NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

Page 1	/14
Page 2	/27
Page 3	/27
Page 4	/32
TOTAL	/100

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

---

(6) 1. Prove the identity $(\sin x + \cos x)^2 = 1 + \sin 2x$.

$$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x \quad 2$$

$$= (\sin^2 x + \cos^2 x) + 2\sin x\cos x$$

$$= \overset{2}{1} + \sin 2x \quad 2$$

$\boxed{6}$

(8) 2. If $f(x) = \dfrac{1}{x+7}$ and $g(x) = x^3 - 8$ find the composite functions $f \circ g$ and $g \circ f$ and give their domains.

$$(f\circ g)(x) = f(g(x)) = f(x^3-8) = \frac{1}{(x^3-8)+7} = \frac{1}{x^3-1}$$

$$(g\circ f)(x) = g(f(x)) = g\left(\frac{1}{x+7}\right)$$

$$= \left(\frac{1}{x+7}\right)^3 - 8$$

3 $\boxed{(f \circ g)(x) = \dfrac{1}{x^3-1}}$

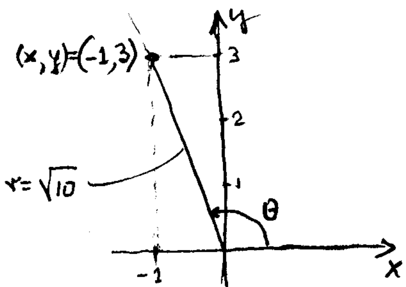
1 $\boxed{\text{Domain of } (f \circ g) \quad (-\infty,1)\cup(1,\infty) \text{ or all } x\neq 1}$

3 $\boxed{(g \circ f)(x) = \dfrac{1}{(x+7)^3} - 8}$

1 $\boxed{\text{Domain of } (g \circ f) \quad (-\infty,-7)\cup(-7,\infty) \text{ or all } x\neq -7}$

$\boxed{8}$

(5) 3. If $\tan\theta = -3$ and $\frac{\pi}{2} < \theta < \pi$, find the following:



$\tan\theta = \frac{y}{x} = -3 = \frac{3}{-1}$

let $x=-1$, $y=3$

$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{10}}$

$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{10}}$

$\sec\theta = \frac{1}{\cos\theta} = -\sqrt{10}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{10}}{3}$

$\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{3}$

$\sin\theta = \frac{3}{\sqrt{10}}$	1 pt each NPC
$\cos\theta = -\frac{1}{\sqrt{10}}$	
$\sec\theta = -\sqrt{10}$	
$\csc\theta = \frac{\sqrt{10}}{3}$	
$\cot\theta = -\frac{1}{3}$	

5

(8) 4. Find a formula for the inverse of the function $f(x) = 5 - 4x^3$.

$x = f^{-1}(y) \iff y = f(x)$

$y = f(x) \rightarrow y = 5 - 4x^3$ 2 $\rightarrow x^3 = \frac{5-y}{4}$

$\rightarrow x = \sqrt[3]{\frac{5-y}{4}}$ 4

$\therefore f^{-1}(y) = \sqrt[3]{\frac{5-y}{4}}$ and $f^{-1}(x) = \sqrt[3]{\frac{5-x}{4}}$

$\boxed{f^{-1}(x) = \sqrt[3]{\frac{5-x}{4}}}$ 2

8

(9) 5. Find the equations of the vertical and horizontal asymptotes of the function $y = \frac{x}{x^2 - x - 2}$.

$f(x) = \frac{x}{x^2-x-2} = \frac{x}{(x+1)(x-2)}$

$\lim_{x \to (-1)^+} \frac{x}{(x+1)(x-2)} = \infty$ $\therefore$ $x=-1$ is a ver. as.

$\lim_{x \to 2^+} \frac{x}{(x+1)(x-2)} = \infty$ $\therefore$ $x=2$ is a ver. as.

$\lim_{x \to \infty} \frac{x}{x^2-x-2} = 0$ $\therefore$ $y=0$ is a hor. as.

$\lim_{x \to -\infty} \frac{x}{x^2-x-2} = 0$ $\therefore$ $y=0$ is a hor. as.

Vertical asymptotes
$x=-1$ 3
$x=2$ 3

Horizontal asymptotes
$y=0$ 3

9

(5) 6. (a)Complete the definition: The function f is continuous at a if $\lim_{x \to a} f(x) = f(a)$ 3

(b) Use (a) to explain why the function $f(x) = \begin{cases} \frac{1}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ is discontinuous at 1.

