NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(16) 1. Find the derivatives of the following functions. It is not necessary to simplify.

(a) $y = \sqrt[3]{1+x^3} = (1+x^3)^{1/3}$

NPC but -1 pt if first answer is correct and there is an error in simplifying

$$\frac{dy}{dx} = \frac{1}{3}(1+x^3)^{-\frac{2}{3}} 3x^2$$

or

$$\boxed{\frac{x^2}{(1+x^3)^{2/3}}}$$

4

(b) $f(x) = \tan^{-1}(\cos^2 x)$

$$f'(x) = \frac{1}{1+\cos^4 x} 2(\cos x)(-\sin x)$$

or

$$\boxed{\frac{-2\sin x\cos x}{1+\cos^4 x}}$$

4

(c) $y = \sqrt{x}\ln x$

$$\frac{dy}{dx} = \sqrt{x}\,\frac{1}{x} + \frac{1}{2\sqrt{x}}\ln x$$

or

$$\boxed{\frac{2+\ln x}{2\sqrt{x}}}$$

4

(d) $F(x) = \sin^{-1} e^{3x}$.

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(8) 2. Find all points (x, y), with $0 \le x \le 2\pi$, on the graph of the function $f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

$f'(x) = 2\cos x + 2\sin x \cos x$

$f'(x) = 0$: $2\cos x(1+\sin x) = 0$ (2)

$\cos x = 0 \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = -1 \rightarrow x = \frac{3\pi}{2}$

$x = \frac{\pi}{2} \rightarrow y = 2+1 = 3$, $x = \frac{3\pi}{2} \rightarrow y = -2+1 = -1$

-1 pt for each additional wrong value of x!

(2) (1) (2) (1)

$(x, y) = \boxed{(\frac{\pi}{2}, 3)\ (\frac{3\pi}{2}, -1)}$ $\boxed{8}$

(9) 3. If $\dfrac{y}{x-y} = x^2 + 1$, find $\dfrac{dy}{dx}$ by implicit differentiation.

$$\frac{(x-y)\frac{dy}{dx} - y\left(1 - \frac{dy}{dx}\right)}{(x-y)^2} = 2x \quad (5)$$

$$(x-y)\frac{dy}{dx} - y + y\frac{dy}{dx} = 2x(x-y)^2$$

$$x\frac{dy}{dx} = 2x(x-y)^2 + y$$

(4)

$$\frac{dy}{dx} = \boxed{\frac{2x(x-y)^2 + y}{x}} \quad \boxed{9}$$

or $y = x^3 + x - x^2 y - y$

$\frac{dy}{dx} = 3x^2 + 1 - x^2\frac{dy}{dx} - 2xy - \frac{dy}{dx}$ (5)

$\frac{dy}{dx} = \frac{3x^2 - 2xy + 1}{2 + x^2}$ (4)

(9) 4. Evaluate each expression:

NPC

(a) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = y \iff \sin y = -\frac{\sqrt{2}}{2}, -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = -\frac{\pi}{4}$

$\boxed{-\frac{\pi}{4}}$ (3)

(b) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = y \iff \tan y = \frac{\sqrt{3}}{3}, -\frac{\pi}{2} < y < \frac{\pi}{2}$

$y = \frac{\pi}{6}$

$\boxed{\frac{\pi}{6}}$ (3)

(c) $\sin\left(\cos^{-1}\frac{4}{5}\right)$ $y = \cos^{-1}\frac{4}{5} \iff \cos y = \frac{4}{5}, 0 \le y \le \pi$

$\sin y = +\sqrt{1-\cos^2 y} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

because $0 \le y \le \pi$

$\boxed{\frac{3}{5}}$ (3)

(6) 5. Find the second derivative of the function $y = \sqrt{1+x^3}$. Do not simplify.

$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^3}} 3x^2$ (2)

$= \frac{3}{2} \frac{x^2}{\ldots}$

or $y = (1+x^3)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(1+x^3)^{-\frac{1}{2}} 3x^2$

$= \frac{3}{2}(1+x^3)^{-\frac{1}{2}} x^2$ (2)

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(10) 6. Find the derivative of the function $y = x^{1/x}$.

$$y = x^{1/x} = e^{\ln x^{1/x}} = e^{\frac{1}{x}\ln x} \quad (4)$$

$$\frac{dy}{dx} = e^{\frac{1}{x}\ln x}\,\frac{x\frac{1}{x} - \ln x}{x^2}$$

$$= e^{\frac{1}{x}\ln x}\,\frac{1-\ln x}{x^2} = x^{1/x}\,\frac{1-\ln x}{x^2}$$

or (6)

$$\boxed{x^{1/x}\,\frac{1-\ln x}{x^2}}$$

or $\ln y = \frac{1}{x}\ln x$ (4)

