x}{(x-1)(x+2)}\, dx =$

$$\frac{3x}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$
$$\rightarrow 3x = A(x+2) + B(x-1)$$
$$x = 1 \rightarrow 3 = 3A + 0 \cdot B \rightarrow A = 1$$
$$x = -2 \rightarrow -6 = 0A - 3B \rightarrow B = 2$$
$$\int \frac{3x}{(x-1)(x+2)} dx = \int \left(\frac{1}{x-1} + \frac{2}{x+2}\right) dx = \ln|x-1| + 2\ln|x+2| + C$$

A. $\ln|x-1| + 2\ln|x+2| + C$

B. $\ln|x-1| - 2\ln|x+2| + C$

C. $\ln|x-1| + \ln|x+2| + C$

D. $2\ln|x-1| + \ln|x+2| + C$

E. $2\ln|x-1| - 2\ln|x+2| + C$

7. From a table of integrals, it appears the integral $\int \frac{\sqrt{9x^2-4}}{12x}\,dx$ is closest in form to $\int \frac{\sqrt{u^2-a^2}}{u}\,du$. With an appropriate substitution, $\int \frac{\sqrt{9x^2-4}}{12x}\,dx =$

Let $u = 3x$ and $a = 2$. Then $x = \frac{1}{3}u$

Then $du = 3dx$, so $dx = \frac{1}{3}du$

$$\int \frac{\sqrt{9x^2-4}}{12x}\,dx = \int \frac{\sqrt{u^2-a^2}}{12\left(\frac{1}{3}u\right)}\,\frac{1}{3}\,du$$

$$= \frac{1}{12}\int \frac{\sqrt{u^2-a^2}}{u}\,du$$

A. $\frac{1}{4}\int \frac{\sqrt{u^2-2^2}}{u}\,du$

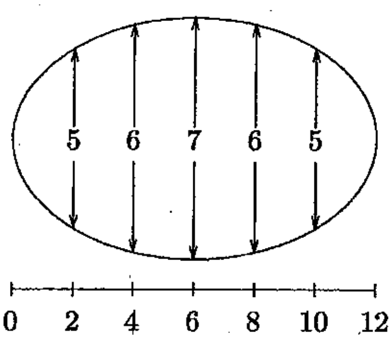
B. $\frac{1}{36}\int \frac{\sqrt{u^2-2^2}}{u}\,du$

C. $\frac{1}{12}\int \frac{\sqrt{u^2-2^2}}{u}\,du$

D. $12\int \frac{\sqrt{u^2-2^2}}{u}\,du$

E. $\frac{2}{3}\int \frac{\sqrt{u^2-2^2}}{u}\,du$

8. A pool, 12 yards long, is shaped like an oval. The distance, in yards, across the pool, at 2 yard intervals, is shown below. Find the DIFFERENCE between T_6, the trapezoidal approximation of the area of the pool and M_3, the midpoint approximation of the area of the pool.



$M_3 = 4(5+7+5) = 68$

A. 12

B. 10

C. 8

D. 5

E. 2.5

$T_6 = \frac{2}{2}\left(0 + 2(5) + 2(6) + 2(7) + 2(6) + 2(5) + 0\right) = 58$

9. $\int_{-1}^{2} \frac{1}{x}\, dx =$ $\cancel{\ln|x|\Big|_{-1}^{2}}$

$\cancel{= \ln 2 - \ln 1}$

$\cancel{= \ln 2}$

A. $\ln\left(\frac{1}{2}\right)$

B. $\ln(2)$

C. $\frac{3}{4}$

D. $\frac{5}{4}$

E. Diverges

$\int_{-1}^{2} \frac{1}{x}\, dx = \int_{-1}^{0} \frac{1}{x}\, dx + \int_{0}^{2} \frac{1}{x}\, dx$

$\int_0^2 \frac{1}{x}\, dx = \lim_{t\to 0^+} \int_t^2 \frac{1}{x}\, dx = \lim_{t\to 0^+} \left(\ln x \Big|_t^2\right) = \lim_{t\to 0^+} (\ln 2 - \ln t) = \infty$

10. $\int_{1}^{\infty} \frac{1}{(2x+2)^3}\, dx =$ $\lim_{t\to\infty} \int_1^t \frac{1}{(2x+2)^3}\, dx$

$= \lim_{t\to\infty} \left(\frac{-1}{4(2x+2)^2} \Big|_1^t \right)$

$= \lim_{t\to\infty} \left(\frac{-1}{4(2t+2)^2} - \frac{-1}{64} \right) = 0 + \frac{1}{64}$

A. $\frac{1}{16}$

B. $\frac{1}{128}$

C. $\frac{1}{32}$

D. $\frac{1}{64}$

E. Diverges

$u = 2x+2 \rightarrow du = 2\,dx \rightarrow dx = \frac{1}{2}du \rightarrow \int \frac{1}{(2x+2)^3}\, dx = \int u^{-3} \frac{1}{2}\, du = \frac{1}{2}\frac{u^{-2}}{-2} + C = \frac{-1}{4u^2} + C$

