1. Which statement is true about these two series?

$$\sum_{n=1}^{\infty} \frac{(-1.5)^n}{n^3} \quad \text{and} \quad \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

A. Both are conditionally convergent.

B. One is divergent and one is absolutely convergent.

C. Both are absolutely convergent.

D. One is absolutely convergent and one is conditionally convergent.

E. $\boxed{\text{One is conditionally convergent and one is is divergent.}}$

RATIO

$$\lim_{n\to\infty}\left|\frac{\frac{(-1.5)^{n+1}}{(n+1)^3}}{\frac{(-1.5)^n}{n^3}}\right| = \lim_{n\to\infty} 1.5\left(\frac{n}{n+1}\right)^3 = 1.5 > 1$$

DIVERGENT

$$\frac{1}{\ln(n+1)} < \frac{1}{\ln(n)}$$

and

$$\lim_{n\to\infty}\frac{1}{\ln n} = 0$$

CONVERGES BY A.S.T.

$$n > \ln n$$

$$\Downarrow$$

$$\frac{1}{n} < \frac{1}{\ln n}$$

and

$\sum \frac{1}{n}$ diverges.

By direct comparison,

$\sum \frac{1}{\ln n}$ DIVERGES

2. Suppose $b_n > 0$ for each n, and

RATIO TEST: $$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}b_{n+1}}{(-1)^n b_n}\right| = \lim_{n\to\infty}\frac{b_{n+1}}{b_n} = \frac{1}{3}. < 1$$

We know that the series $\sum_{n=0}^{\infty}(-1)^n b_n$

A. $\boxed{\text{must be absolutely convergent.}}$

B. must be divergent.

C. must be conditionally convergent.

D. converges or diverges; the given information is inconclusive.

E. converges; either conditional or absolute convergence can occur.

3. How many of these series converge?

$$\sum_{k=1}^{\infty} \frac{k!}{(k+2)!}, \quad \sum_{\ell=0}^{\infty} \frac{1}{\ell+2^{\ell}}, \quad \sum_{m=1}^{\infty} \frac{m!}{m^m}, \quad \sum_{n=1}^{\infty} \frac{n^3 3^n}{2^{2n}}$$

A. only two
B. $\boxed{\text{all four}}$
C. only one
D. only three
E. none

$$\frac{1}{\ell+2^{\ell}} < \left(\frac{1}{2}\right)^{\ell}$$

$\sum \left(\frac{1}{2}\right)^{\ell}$ converges $[|r| = \frac{1}{2} < 1]$

$\Rightarrow \sum \frac{1}{\ell+2^{\ell}}$ converges

$$\frac{n^3 3^n}{2^{2n}} = n^3 \left(\frac{3}{4}\right)^n.$$

Root/Ratio Test gives $3/4 < 1$

Converges

$$\frac{k!}{(k+2)!} = \frac{1}{(k+1)(k+2)} < \frac{1}{k^2}$$

$\sum \frac{1}{k^2}$ converges $[p = 2 > 1]$

$\Rightarrow \sum \frac{k!}{(k+2)!}$ converges

$$\frac{m!}{m^m} = \frac{m(m-1)(m-2)\dots(2)(1)}{m(m)(m)\dots(m)(m)} < \frac{2}{m^2}.$$

$\sum \frac{2}{m^2}$ converges $[p = 2 > 1]$

OR

Ratio:

$$\frac{(m+1)!}{(m+1)^{m+1}} \frac{m^m}{m!} = \frac{(m+1)m^m}{(m+1)^{m+1}} = \left(\frac{m}{m+1}\right)^m = \left(1+\frac{1}{m}\right)^{-m} = e^{-1} < 1$$

4. According to the Alternating Series Estimation Theorem, for which n is the following true?

$$\left| \ln 2 - \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots + \frac{(-1)^{n+1}}{n}\right) \right| \le 0.01$$

Hint: $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ with interval of convergence $(-1, 1]$

A. $n \ge 10$
B. $n \ge 27$
C. $\boxed{n \ge 99}$
D. $n \ge 11$
E. $n \ge 9$

$$\frac{1}{n+1} \le 0.01$$

$$\frac{1}{n+1} \le \frac{1}{100}$$

$$100 \le n+1$$

$$99 \le n$$

5. Which of these x values is in the interval of convergence of the following power series?

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x+2)^n}{3n+1}$$

Center: $a=-2$

A. $\boxed{x=-1}$

B. $x=0$

C. $x=1$

D. $x=-3$

E. $x=2$

Ratio: $\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x+2)^{n+1}}{3(n+1)+1}\cdot\frac{3n+1}{(-1)^n(x+2)^n}\right| = \lim_{n\to\infty}\frac{3n+1}{3n+4}|x+2| = |x+2| < 1$

radius of convergence $=1$

Endpoints: $x=-3$: $\sum(-1)^n\frac{(-3+2)^n}{3n+1} = \sum\frac{1}{3n+1}$ diverges

$x=-1$: $\sum(-1)^n\frac{(-1+2)^n}{3n+1} = \sum\frac{(-1)^n}{3n+1}$ Converges (A.S.T.)

