1. Determine which sequences converge.

I. $\left\{\dfrac{n^5}{5^n}\right\}$

II. $\left\{\dfrac{n}{(\ln n)^2}\right\}$

III. $\left\{\dfrac{\cos n}{n}\right\}$

A. I and II converge, III diverges.

B. I converges, II and III diverge.

C. I and III converge, II diverges.

D. III converges, I and II diverge.

E. II and III converge, I diverges.

2. Evaluate $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 3^n}{5^n}$

A. 31/6

B. 17/3

C. 13/6

D. 14/3

E. 25/6

$$= \sum_{n=0}^{\infty} \frac{2^n}{5^n} + \sum_{n=0}^{\infty} \frac{3^n}{5^n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n + \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n$$

$$= \frac{1}{1 - 2/5} + \frac{1}{1 - 3/5} = \frac{5}{3} + \frac{5}{2} = \frac{25}{6}$$

3. For a series $\sum_{n=1}^{\infty} a_n$ of positive terms, which statements are true.

I. If $\lim_{n\to\infty} n^2 a_n = L$, where $L \neq 0, \infty$, the series converges.

II. If $\lim_{n\to\infty} \frac{a_n}{e^n} = L$, where $L \neq 0, \infty$, the series converges.

III. If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$, where $L \neq 0, \infty$, the series converges.

IV. If $\lim_{n\to\infty} a_n = 0$, the series converges.

A. I.

B. I, II, III.

C. I, IV.

D. I, III.

E. I, II, III, IV.

4. Determine which series converge.

I. $\sum_{n=0}^{\infty} \frac{n^5}{5^n}$

II. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

III. $\sum_{n=1}^{\infty} \frac{\cos(\frac{1}{n})}{n}$

A. I and II converge, III diverges.

B. I converges, II and III diverge.

C. I and III converge, II diverges.

D. III converges, I and II diverge.

E. II and III converge, I diverges.

I CONVERGES BY RATIO TEST.

II DIVERGES BY INTEGRAL TEST.

III DIVERGES BY LIMIT COMPARISON WITH $\sum_{n=1}^{\infty} \frac{1}{n}$.

5. For what values of p does $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{(1+e^n)^p}$ converge.

A. $0 < p < 1$

B. $p > 1$

C. $p > 0$

D. $p < e$

E. $p \le 1$

THE SERIES

(1) DIVERGES IF $P \le 0$ SINCE

$$\lim_{m\to\infty} \frac{e^m}{(1+e^m)^P} \neq 0.$$

(2) DIVERGES IF $0 < P \le 1$
BY THE INTEGRAL TEST.

(3) CONVERGES IF $P > 1$
BY THE INTEGRAL TEST.

6. Evaluate $\displaystyle\lim_{n\to\infty} \frac{(3n^3+4n^2+1)^{2/3}}{3n^2+2}$.

A. 0

B. $1/3$

C. $1/\sqrt[3]{3}$

D. $\sqrt[3]{9}$

E. $+\infty$

$$= \lim_{m\to\infty} \frac{m^2\left(3+4/m+1/m^3\right)^{2/3}}{3m^2+2}$$

$$= \lim_{m\to\infty} \frac{\left(3+4/m+1/m^3\right)^{2/3}}{3+2/m^2}$$

$$= \frac{3^{2/3}}{3} = \frac{1}{3^{1/3}}$$

7. If $S = \sum_{n=1}^{\infty} (-1)^n \dfrac{n}{2^n + 1}$, find the smallest N such that we can be sure that $|S_N - S| < \frac{1}{10}$, where S_N is the Nth partial sum.

A. $N = 4$

B. $N = 5$

C. $N = 6$

D. $N = 7$

E. $N = 8$

$$|S_N - S| \leq \frac{N+1}{2^{N+1}+1}$$

WE WANT THE SMALLEST N SUCH THAT $\dfrac{N+1}{2^{N+1}+1} < \dfrac{1}{10}$

OR $\dfrac{2^{N+1}+1}{N+1} > 10$

$N=4 \quad \dfrac{33}{5} = 6\,3/5$

$N=5 \quad \dfrac{65}{6} = 10\,5/6$

8. For the series $\sum_{n=1}^{\infty} \dfrac{n2^{2n+1}}{3^n}$, let $L = \lim_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right|$. Which statement below is true?

