NAME Grading Key.

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(10) 1. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 3x^2 + 3x$ on the interval $[-1, 2]$.

$$f'(x) = 3x^2 - 6x + 3$$
$$0 = 3(x^2 - 2x + 1)$$
$$= 3(x-1)^2$$

4 pts $x = 1$ (critical value)

$f(-1) = -7$

$f(1) = 1$

$f(2) = 2$

2 1

abs. max. $\boxed{f(2) = 2}$

abs. min. $\boxed{f(-1) = -7}$

2 1

(8) 2. Suppose f is continuous on $[2,5]$, f is differentiable on $(2,5)$ and $1 \le f'(x) \le 4$ for all x in $(2,5)$. Show that $3 \le f(5) - f(2) \le 12$. (Hint: Use the Mean Value Theorem.)

4 pts $f'(c) = \dfrac{f(5) - f(2)}{5 - 2}$ for some c in $(2, 5)$.

4 pts
$1 \le \dfrac{f(5) - f(2)}{5 - 2} \le 4$ because $1 \le f'(x) \le 4$ for all x in $(2,5)$.

$\therefore \quad 3 \le f(5) - f(2) \le 12$.

Name: For #3, if the ans. is wrong, Give $\underline{2}$ pts for some correct start.

(20) 3. Find each of the following limits as a real number, $+\infty$, $-\infty$ or DNE (does not exist).

(a) $\lim_{x\to 0} \frac{\sin x - x}{x^3} \overset{H}{=} \lim_{x\to 0} \frac{\cos x - 1}{3x^2} \overset{H}{=} \lim_{x\to 0} \frac{-\sin x}{6x}$

$\overset{H}{=} \lim_{x\to 0} \frac{-\cos x}{6} = -\frac{1}{6}$

$\boxed{-\frac{1}{6}}$ 5pts

(b) $\lim_{x\to 1^+} x^{\frac{1}{1-x}} = \lim_{x\to 1^+} e^{\frac{\ln x}{1-x}}$

$= e^{\lim_{x\to 1^+} \frac{\ln x}{1-x}}$

$\overset{H}{=} e^{\lim_{x\to 1^+} \frac{1/x}{-1}}$

$= e^{-1}$

0 pts if answer is correct but there is no work or work is nonsense

$\boxed{e^{-1} \text{ or } \frac{1}{e}}$ 5pts

(c) $\lim_{x\to\infty} x\tan\frac{1}{x} = \lim_{x\to\infty} \frac{\tan\frac{1}{x}}{1/x} \overset{H}{=} \lim_{x\to\infty} \frac{\sec^2\frac{1}{x}\left(-\frac{1}{x^2}\right)}{(-1/x^2)}$

$= \lim_{x\to\infty} \sec^2\frac{1}{x} = 1$

$\boxed{1}$ 5pts

(d) $\lim_{x\to 0^+} (\ln x - \ln\sin x) = \lim_{x\to 0^+} \ln\frac{x}{\sin x} = \ln\left(\lim_{x\to 0^+} \frac{x}{\sin x}\right)$.

$\overset{H}{=} \ln\left(\lim_{x\to 0^+} \frac{1}{\cos x}\right)$

$= \ln 1 = 0$

$\boxed{0}$ 5pts

(10) **4. The number $x = -1$ is a critical number of the function $f(x) = xe^x$. Decide whether f has a local maximum or a local minimum or neither at $x = -1$. Give reasons for your answer.**

$f'(x) = xe^x + e^x$

2pts $f''(x) = xe^x + 2e^x$

$f''(-1) = -e^{-1} + 2e^{-1}$

4pts $= e^{-1} = \frac{1}{e} > 0$

or

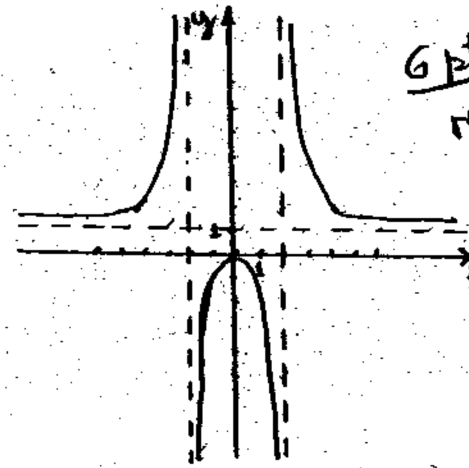
$f'(x) = (x+1)e^x$ 2pts

$\left.\begin{array}{l} f'(x) < 0 \text{ for } x < -1 \\ f'(x) > 0 \text{ for } x > -1 \end{array}\right\}$ 4pts

$\boxed{\text{Local Minimum}}$ 4pts

Name: ______________________

(20) 5. Let $f(x) = \frac{x^2}{x^2-4}$. Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation of each asymptote. Write NONE where appropriate.



