NAME GRADING KEY

STUDENT ID ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(6) 1. Find the domain of the function $g(x) = \sqrt[4]{x^2 - 6x}$.

$x^2 - 6x \geq 0$ ②

$x(x-6) \geq 0$

$x(x-6)$ + + + + 0 − − − 0 + + +

0 6

−1pt if one or both end points are omitted

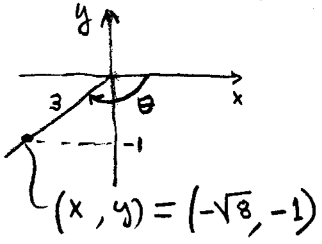
② ②

$(-\infty, 0] \cup [6, \infty)$ $\boxed{6}$

or $-\infty < x \leq 0$ and $6 \leq x < \infty$

(5) 2. Find $\tan\theta$ if $\sin\theta = -\frac{1}{3}$ and $-\pi < \theta < -\frac{\pi}{2}$.

Since $-\pi < \theta < -\frac{\pi}{2}$, the terminal side of θ is in the third quadrant, where both x and y are negative.



$\sin\theta = \frac{y}{r} = -\frac{1}{3}$: Let $y = -1$, $r = 3$ $\rightarrow$ $x = -\sqrt{3^2 - 1} = -\sqrt{8}$

$\tan\theta = \frac{y}{x} = \frac{-1}{-\sqrt{8}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$

$\tan\theta = \frac{1}{2\sqrt{2}}$ or $\frac{1}{\sqrt{8}}$ $\boxed{5}$

−2pts for wrong sign

(8) 3. If $f(x) = \ln x$ and $g(x) = x^2 + 1$, find the composite functions $f \circ g$ and $g \circ f$ and their domains.

$(f \circ g)(x) = \ln(x^2+1)$ ③ , domain : $(-\infty, \infty)$ ① $\boxed{4}$

$(g \circ f)(x) = (\ln x)^2 + 1$ ③ , domain : $(0, \infty)$ ① $\boxed{4}$

Name: ______________________________

(7) 4. Solve each equation for x.

(a) $e^{x-1} = 2$

$\ln e^{x-1} = \ln 2 \longrightarrow x - 1 = \ln 2$

$x = 1 + \ln 2$

$\boxed{x = 1 + \ln 2}$ $\boxed{3}$

(b) $\ln \ln x = 2$

$e^{\ln \ln x} = e^2 \longrightarrow \ln x = e^2$ ②

$e^{\ln x} = e^{e^2} \rightarrow x = e^{e^2}$

$\boxed{x = e^{e^2}}$ $\boxed{4}$

(8) 5. Write the equation of the graph that results by

(a) shifting the graph of $y = \ln x$ three units upward.

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$\boxed{y = \ln x + 3}$ $\boxed{2}$

(b) reflecting the graph of $y = 1 + \ln x$ about the x-axis.

$\boxed{y = -1 - \ln x}$ $\boxed{2}$

(c) stretching the graph of $y = \sin x$ vertically by a factor of 3.

$\boxed{y = 3 \sin x}$ $\boxed{2}$

(d) reflecting the graph of $y = 3 \ln x$ about the x-axis and then about the y-axis.

$\boxed{y = -3 \ln(-x)}$ $\boxed{2}$

(5) 6. True or False. (Circle T or F)

(a) The function $f(x) = |x|$ is continuous at $x = 0$. [✓ T] F

(b) The function $f(x) = |x|$ is differentiable at $x = 0$. T [✓ F]

(c) The function $f(x) = |x|$ is differentiable at $x = 1$. [✓ T] F

(d) The function $g(x) = \frac{|x|}{x}$ is continuous at $x = 0$. T [✓ F]

(e) The function $h(x) = \ln(x - 1)$ is continuous at $x = 0$. T [✓ F]

$\boxed{5}$

(7) 7. The function $f(x) = \begin{cases} \frac{x^2-4}{x+2} & \text{if } x \neq -2 \\ c & \text{if } x = -2 \end{cases}$ is continuous at $x = -2$ if $c =$

f is continuous at $x = -2$ if $\lim_{x \to -2} f(x) = f(-2)$

$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{x^2-4}{x+2} = \lim_{x \to -2} \frac{(x+2)(x-2)}{x+2} = \lim_{x \to -2} (x-2) = -4$ ④

$f(-2) = c$

$-4 = c$

③

$\boxed{c = -4}$ $\boxed{7}$

(8) 8. Prove that $\lim_{x\to 0} x^4 \sin\frac{1}{x} = 0$. Name the theorem you are using.

