Exam 3, Math 162, Spring 2014 (Version 01)

1	2	3	4	5	6	7	8	9	10	11
T	F	T	T	T	F	F	T	F	F	F

#12. Ratio test: $\dfrac{(n+1)|x-3|^{n+1}}{2^{n+2}} \cdot \dfrac{2^{n+1}}{n|x-3|^n} = \dfrac{1}{2}\,\dfrac{n+1}{n}\,|x-3|$

$\xrightarrow[(n\to\infty)]{} \frac{1}{2}|x-3|$. Need $|x-3| < 2$ or $-2 < x-3 < 2$

or $1 < x < 5$. Test end points.

$x=1$: $\displaystyle\sum_{n=1}^{\infty} \frac{n(-1)^n 2^n}{2^{n+1}} = \frac{1}{2}\sum_{n=1}^{\infty} n(-1)^n$. Divergent.

$x=5$: $\displaystyle\sum_{n=1}^{\infty} \frac{n\,2^n}{2^{n+1}} = \frac{1}{2}\sum_{n=1}^{\infty} n$. Divergent.

Interval of Convergence: $(1, 5)$. $\boxed{D}$

#13. Differentiate term by term.

$f^{(4)}(0) = 3 \cdot 3 \cdot 2 \cdot 1 = 18$. Ans $\boxed{D}$

#14. $S = \sum_{n=1}^{\infty} \frac{n}{2^n}$, $b_n = \frac{n}{2^n}$.

$S_n = \sum_{k=1}^{n} \frac{k}{2^k}$.

$|S - S_n| \leq b_{n+1}$ Need $\frac{(n+1)}{2^{n+1}} < \frac{1}{10}$

$n=4$ gives: $\frac{5}{32}$ Not $< \frac{1}{10}$, $n=4$ does not work

$n=5$ gives: $\frac{6}{64} < \frac{1}{10}$. So, $n=5$ works.

$\boxed{B}$

#15. ~~First~~ $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$. $\boxed{f(0) = 1.}$

$f'(x) = \frac{3}{2}(1+x)^{1/2}$, $f''(x) = \frac{1}{2} \cdot \frac{3}{2}(1+x)^{-1/2} = \frac{3}{4}(1+x)^{-1/2}$.

$f'''(x) = \frac{3}{4}\left(-\frac{1}{2}\right)(1+x)^{-3/2}$. $\boxed{f'(0) = \frac{3}{2}}$, $\boxed{f''(0) = \frac{3}{4}}$, $\boxed{f'''(0) = \frac{-3}{8}}$

three terms

$= 1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3$. $\boxed{\text{Ans} \quad B}$

#16 $\dfrac{x}{1+3x} = x \sum_{n=0}^{\infty} (-3x)^n = x \sum_{n=0}^{\infty} (-1)^n 3^n x^n$

$= \sum_{n=0}^{\infty} (-1)^n 3^n x^{n+1} = x - 3x^2 + 9x^3 + \cdots$

$c_3 = 9$. $\boxed{\text{Ans C}}$

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#17. Ratio test gives convergence for $|x| < 1$.

Test $x = 1$ and $x = -1$ separately.

$x = 1$: $\sum_{n=1}^{\infty} \dfrac{1}{n(n+1)}$ Converges by comparison test with $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$.

$x = -1$: $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n(n+1)}$. Converges, Alternating Series

$\boxed{\text{Ans A}}$

#18. $\dfrac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$

Differentiate:

$\dfrac{1}{(1+x)^2} = -\left[-1 + 2x - 3x^2 + 4x^3 - 5x^4 + \cdots\right] = 1 - 2x + 3x^2 - 4x^3 \cdots$

$\boxed{\text{Ans. E.}}$