6 pts
HPC for graph

symmetry $f(-x) = f(x)$.

$$\lim_{x\to\pm\infty} \frac{x^2}{x^2-4} = \lim_{x\to\infty} \frac{1}{1-4/x^2} = 1$$

$$x^2-4 = (x-2)(x+2) = 0$$

$x = \pm 2$. asymptotes.

$$f'(x) = \frac{(x^2-4)2x - x^2\cdot 2x}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2}$$

critical pt $x=0, y=0$.

$$f''(x) = -\frac{(x^2-4)^2\cdot 8 - 8x\cdot 2(x^2-4)2x}{(x^2-4)^4}$$

$$= \frac{8(3x^2+4)}{(x^2-4)^3} \neq 0$$

$f''(0) < 0 \therefore (0,0)$ Local Max

$f'(x) > 0$ if $x<0$ (incr).

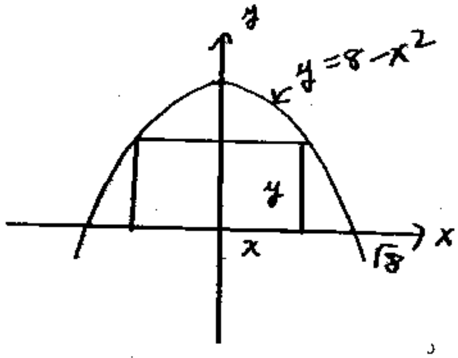
$f'(x) < 0$ if $x>0$ (decr)

$f''(x) > 0$ if $x < -2$ or $x > 2$. (concave up).

$f''(x) < 0$ if $-2<x<2$ (concave down)

domain	$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$	1
intercepts	$(0,0)$	1
symmetry	about y-axis	1
horizontal asymptotes	$y=1$.	1
vertical asymptotes	$x=2$ and $x=-2$.	1
intervals of increase	$(-\infty,-2)$ and $(-2,0)$	1
intervals of decrease	$(0,2)$ and $(2,\infty)$	1
local maxima	$(0,0)$ or $f(0)=0$	1
local minima	NONE	1
intervals of concave down	$(-2,2)$	2
intervals of concave up	$(-\infty,-2)$ and $(2,\infty)$	2
points of inflection	NONE	1

(12) 6. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices on the parabola $y = 8 - x^2$.



$A = 2xy, \quad y = 8 - x^2$

$A = 2x(8 - x^2)$ 4 pts

$= 16x - 2x^3$, where $0 \le x \le \sqrt{8}$

$\frac{dA}{dx} = 16 - 6x^2$

$0 = 16 - 6x^2$ 2 pts

$x = \sqrt{\frac{16}{6}} = \sqrt{\frac{8}{3}}$

$y = \frac{16}{3}$

Test: On $0 \le x \le \sqrt{8}$

$\frac{dA}{dx} > 0$ if $x < \sqrt{\frac{8}{3}}$

$\frac{dA}{dx} < 0$ if $x > \sqrt{\frac{8}{3}}$

$\therefore$ Max.

(Take off 1 pt if the "2" is missing in A and/or base)

base = $\boxed{2\sqrt{\frac{8}{3}}}$ 3 pts

height = $\boxed{\frac{16}{3}}$ 3 pts

(5) 7. Find the most general antiderivative of $f(x) = 4 \sec x \tan x - \frac{3}{x}$.

Take off 1 pt if "C" is missing.

$\boxed{4 \sec x - 3 \ln|x| + C}$ 5 pts

(5) 8. If $\int_2^8 f(x)dx = 1.7$ and $\int_5^8 f(x)dx = 2.5$, find $\int_2^5 f(x)dx$.

$\int_2^8 f(x)dx = \int_2^5 f(x)dx + \int_5^8 f(x)dx$

$1.7 = \int_2^5 f(x)dx + 2.5$

−2 pts if answer is wrong but is clear from work that it is a subtraction error

$\int_2^5 f(x)dx = \boxed{-0.8}$ 5 pts

(10) 9. Find f if $f''(x) = \sqrt{x}$, $f(1) = 1$, and $f'(1) = 2$.

$f'(x) = \frac{2}{3}x^{3/2} + C$ 2 pts

$2 = \frac{2}{3} + C, \quad C = \frac{4}{3}$. 2 pts

← If C is wrong grade the rest with consistency rule.

$f'(x) = \frac{2}{3}x^{3/2} + \frac{4}{3}$

$f(x) = \frac{4}{15}x^{5/2} + \frac{4}{3}x + D$ 2 pts

$1 = \frac{4}{15} + \frac{4}{3} + D$

$D = -\frac{9}{15} = -\frac{3}{5}$ 2 pts

$f(x) = \boxed{\frac{4}{15}x^{5/2} + \frac{4}{3}x - \frac{3}{5}}$ 2 pts