$\lim_{x \to 1} f(x)$ does not exist 2

In fact $\lim_{x \to 1^+} f(x) = \infty$

5

(18) 7. For each of the following, fill in the boxes below with a finite number, or one of the symbols $+\infty, -\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

3pts each NPC

(a) $\lim_{x\to 0^-} \frac{3x}{|x|} = \lim_{x\to 0^-} \frac{3x}{-x} = \lim_{x\to 0^-} (-3) = -3$

$\boxed{-3}$

(b) $\lim_{x\to(-\frac{\pi}{2})^-} \sec x = \lim_{x\to(-\frac{\pi}{2})^-} \frac{1}{\cos x} = -\infty$

$\boxed{-\infty}$

(c) $\lim_{x\to\infty} \frac{2+\sqrt{x}}{2-\sqrt{x}} = \lim_{x\to\infty} \frac{\frac{2}{\sqrt{x}}+1}{\frac{2}{\sqrt{x}}-1} = \frac{1}{-1} = -1$

$\boxed{-1}$

(d) $\lim_{x\to-\infty} (x^3 - 5x^2) = \lim_{x\to-\infty} x^3\left(1-\frac{5}{x}\right) = -\infty$

$\boxed{-\infty}$

(e) $\lim_{x\to 0} \frac{\tan x}{4x} = \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{4x} = \lim_{x\to 0} \frac{1}{4}\frac{\sin x}{x}\cdot\frac{1}{\cos x}$

$= \frac{1}{4}\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x} = \frac{1}{4}\cdot 1\cdot 1$

$= \frac{1}{4}$

$\boxed{\frac{1}{4}}$

(f) $\lim_{x\to 0} \ln\left(\frac{e^x+2}{3}\right) = \ln\left(\lim_{x\to 0}\frac{e^x+2}{3}\right) =$

$= \ln 1 = 0$

$\boxed{0}$

$\boxed{18}$

(9) 8. Evaluate the following:

3 pts each NPC

(a) $\cos(\pi \ln e^{1/4}) = \cos\left(\pi\,\frac{1}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\boxed{\frac{1}{\sqrt{2}}}$

(b) $\tan(\pi e^{-\ln 4}) = \tan\left(\pi \frac{1}{e^{\ln 4}}\right) = \tan\frac{\pi}{4} = 1$

$\boxed{1}$

(c) $\sin\left(\frac{\pi}{\ln e^{-2}}\right) = \sin\left(\frac{\pi}{-2}\right) = \sin\left(-\frac{\pi}{2}\right) = -1$

$\boxed{-1}$

$\boxed{9}$

(10) 9. Find the derivative of the function $f(x) = \sqrt{1+2x}$ using the definition of the derivative: $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$. (0 credit for using a formula for the derivative).

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{1+2(x+h)}-\sqrt{1+2x}}{h} \quad 4$$

$$= \lim_{h\to 0} \frac{\sqrt{1+2(x+h)}-\sqrt{1+2x}}{h} \cdot \frac{\sqrt{1+2(x+h)}+\sqrt{1+2x}}{\sqrt{1+2(x+h)}+\sqrt{1+2x}} \; 2 = \lim \frac{1+2(x+h)-(1+2x)}{h\left[\sqrt{1+2(x+h)}+\sqrt{1+2x}\right]}$$

$$= \lim_{h\to 0} \frac{2h}{h\left[\sqrt{1+2(x+h)}+\sqrt{1+2x}\right]} = \lim_{h\to 0} \frac{2}{\sqrt{1+2(x+h)}+\sqrt{1+2x}} \; 2 = \frac{1}{\sqrt{1+2x}} \; 2$$

$\boxed{10}$

(6) 10. Find an equation of the tangent line to the curve $y = \dfrac{3}{x^2} - \dfrac{4}{x^3}$ at the point $(-1, 7)$.

$$y = 3x^{-2} - 4x^{-3}$$

$$\frac{dy}{dx} = -6x^{-3} + 12x^{-4} \; 2 \qquad \left.\frac{dy}{dx}\right|_{x=-1} = -6(-1)^{-3} + 12(-1)^{-4} = 6 + 12 = 18 \; 2$$

$$y - 7 = 18(x+1)$$

2

$\boxed{y-7 = 18(x+1)}$ $\boxed{6}$

(16) 11. Find the derivatives of the following functions. (It is not necessary to simplify).

(a) $f(x) = x^2\sqrt{x} + \dfrac{1}{x\sqrt{x}}$.

$$= x^2 x^{\frac{1}{2}} + x^{-1}x^{-\frac{1}{2}} = x^{5/2} + x^{-3/2}$$

$$f'(x) = \frac{5}{2}x^{3/2} - \frac{3}{2}x^{-5/2}$$

$\boxed{\frac{5}{2}x^{3/2} - \frac{3}{2}x^{-5/2}}$ $\boxed{4}$

(b) $y = e^x \cot x$.

$$\frac{dy}{dx} = e^x(-\csc^2 x) + e^x \cot x$$

$\boxed{-e^x\csc^2 x + e^x \cot x}$ $\boxed{4}$

(c) $f(x) = \dfrac{x^2 + \sin x}{1 + \cos x}$.

or

$$f'(x) = \frac{(1+\cos x)(2x+\cos x) - (x^2+\sin x)(-\sin x)}{(1+\cos x)^2}$$

$\boxed{\dfrac{(1+\cos x)(2x+\cos x) + (x^2+\sin x)(\sin x)}{(1+\cos x)^2}}$ $\boxed{4}$

(d) $g(t) = 4\sec t + \tan t$.

$$g'(t) = 4\sec t \tan t + \sec^2 t$$

$\boxed{4\sec t \tan t + \sec^2 t}$ $\boxed{4}$

3 pts each

NPC except: −1 pt if initial answer is correct and mistake occurs in simplifying or copying