NAME GRADING KEY

STUDENT ID __________

RECITATION INSTRUCTOR __________

RECITATION TIME __________

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(10) 1. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 3x + 1$ on the interval $[0, 3]$.

$f'(x) = 3x^2 - 3$

not in $(0,3)$

$f'(x) = 0$: $3x^2 - 3 = 0 \to x^2 - 1 = 0 \to \cancel{x = -1}, x = 1$

④ $x = 1$ is a critical number in $(0,3)$

$f(0) = 1$

$f(1) = 1 - 3 + 1 = -1$ abs. min.

$f(3) = 27 - 9 + 1 = 19$ abs. max.

② ①

abs. max. $\boxed{f(3) = 19}$

abs. min. $\boxed{f(1) = -1}$

② ①

$\boxed{10}$

(8) 2. Suppose that f is continuous on $[1, 4]$ and differentiable on $(1, 4)$.

(a) The Mean Value Theorem asserts that there is a number c in (a, b) such that

$\boxed{f(4) - f(1) = 3f'(c)}$

NPC $\boxed{4}$

(b) If $f(1) = 2$ and $f'(x) \geq 4$ for all x in $(1, 4)$, find the smallest possible value for $f(4)$.

$f(4) - f(1) \geq 3 \cdot 4$

$f(4) \geq 2 + 12 = 14$

$\boxed{f(4) \geq 14}$

NPC $\boxed{4}$

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In 3: 0 points if answer is correct but there is no work or work is nonsense

(20) 3. Find each of the following as a real number, $+\infty$, $-\infty$ or write DNE (does not exist).

(a) $\lim_{x\to\frac{\pi}{4}} \frac{\tan x}{x} = \frac{1}{\frac{\pi}{4}} = \frac{4}{\pi}$

$\boxed{\frac{4}{\pi}}$ NPC $\boxed{5}$

(b) $\underset{x\to0}{\lim}\ \underset{\infty-\infty}{(\csc x - \cot x)} = \lim\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$ ②

$= \underset{\frac{0}{0}}{\lim_{x\to0} \frac{1-\cos x}{\sin x}} \overset{H}{=} \lim_{x\to0} \frac{\sin x}{\cos x} = 0$

$\boxed{0}$ $\boxed{5}$

(c) $\underset{\infty\cdot0}{\lim_{x\to-\infty} x^2e^x} = \lim_{x\to-\infty} \underset{\frac{\infty}{\infty}}{\frac{x^2}{e^{-x}}} \overset{H}{=} \lim_{x\to-\infty} \underset{\frac{\infty}{\infty}}{\frac{2x}{-e^{-x}}}$ ②

$\overset{H}{=} \lim_{x\to-\infty} \frac{2}{e^{-x}} = 0$

$\boxed{0}$ $\boxed{5}$

(d) $\underset{1^{\infty}}{\lim_{x\to0}(\cos x)^{1/x}} = \lim_{x\to0} e^{\frac{1}{x}\ln(\cos x)} = e^{\lim_{x\to0}\frac{\ln(\cos x)}{x}}$ ②

$\lim_{x\to0} \underset{\frac{0}{0}}{\frac{\ln(\cos x)}{x}} \overset{H}{=} \lim_{x\to0} \frac{-\frac{\sin x}{\cos x}}{1} = 0 \quad \therefore \lim_{x\to0}(\cos x)^{1/x} = e^0 = 1$

or Set $y = (\cos x)^{1/x}$ $\quad \ln y \overset{②}{=} \frac{1}{x}\ln(\cos x)$

$\lim_{x\to0} \ln y = \lim_{x\to0} \frac{\ln(\cos x)}{x} = \lim_{x\to0} \frac{-\frac{\sin x}{\cos x}}{1} = 0$

$\lim_{x\to0} y = \lim_{x\to0} e^{\ln y} = e^0 = 1$

$\boxed{1}$ $\boxed{5}$

(12) 4. The numbers 1 and 3 are the only critical numbers of the function $f(x) = x^3 - 6x^2 + 9x$. Showing all necessary work, decide whether f has a local maximum or a local minimum,

(a) at $x = 1$, using the first derivative test,

$f'(x) = 3x^2 - \overset{②}{12}x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$

$f'(x) \quad + + + + \underset{1}{0} - - - - - - \underset{3}{0} + + + + \rightarrow x$

At $x=1$, f' changes sign from positive to negative ②

∴ at $x=1$ f has a local maximum ②

loc. max

(b) at $x = 3$, using the second derivative test.

$f''(x) = \underset{②}{6x} - 12 \qquad f''(3) = 6 > 0$ ②

∴ at $x=3$ f has a local minimum ②

loc. min

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(18) 5. Let $f(x) = \frac{x+1}{x-1}$. Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation for each asymptote. Write NONE where appropriate.