A. $L = \dfrac{2}{3}$ and the series converges.

B. $L = \dfrac{2}{3}$ and the series diverges.

C. $L = \dfrac{4}{3}$ and the series converges.

D. $L = \dfrac{4}{3}$ and the series diverges.

E. $L = 1$ and the series converges.

$$\lim_{m\to\infty} \left|\frac{a_{m+1}}{a_m}\right|$$

$$= \lim_{m\to\infty} \frac{(m+1)\,2^{2(m+1)+1}}{3^{m+1}} \cdot \frac{3^m}{m\,2^{2m+1}}$$

$$= \frac{4}{3} \lim_{m\to\infty} \frac{m+1}{m} = \frac{4}{3}$$

9. Find the interval of convergence for $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n3^n}$.

A. $(-\frac{1}{3}, \frac{1}{3}]$

B. $(\frac{-1}{3}, \frac{1}{3})$

C. $(-3, 3]$

D. $[-3, 3)$

E. $[-3, 3]$

$$\lim_{n\to\infty} \frac{|x|^{n+1}}{(n+1)3^{n+1}} \frac{n\,3^n}{|x|^n}$$

$$= \frac{|x|}{3} \lim_{n\to\infty} \frac{n}{n+1} = \frac{|x|}{3}.$$ BY RATIO TEST, SERIES CONV. IF $|x| < 3$ AND DIV. IF $|x| > 3$. $R = 3$

$x = -3$. $\sum_{n=1}^{\infty} \frac{1}{n}$ DIV. (INTEGRAL TEST)

$x = 3$. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ CONV. (ALT. SER. TEST)

10. Find the power series representation of $f(x) = \frac{x}{3+4x}$ centered at 0.

A. $\sum_{n=0}^{\infty} (-1)^n \cdot \frac{4^n}{3^{n+1}} x^{n+1}$

B. $\sum_{n=0}^{\infty} (\frac{4}{3})^n x^{n+1}$

C. $\sum_{n=0}^{\infty} (-1)^n \cdot \frac{3^{n-1}}{4^n} x^{n+1}$

D. $\sum_{n=0}^{\infty} 3 \cdot 4^n x^{n+1}$

E. $\sum_{n=0}^{\infty} 3(-4)^n x^{n+1}$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$\frac{x}{3+4x} = \frac{x}{3} \frac{1}{1+(4x/3)}$$

$$= \frac{x}{3} \sum_{n=0}^{\infty} (-1)^n \left(\frac{4x}{3}\right)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{4^n x^{n+1}}{3^{n+1}}$$

11. Which statement about the series $\sum_{n=1}^{\infty} \frac{\sqrt{2n^2+4n}}{n^3+1}$ is true?

A. It diverges, by using the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

B. It converges, by using the Limit Comparison Test with $\sum \frac{1}{n^2}$.

C. It converges by the Ratio Test.

D. It diverges by using the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

E. It converges by the Alternating Series Test.

12. Find the power series representation of $\frac{d}{dx}\left(\frac{x}{1-2x^3}\right)$, centered at 0.

$$\frac{x}{1-2x^3} = x\left(\frac{1}{1-2x^3}\right)$$

$$= x \sum_{m=0}^{\infty} (2x^3)^m$$

$$= \sum_{m=0}^{\infty} 2^m x^{3m+1}$$

A. $\sum_{n=0}^{\infty} 2^{3n} 3n x^{3n-1}$

B. $\sum_{n=0}^{\infty} 2^n 3n x^{3n+1}$

C. $\sum_{n=0}^{\infty} 2^n 3n x^{3n-1}$

D. $\sum_{n=0}^{\infty} 6n x^{3n-1}$

E. $\sum_{n=0}^{\infty} 2^n (3n+1) x^{3n}$

$$\frac{d}{dx}\left(\frac{x}{1-2x^3}\right) = \sum_{m=0}^{\infty} 2^m (3m+1) x^{3m}$$