$-1 \leq \sin\frac{1}{x} \leq 1$ ②

$-x^4 \leq x^4 \sin\frac{1}{x} \leq x^4$ ②

$\lim_{x\to 0}(-x^4) = 0$ ① $\lim_{x\to 0} x^4 = 0$ ① $\therefore \lim_{x\to 0} x^4 \sin\frac{1}{x} = 0$ by the Squeeze Theorem ②

(18) 9. For each of the following, fill in the boxes with a finite number or one of the symbols ∞, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

(a) $\lim_{x\to\infty} \frac{x^7-1}{x^6+1} = \lim_{x\to\infty} \frac{x - \frac{1}{x^6}}{1 + \frac{1}{x^6}} = \infty$ NPC

$\boxed{\infty}$ $\boxed{3}$

(b) $\lim_{t\to(-5)^-} \frac{1}{t+5} = -\infty$

as $t \to -5$ $t+5 \to 0$

$t < -5 \to t+5 < 0$

$\boxed{-\infty}$ $\boxed{3}$

(c) $\lim_{\theta\to 0} \frac{\sin^2\theta}{\theta} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \sin\theta$

$= \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \lim_{\theta\to 0} \sin\theta = 1\cdot 0 = 0$

$\boxed{0}$ $\boxed{3}$

(d) $\lim_{x\to 0} \sin\frac{1}{x}$

as $x \to 0^+$ $\frac{1}{x} \to \infty$

and $\sin\frac{1}{x}$ oscilates between -1 and 1 infinitely many times

$\boxed{\text{DNE}}$ $\boxed{3}$

(e) $\lim_{t\to 0} \frac{\sqrt{t+4}-2}{t} = \lim_{t\to 0} \frac{\sqrt{t+4}-2}{t} \cdot \frac{\sqrt{t+4}+2}{\sqrt{t+4}+2}$

$= \lim_{t\to 0} \frac{t+4-4}{t(\sqrt{t+4}+2)} = \lim_{t\to 0} \frac{1}{\sqrt{t+4}+2} =$

$\boxed{\frac{1}{4}}$ $\boxed{3}$

(f) $\lim_{\theta\to\frac{\pi}{6}} \frac{\sin\theta}{1+\cos\theta} =$

$= \frac{\lim_{\theta\to\frac{\pi}{6}} \sin\theta}{\lim_{\theta\to\frac{\pi}{6}} (1+\cos\theta)} = \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$ ← or → $=$ $\boxed{\frac{1}{2+\sqrt{3}}}$ $\boxed{3}$

(You must give the exact values of the trigonometric functions where necessary).

Name: ______________________ [illegible] $h \to 0$ [illegible] it shouldn't

(10) 10. Find the derivative of $f(x) = \dfrac{1}{\sqrt{x}}$ using the definition of the derivative:

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$. (0 credit for using a formula for the derivative).

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \text{ ④} = \lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \text{ ①}$$

$$= \lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \, \frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}} \text{ ②} = \lim_{h\to 0} \frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\,(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}\,(\sqrt{x}+\sqrt{x+h})} \text{ ②} = -\frac{1}{2x\sqrt{x}} \text{ ①}$$

(6) 11. For what values of x does the graph of $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent?

$f'(x) = 6x^2 - 6x - 6$ ②

$f'(x) = 0 : \quad 6x^2 - 6x - 6 = 0$ ②

$x^2 - x - 1 = 0$

$x = \dfrac{1 \pm \sqrt{1-4(-1)}}{2} = \dfrac{1 \pm \sqrt{5}}{2}$ or

① ①

$\boxed{x = \frac{1+\sqrt{5}}{2}, \ \frac{1-\sqrt{5}}{2}}$ $\boxed{6}$

(12) 12. Find the derivatives of the following functions. (It is not necessary to simplify).

(a) $f(x) = \dfrac{1}{x\sqrt{x}} = x^{-\frac{3}{2}}$

$f'(x) = -\frac{3}{2} x^{-\frac{5}{2}}$

$\boxed{-\frac{3}{2} x^{-\frac{5}{2}}}$ $\boxed{3}$

(b) $y = \cos x - 2\tan x$

$\dfrac{dy}{dx} = -\sin x - 2\sec^2 x$

$\boxed{-\sin x - 2\sec^2 x}$ $\boxed{3}$

(c) $g(t) = e^t \sec t$

$g'(t) = e^t \sec t \tan t + e^t \sec t$

$\boxed{e^t \sec t \tan t + e^t \sec t}$ $\boxed{3}$

(d) $y = \dfrac{x^2 \sin x}{e^x + 1}$

$\boxed{\dfrac{(e^x+1)(x^2\cos x + 2x\sin x) - x^2 \sin x\, e^x}{(e^x+1)^2}}$ $\boxed{3}$

↗

NPC except: -1 pt if initial answer is correct and mistake occurs in simplifying or copying.