$y = \frac{x+1}{x-1}$ Domain all $x \neq 1$

$y=0 \rightarrow x=-1 \quad (-1,0)$

$x=0 \rightarrow y=-1 \quad (0,-1)$

$f(-x) = \frac{-x+1}{-x-1} = \frac{x-1}{x+1}$

no symmetry

$\lim_{x \to \infty} \frac{x+1}{x-1} = 1 \quad \lim_{x \to -\infty} \frac{x+1}{x-1} = 1$

h.a. $y=1$

$\lim_{x \to 1^+} \frac{x+1}{x-1} = \infty, \quad \lim_{x \to 1^-} \frac{x+1}{x-1} = -\infty$

v.a. $x=1$

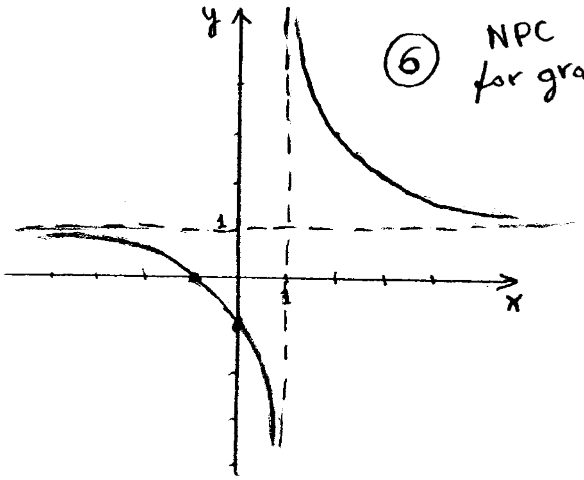
$f'(x) = \frac{(x-1)1-(x+1)\cdot 1}{(x-1)^2} = -\frac{2}{(x-1)^2}$

$f'(x) \quad ------\underset{1}{|}----\rightarrow x$

$f'(x)=0: \; -\frac{2}{(x-1)^2}=0$ never

$f''(x) = \frac{4}{(x-1)^3}$

$f''(x) \quad ----\underset{1}{|}++++\rightarrow x$



⑥ NPC for graph

domain	all $x \neq 1$ or $(-\infty,1)\cup(1,\infty)$	①
intercepts	$(-1,0)$ $(0,-1)$	①
symmetry	NONE	①
horizontal asymptotes	$y=1$	①
vertical asymptotes	$x=1$	①
intervals of increase	NONE	①
intervals of decrease	$(-\infty,1)$ and $(1,\infty)$	①
local maxima	NONE	①
local minima	NONE	①
intervals of concave down	$(-\infty,1)$	①
intervals of concave up	$(1,\infty)$	①
points of inflection	NONE	①

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(8) 6. Find two positive numbers x and y whose product is 100 and whose sum is a minimum. You must use Calculus.

$xy = 100 \qquad S = x + y$

② $S = x + \frac{100}{x}$, $0 < x < \infty$ ③ ①

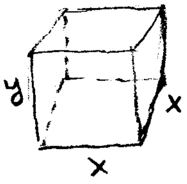
② $\frac{dS}{dx} = 1 - \frac{100}{x^2} \qquad \frac{dS}{dx} = 0 : 1 - \frac{100}{x^2} = 0 \rightarrow x = 10, \ y = 10$

$\frac{dS}{dx}$ 0 − − − − − 0 + + + + 10 min

$x = 10 \quad , y = 10$

$\boxed{8}$

(12) 7. A box with square base and open top must have a volume of 4 m^3. Find the dimensions of the box that minimize the amount of material used.



$x^2 y = 4$ ② Let A = area of material of box

$A = x^2 + 4xy$ ②

$A = x^2 + 4x \frac{4}{x^2} = x^2 + \frac{16}{x}$ ② , $0 < x < \infty$

$\frac{dA}{dx} = 2x - \frac{16}{x^2}$ ②

$\frac{dA}{dx} = 0 : \quad 2x - \frac{16}{x^2} = 0 \rightarrow x^3 = 8 \rightarrow x = 2$ ③

$y = 1$ ①

$\frac{dA}{dx}$ 0 − − − − 0 + + + 2 min

edge of base = 2 m
height = 1 m

(12) 8. Find f if $f''(x) = 3e^{-x} + 5\cos x$, $f(0) = 1$ and $f'(0) = 2$.

$f'(x) = -3e^{-x} + 5\sin x + C_1$ ③

$f'(0) = 2: \quad 2 = -3e^0 + 5\sin 0 + C_1 \longrightarrow C_1 = 5$

$f'(x) = -3e^{-x} + 5\sin x + 5$ ③

$f(x) = 3e^{-x} - 5\cos x + 5x + C_2$ ③

$f(0) = 1: \quad 1 = 3e^0 - 5\cos 0 + 5 \cdot 0 + C_2 \longrightarrow C_2 = 3$

$f(x) = 3e^{-x} - 5\cos x + 5x + 3$ ③

$f(x) = 3e^{-x} - 5\cos x + 5x + 3$

$\boxed{12}$