11. Find the length of the curve $y = 3 + 2x^{3/2}$, $1 \le x \le 2$.

arc length $= \int_a^b \sqrt{1 + (dy/dx)^2}\, dx$

$1 + \frac{dy}{dx} = 1 + (3x^{1/2})^2 = 1 + 9x$

$\int_1^2 \sqrt{1+9x}\, dx = \frac{1}{9}\cdot\frac{2}{3}(1+9x)^{3/2}\Big|_1^2$

$= \frac{2}{27}\left(19^{3/2} - 10^{3/2}\right)$

A. $\frac{2}{27}\left(19^{3/2} - 10^{3/2}\right)$

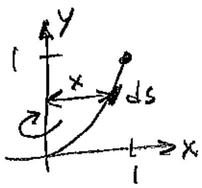
B. $\frac{2}{27}\left(21^{3/2} - 13^{3/2}\right)$

C. $\frac{1}{3}\left(21^{3/2} - 13^{3/2}\right)$

D. $4\sqrt{2} - 1$

E. $4\sqrt{2} + 1$

12. The curve $y = x^5$, $0 \le x \le 1$ is rotated about the y–axis. The surface area of the resulting surface of revolution is



$$ds = \sqrt{1 + (dy/dx)^2}\, dx = \sqrt{1 + (5x^4)^2}\, dx$$

$$\text{Surface area} = \int_0^1 2\pi x \sqrt{1 + 25x^8}\, dx$$

A. $\int_0^1 2\pi x\sqrt{1+x^{10}}\,dx$

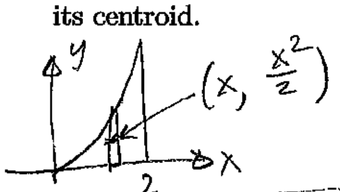
B. $\int_0^1 2\pi x^5\sqrt{1+x^{10}}\,dx$

C. $\int_0^1 2\pi x\sqrt{1+25x^8}\,dx$

D. $\int_0^1 2\pi x^5\sqrt{1+25x^8}\,dx$

E. $\int_0^1 2\pi x\sqrt{1+5x^4}\,dx$

13. A plane region is bounded by $y = x^2, y = 0$ and $x = 2$. Find the y–coordinate, $\overline{y}$, of its centroid.



$$m = \int_0^2 \rho x^2\, dx = \rho \frac{x^3}{3}\Big|_0^2 = \frac{8}{3}\rho.$$

$$M_x = M_{y=0} = \int_0^2 \rho \frac{x^2}{2} x^2\, dx = \int_0^2 \frac{\rho}{2} x^4\, dx = \frac{\rho}{10} x^5 \Big|_0^2 = \frac{32\rho}{10} = \frac{16\rho}{5}.$$

$$\overline{y} = \frac{M_x}{m} = \frac{\frac{16\rho}{5}}{\frac{8}{3}\rho} = \frac{6}{5}$$

A. $\overline{y} = \frac{4}{5}$

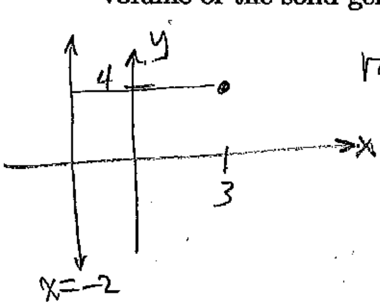
B. $\overline{y} = \frac{2}{5}$

C. $\overline{y} = \frac{7}{5}$

D. $\overline{y} = 1$

E. $\overline{y} = \frac{6}{5}$

14. A plane region in the first quadrant has centroid $(3, 4)$ and area 7 square units. The volume of the solid generated by revolving the region about the line $x = -2$ is



radius $= 3 - (-2) = 5$

Volume of solid of revolution

$$= 2\pi(5)(7) = 70\pi$$

A. 84π cubic units

B. 70π cubic units

C. 56π cubic units

D. 42π cubic units

E. 35π cubic units

Theorem of Pappus

15. Determine whether the sequence $a_n = \dfrac{n^2+1}{n^2}$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$$

A. Converges to 2
B. Converges to 1
C. Converges to 0
D. Converges to 1/2
E. Diverges

16. Determine whether the sequence $a_n = \sin(n/3)$ converges or diverges. If it converges, find the limit.

$\lim_{n\to\infty} \sin\left(\frac{n}{3}\right)$ does not exist.

$\therefore \left\{\sin\left(\frac{n}{3}\right)\right\}$ diverges

A. Converges to 0
B. Converges to 1
C. Converges to $\pi/3$
D. Converges to $\dfrac{\sqrt{3}}{2}$
E. Diverges.

17. Determine whether the sequence $a_n = \dfrac{2^{n-1}}{3^{n+2}}$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty} \frac{2^{n-1}}{3^{n+2}} = \lim_{n\to\infty} \frac{(2)^n(2)^{-1}}{(3)^n(3)^2}$$

$$= \lim_{n\to\infty} \frac{1}{18}\left(\frac{2}{3}\right)^n = 0$$

A. Converges to $\dfrac{2}{3}$
B. Converges to 3
C. Converges to $\dfrac{1}{54}$
D. Converges to 0
E. Diverges.