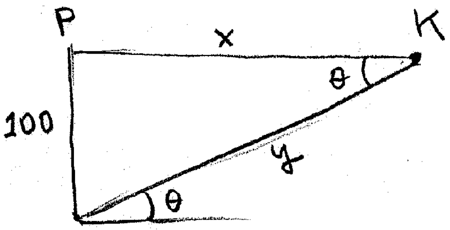
$$\frac{1}{y}\frac{dy}{dx} = \frac{x\frac{1}{x} - \ln x}{x^2}$$

$$\frac{dy}{dx} = y\,\frac{1-\ln x}{x^2} \quad (3)$$

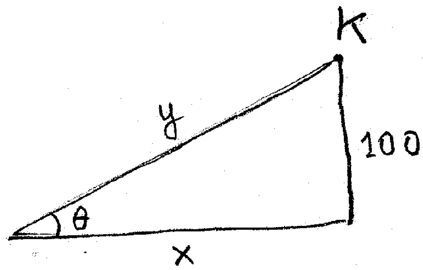
$$= x^{1/x}\,\frac{1-\ln x}{x^2}$$

$\boxed{10}$

(14) 7. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/sec. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string have been let out?



or



$$\frac{dx}{dt} = 8 \text{ ft/sec} \quad (1)$$

Find $\frac{d\theta}{dt}$ when $y = 200$ ft.

$$\cot\theta = \frac{x}{100} \quad (3)$$

$$-\csc^2\theta\,\frac{d\theta}{dt} = \frac{1}{100}\frac{dx}{dt} \quad (4)$$

When $y = 200$:

$$\sin\theta = \frac{100}{200} = \frac{1}{2}$$

$\therefore \csc\theta = 2$ (2)

$$-2^2\,\frac{d\theta}{dt} = \frac{1}{100}\,8 \rightarrow \frac{d\theta}{dt} = -\frac{1}{50} \text{ rads/sec}$$

or (4)

$$\boxed{\frac{1}{50} \text{ rads/sec}}$$

or $\tan\theta = \frac{100}{x}$ (3)

$$\sec^2\theta\,\frac{d\theta}{dt} = -\frac{100}{x^2}\frac{dx}{dt} \quad (4)$$

$$\frac{y^2}{\cancel{x^2}}\frac{d\theta}{dt} = -\frac{100}{\cancel{x^2}}\frac{dx}{dt}$$

When $y = 200$:

$$200^2\,\frac{d\theta}{dt} = -100\cdot 8$$

$$\frac{d\theta}{dt} = -\frac{1}{50} \text{ rads/sec} \quad (6)$$

(6) 8. Find the differential of $y = \ln\sqrt{1+x^2}$.

$$dy = \frac{dy}{dx}dx = \frac{1}{\sqrt{1+x^2}}\,\frac{1}{2\sqrt{1+x^2}}\,2x\,dx$$

0 credit if derivative is wrong.
-1 pt if dx is missing

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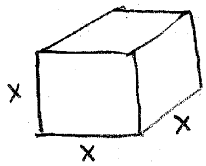
(12) 9. The volume of a melting cube of ice is decreasing at a rate of 10 cm^3/min. How fast is the surface area of the ice cube decreasing when the length of an edge is 30 cm?



Let x = length of an edge
V = volume of cube
S = surface area of cube

$\frac{dV}{dt} = -10$ cm^3/min

Find $\frac{dS}{dt}$ when $x = 30$ cm

$S = 6x^2$ ①

$\frac{dS}{dt} = 12x\frac{dx}{dt}$ ②

We need to find $\frac{dx}{dt}$ when $x=30$.

$V = x^3$ ①

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$ ②

When $x=30$: $-10 = 3\cdot 30^2\frac{dx}{dt} \rightarrow \frac{dx}{dt} = -\frac{10}{3\cdot 30^2} = -\frac{1}{270}$ ②

$\frac{dS}{dt} = 12\cdot 30\cdot\left(-\frac{1}{270}\right) = -\frac{12}{9} = -\frac{4}{3}$ cm^2/min ④

or $S = 6x^2$ ①, $V = x^3$ ①

$S = 6V^{2/3}$ ④

$\frac{dS}{dt} = 6\cdot\frac{2}{3}V^{-\frac{1}{3}}\frac{dV}{dt}$ ② $= \frac{4}{x}\frac{dV}{dt}$

When $x = 30$:

$\frac{dS}{dt} = \frac{4}{30}(-10) = -\frac{4}{3}$ ④

-1 pt for each minor numerical error

$\boxed{\frac{4}{3}\ cm^2/min}$ $\boxed{12}$

(10) 10. (a) Find the linearization $L(x)$ of the function $f(x) = e^{-2x}$ at $a = 0$.

$L(x) = f(a) + f'(a)(x-a)$ ②

$f(x) = e^{-2x}$ $f'(x) = -2e^{-2x}$

$f(0) = 1$ $f'(0) = -2$

$L(x) = 1 - 2x$

③ $L(x) = 1 - 2x$ $\boxed{5}$

(b) Use a linear approximation to estimate the number $e^{-0.2}$.

$f(x) \approx L(x)$ for x near a

or $e^{-x} \approx 1 - x$ for x near 0

$e^{-0.2} \approx 1 - (0.2) = 0.8$