Interval of convergence: $(-3,-1]$

6. Find the Maclaurin series for

$$f(x)=\frac{3x^2}{(1+x^3)^2}$$

Hint: f is the derivative of a function with an easy Maclaurin formula

A. $\sum_{n=2}^{\infty}(-1)^n 3x^{6n}$

B. $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{3n+1}}{3n+1}$

C. $\sum_{n=1}^{\infty}(-1)^n n x^{3n-3}$

D. $\boxed{\sum_{n=1}^{\infty}(-1)^{n+1}3nx^{3n-1}}$

E. $\sum_{n=0}^{\infty}(-1)^{n+1}x^{3n}$

$$f(x)=\frac{\partial}{\partial x}\left[-\frac{1}{1+x^3}\right]$$

$$=\frac{\partial}{\partial x}\left[-\sum_{n=0}^{\infty}(-x^3)^n\right]$$

$$=\frac{\partial}{\partial x}\sum_{n=0}^{\infty}(-1)^{n+1}x^{3n}$$

$$=\sum_{n=1}^{\infty}(-1)^{n+1}3n\,x^{3n-1}$$

7. Find the first few terms of the Taylor series centered at $a = \frac{1}{4}$ for the function $f(x) = \sqrt{x}$

A. $\frac{1}{2} + \frac{1}{2}\left(x - \frac{1}{4}\right) - \frac{1}{4}\left(x - \frac{1}{4}\right)^2 + \frac{3}{8}\left(x - \frac{1}{4}\right)^3 - \ldots$

B. $\frac{1}{2} + \frac{1}{2}\left(x - \frac{1}{4}\right) - \frac{1}{8}\left(x - \frac{1}{4}\right)^2 + \frac{1}{16}\left(x - \frac{1}{4}\right)^3 - \ldots$

C. $\frac{1}{2} + \left(x - \frac{1}{4}\right) - 2\left(x - \frac{1}{4}\right)^2 + 12\left(x - \frac{1}{4}\right)^3 - \ldots$

D. $\frac{1}{2} + \left(x - \frac{1}{4}\right) - \frac{1}{2}\left(x - \frac{1}{4}\right)^2 + \frac{1}{6}\left(x - \frac{1}{4}\right)^3 - \ldots$

E. $\boxed{\frac{1}{2} + \left(x - \frac{1}{4}\right) - \left(x - \frac{1}{4}\right)^2 + 2\left(x - \frac{1}{4}\right)^3 - \ldots}$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f''(x) = -\frac{1}{4} x^{-3/2}$

$f'''(x) = \frac{3}{8} x^{-5/2}$

At $x = 1/4$

$1/2$

$\frac{1}{2\left(\frac{1}{2}\right)} = 1$

$-\frac{1}{4\left(\frac{1}{2}\right)^3} = -2$

$\frac{3}{8\left(\frac{1}{2}\right)^5} = 12$

$\frac{1}{2} + \frac{1}{1!}\left(x - \frac{1}{4}\right) - \frac{2}{2!}\left(x - \frac{1}{4}\right)^2 + \frac{12}{3!}\left(x - \frac{1}{4}\right)^3 - \ldots$

8. Use a Maclaurin series to choose the best estimate of

$$\int_0^{0.1} \sin(x^2)\, dx$$

Hint: $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ and $\boxed{\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}}$ for all x

A. $\boxed{\frac{(0.1)^3}{3} - \frac{(0.1)^7}{42}}$

B. $\frac{(0.1)^4}{2} - \frac{(0.1)^8}{24}$

C. $\frac{(0.1)^2}{2} - \frac{(0.1)^4}{24}$

D. $(0.1)^2 - \frac{(0.1)^6}{6}$

E. $(0.1)^3 - \frac{(0.1)^5}{6}$

$$\sin(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}$$

$$\int_0^{0.1} \sin(x^2)\,dx = \int_0^{0.1} \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}\,dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{(4n+3)(2n+1)!}\Bigg]_{x=0}^{x=0.1}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(0.1)^{4n+3}}{(4n+3)(2n+1)!}$$

Use first two terms

9. Find the Maclaurin series for $f(x) = \dfrac{1}{\sqrt{1+x}}$. $= (1+x)^{-1/2} = \sum_{n=0}^{\infty} \binom{-1/2}{n} x^n$

