NAME GRADING KEY

STUDENT ID ______________________

RECITATION INSTRUCTOR ______________________

RECITATION TIME ______________________

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DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

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(16) 1. Find the derivatives of the following functions. (It is not necessary to simplify).

NPC

(a) $y = \sqrt[4]{1 + 2x + x^3}$

$\boxed{\frac{1}{4}(1+2x+x^3)^{-\frac{3}{4}}(2+3x^2)}$ 4

(b) $f(x) = e^{x \cos x}$

$\boxed{e^{x\cos x}(-x\sin x + \cos x)}$ 4

(c) $y = \tan^2(3x)$

$\boxed{2\tan(3x)\sec^2(3x)\cdot 3}$ 4

(d) $y = [\ln(1 + e^x)]^2$

$\boxed{2\ln(1+e^x)\frac{1}{1+e^x}e^x}$ 4

16

(6) 2. The position function of a particle is given by $s = t^3 - (4.5)t^2 - 7t,\ t \geq 0$.

(a) When does the particle reach a velocity of 5 m/sec?

$v(t) = 3t^2 - 9t - 7$

$t?$ $v(t) = 5 \rightarrow 3t^2 - 9t - 7 = 5$

$3t^2 - 9t - 12 = 0 \rightarrow t^2 - 3t - 4 = 0$

$(t-4)(t+1) = 0 \rightarrow t = 4, -1$

3

$\boxed{t = 4\text{ sec}}$

(b) When is the acceleration 0?

$a(t) = v'(t) = 6t - 9$

$a(t) = 0$; $6t - 9 = 0 \longrightarrow t = \frac{3}{2}$

3

$\boxed{t = \frac{3}{2}\text{ sec}}$

$\boxed{6}$

(6) 3. Find $\frac{dy}{dx}$ by implicit differentiation, if

$\sin x + \cos y = \sin x \cos y$.

$\cos x - \sin y \cdot \frac{dy}{dx} = \sin x(-\sin y)\frac{dy}{dx} + \cos x \cos y$ 3

$(\sin x \sin y - \sin y)\frac{dy}{dx} = \cos x \cos y - \cos x$

$\boxed{\frac{dy}{dx} = \frac{\cos x(\cos y - 1)}{\sin y(\sin x - 1)}}$ 3

$\boxed{6}$

(8) 4. Find the equation of the tangent line to the curve

$x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

$2x + 2x\frac{dy}{dx} + 2y - 2y\frac{dy}{dx} + 1 = 0$ 3

At $(x,y) = (1,2)$: $2\cdot 1 + 2\cdot 1\frac{dy}{dx} + 2\cdot 2 - 2\cdot 2\frac{dy}{dx} + 1 = 0$

$-2\frac{dy}{dx} = -2 - 4 - 1$ $\quad \frac{dy}{dx} = \frac{7}{2}$ 3

eq. of tan. line at $(1,2)$:

$y - 2 = \frac{7}{2}(x - 1)$ 2

$\boxed{y - 2 = \frac{7}{2}(x-1)}$

$\boxed{8}$

(6) 5. Use a linear approximation to estimate $e^{-0.015}$.

$f(x) \approx f(a) + f'(a)(x - a)$, for x near a

Let $f(x) = e^x$ 2 and $a = 0$ 2

$e^x \approx e^0 + e^0(x - 0)$ for x near 0

$e^x \approx 1 + x$, for x near 0

$e^{-0.015} \approx 1 - 0.015 = 0.985$ 2

$\boxed{0.985}$

$\boxed{6}$

(5) 6. Find the differential dy of $y = \ln(\sec x + \tan x)$.

$dy = \frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x)\,dx$

or

-1 pt for missing dx

$= \frac{\sec x}{\sec x + \tan x}(\tan x + \sec x) = \sec x\,dx$

$\boxed{dy = \sec x\,dx}$ 5

(9) 7. Find the exact value (in radians) of

NPC

(a) $\sin^{-1}(\frac{\sqrt{3}}{2}) = y \iff \frac{\sqrt{3}}{2} = \sin y, \ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\therefore y = \frac{\pi}{3}$

$\boxed{\frac{\pi}{3}}$ 3

(b) $\tan^{-1}(\frac{1}{\sqrt{3}}) = y \iff \frac{1}{\sqrt{3}} = \tan y, \ -\frac{\pi}{2} < y < \frac{\pi}{2}$

$y = \frac{\pi}{6}$

$\boxed{\frac{\pi}{6}}$ 3

(c) $\sin^{-1}(\sin \frac{7\pi}{3}) = y \iff \sin\frac{7\pi}{3} = \sin y, \ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = \frac{\pi}{3}$

$\boxed{\frac{\pi}{3}}$ 3

$\boxed{9}$

(6) 8. Simplify the expression $\sin(2\sin^{-1}(x))$ and write it in terms of x without using trigonometric and inverse trigonometric functions.