A. $1 - \frac{1}{2}x + \sum_{n=2}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n n!} x^n$

B. $\sum_{n=0}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n+1)}{2^n n!} x^n$

C. $1 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n-1)(2n-3)}{2^n} x^n$

D. $\sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)}{2^n} x^n$

E. $\boxed{1 + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n n!} x^n}$

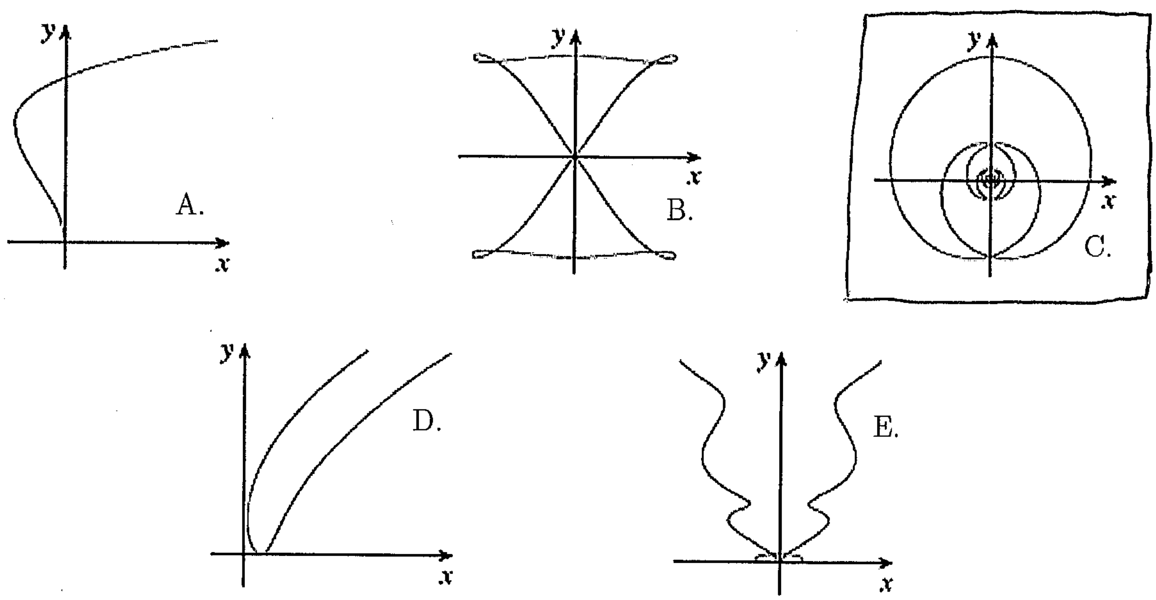
$$\binom{-1/2}{n} = \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})\cdots(-\frac{1}{2}-n+1)}{n!}$$

$$= \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})\cdots(-(\frac{2n-1}{2}))}{n!}$$

$$= \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n n!}$$

10. Match the parametric equations with the correct graph. (x,y) is in all four quadrants

$$x = \frac{\sin 2t}{4+t^2}, \quad y = \frac{\cos 2t}{4+t^2} \qquad \lim_{t\to\pm\infty}(x,y) = (0,0)$$



11. Consider the parametric equations

$$x = 2t^3 + 3t^2 - 12t, \quad y = 2t^3 + 3t^2 + 2.$$

Find a point (x, y) where the tangent is vertical.

A. $(52, -2)$
B. $\boxed{(-7, 7)}$
C. $(13, 3)$
D. $(4, 30)$
E. $(0, 2)$

$$\frac{dx}{dt} = 6t^2 + 6t - 12 = 6(t^2 + t - 2) = 6(t+2)(t-1)$$

$t = 1$ OR $t = -2$

$$\frac{dy}{dt} = 6t^2 + 6t = 6t(t+1)$$

$$\left.\frac{dy}{dt}\right|_{t=1} = 12 \neq 0.$$

At $t=1$:
$x = 2+3-12 = -7$
$y = 2+3+2 = 7.$

$\left.\frac{dy}{dx}\right|_{t=1}$ is infinite (undefined)

$$\lim_{t \to 1^{+/-}} \frac{dy}{dx} = \text{"}\frac{12}{0^{+/-}}\text{"} = \pm\infty$$

12. A curve C is defined by the parametric equations $x = t^3 - 12t$, $y = t^2 - 1$. Find all the values of t for which C is concave down.

A. $\boxed{t < -2 \text{ and } t > 2}$
B. $t < -2$
C. all real t
D. $-2 < t < 2$
E. $t > 2$

$$\frac{dx}{dt} = 3t^2 - 12 \qquad \frac{dy}{dt} = 2t$$

$$\frac{dy}{dx} = \frac{2t}{3t^2 - 12}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{2t}{3t^2-12}\right)}{3t^2-12}$$

$$= \frac{(3t^2-12)(2) - (2t)(6t)}{(3t^2-12)^3} = \frac{-6(t^2+4)}{(3t^2-12)^3}$$

NUMERATOR IS ALWAYS NEGATIVE.
WHEN IS DENOMINATOR POSITIVE?

$3t^2 - 12 > 0$
$3t^2 > 12$
$t^2 > 4$