Let $y = \sin^{-1} x \overset{2}{\iff} x = \sin y, \ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\sin(2\sin^{-1}(x)) = \sin 2y = 2\sin y \cos y$

$\sin y = x \quad \cos y = \pm\sqrt{1-\sin^2 y} = \cancel{\pm}\sqrt{1-x^2}$ 4

$\therefore \sin(2\sin^{-1} x) = 2x\sqrt{1-x^2}$

4 $\boxed{2x\sqrt{1-x^2}}$

$\boxed{6}$

(12) 9. Find the derivatives of the following functions. (It is not necessary to simplify).

NPC

(a) $y = \sin^{-1}(\sqrt{\sin x})$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{\sin x})^2}} \cdot \frac{1}{2\sqrt{\sin x}} \cos x$$

or

$\boxed{\frac{1}{\sqrt{1-\sin x}} \frac{1}{2\sqrt{\sin x}} \cdot \cos x}$ 4

(b) $y = \tan^{-1}(\sin^{-1}(\sqrt{x}))$

$$\frac{dy}{dx} = \frac{1}{(\sin^{-1}\sqrt{x})^2+1} \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$$

or

$\boxed{\frac{1}{[(\sin^{-1}\sqrt{x})^2+1]\sqrt{1-x} \cdot 2\sqrt{x}}}$ 4

(c) $y = x^{x \ln x} = (e^{\ln x})^{x \ln x} = e^{x(\ln x)^2}$

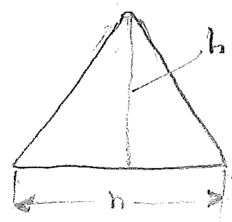
$$\frac{dy}{dx} = e^{x(\ln x)^2}\left[x \cdot 2\ln x \cdot \frac{1}{x} + (\ln x)^2\right]$$

or

$\boxed{x^{x\ln x}\left[2\ln x + (\ln x)^2\right]}$ 4

$\boxed{12}$

(13) 10. Gravel is being dumped from a conveyer belt at a rate of 30 ft^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?



Let V = volume of pile
r = radius of base
h = height of pile

Given : $\frac{dV}{dt} = 30\ ft^3/min$, $r = \frac{h}{2}$

Find : $\frac{dh}{dt}$ when $h = 10$ ft

$V = \frac{1}{3}\pi r^2 h$, $r = \frac{h}{2}$ → $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{12} h^3$ (4) ← or

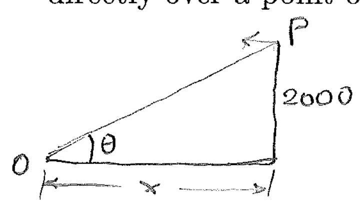
$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$ ← or → $\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt}$ (4)

When $h = 10$:

$\frac{dh}{dt} = \frac{4}{\pi(100)} \cdot 30 = \frac{120}{(100)\pi} = \frac{6}{5\pi}$ ft/min (5)

$\boxed{\frac{6}{5\pi}\ ft/min}$ 13

(13) 11. An airplane is flying at 150 ft/sec at an altitude of 2000 ft in a direction that will take it directly over the observer at the ground level. Find the rate of change of the angle between the line from the observer to the plane and the horizontal, when the plane is directly over a point on the ground that is 2000 ft from the observer.



Given $\frac{dx}{dt} = -150$ ft/sec

Find $\frac{d\theta}{dt}$ when $x = 2000$ ft

$\tan\theta = \frac{2000}{x}$ (4)

$\sec^2\theta \frac{d\theta}{dt} = -\frac{2000}{x^2}\frac{dx}{dt}$

$\frac{d\theta}{dt} = -\frac{1}{\sec^2\theta}\frac{2000}{x^2}\frac{dx}{dt}$ (4)

When $x = 2000$

$\theta = \frac{\pi}{4}$, $\sec^2\theta = \frac{1}{\cos^2\theta} = 2$

$\frac{d\theta}{dt} = -\frac{1}{2}\frac{2000}{(2000)^2}(-150)$

$= \frac{150}{4000} = \frac{15}{400} = \frac{3}{80}$ rads/sec (5)

or

$\cot\theta = \frac{x}{2000}$ (4)

$-\csc^2\theta \frac{d\theta}{dt} = \frac{1}{2000}\frac{dx}{dt}$

$\frac{d\theta}{dt} = -\frac{\sin^2\theta}{2000}\frac{dx}{dt}$ (4)

When $x = 2000$

$\theta = \frac{\pi}{4}$, $\sin^2\theta = \frac{1}{2}$

$\frac{d\theta}{dt} = -\frac{\frac{1}{2}}{2000}(-150) =$

$= \frac{150}{4000} = \frac{15}{400} = \frac{3}{80}$ rads/sec (5)

$\boxed{\frac{3}{80}\ rads/sec}$ 13