MA 165 EXAM 3 Fall 2003

NAME GRADING KEY

STUDENT ID

RECITATION INSTRUCTOR

RECITATION TIME

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(10) 1. Find the absolute maximum and absolute minimum values of the function $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.

$f'(x) = 12x^3 - 12x^2$

$f'(x) = 0$: $12x^3 - 12x^2 = 0 \rightarrow 12x^2(x-1) = 0 \rightarrow x = 0, 1$ ← critical numbers in $(-1, 2)$ ④ *

$f(-1) = 7$

$f(0) = 0$

$f(1) = -1$ ← abs. min.

$f(2) = 3 \cdot 16 - 4 \cdot 8 = 16$ abs. max.

abs. max. $\boxed{f(2) = 16}$ ② ①

abs. min. $\boxed{f(1) = -1}$ ② ①

$\boxed{10}$

(3) 2. True or False. (Circle T or F). The function $f(x) = |x - 1|$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 3]$. T [✓ F]

f is not differentiable at $x = 1 \in (0, 3)$

$\boxed{3}$ NPC

(6) 3. Find all numbers c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \sqrt{x+1}$ on the interval $[0, 3]$.

③ $\frac{f(3) - f(0)}{3 - 0} = f'(c)$, $0 < c < 3$ $f'(x) = \frac{1}{2\sqrt{x+1}}$

$\underbrace{\frac{2-1}{3}}_{①} = \underbrace{\frac{1}{2\sqrt{c+1}}}_{①} \rightarrow 2\sqrt{c+1} = 3$

$4c + 4 = 9$

$c = \frac{5}{4}$ ①

$\boxed{\frac{5}{4}}$

$\boxed{6}$

* If one cr. number is missing or wrong, credit only for correct answers in boxes.

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In 4a,c,d : 0 points if answer is correct but there is no work or work is wrong

(20) 4. Find each of the following as a real number, $+\infty$, $-\infty$ or DNE (does not exist). ↖ write

(a) $\lim_{x\to 0}\frac{e^x-1}{\sin x} \overset{L'H}{=} \lim_{x\to 0}\frac{e^x}{\cos x} = \frac{1}{1} = 1$

$\frac{0}{0}$

$\boxed{1}$ $\boxed{5}$

(b) $\lim_{x\to(\frac{\pi}{2})^-}\frac{\tan x}{\csc x} = \infty$

$\lim_{x\to(\frac{\pi}{2})^-}\tan x = \infty$ $\lim_{x\to(\frac{\pi}{2})^-}\csc x = \lim_{x\to(\frac{\pi}{2})^-}\frac{1}{\sin x} = 1$

$\boxed{\infty}$ $\boxed{5}$

(c) $\lim_{x\to\infty} x^{1/x} = \lim_{x\to\infty}(e^{\ln x})^{\frac{1}{x}} = e^{\lim_{x\to\infty}\frac{\ln x}{x}} = e^0 = 1$ ↗∴

$\lim_{x\to\infty}\frac{\ln x}{x} \overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = \frac{0}{1} = 0$

$\frac{\infty}{\infty}$

$\boxed{1}$ $\boxed{5}$

(d) $\lim_{x\to 0}\frac{\cos x - 1 + \frac{x^2}{2}}{x^3} \overset{L'H}{=} \lim_{x\to 0}\frac{-\sin x + x}{3x^2} =$

$\frac{0}{0}$ $\frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0}\frac{-\cos x + 1}{6x} \overset{L'H}{=} \lim_{x\to 0}\frac{\sin x}{6} = 0$

$\frac{0}{0}$

$\boxed{0}$ $\boxed{5}$

(6) 5. The number e^{-1} is the only critical number of the function $f(x) = x\ln x$. Showing all necessary work, use the second derivative test to decide whether f has a local maximum or a local minimum at e^{-1}.

$f'(x) = x\cdot\frac{1}{x} + \ln x = 1 + \ln x$ $f''(x) = \frac{1}{x}$ ②

$f''(e^{-1}) = \overset{①}{e} \overset{①}{>} 0$

②

loc. min. $\boxed{6}$

(5) 6. Find the most general antiderivative of $f(x) = 3e^x + 7\sec^2 x$.

② ② ①

$\boxed{3e^x + 7\tan x + C}$ $\boxed{5}$

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(18) 7. Let $f(x) = xe^{-x}$. Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation for each asymptote. Write NONE where appropriate.

$y = xe^{-x}$ Domain: all x

$y=0 \rightarrow xe^{-x}=0 \rightarrow x=0$ (0,0)

$x=0 \rightarrow y=0$ (0, 0)

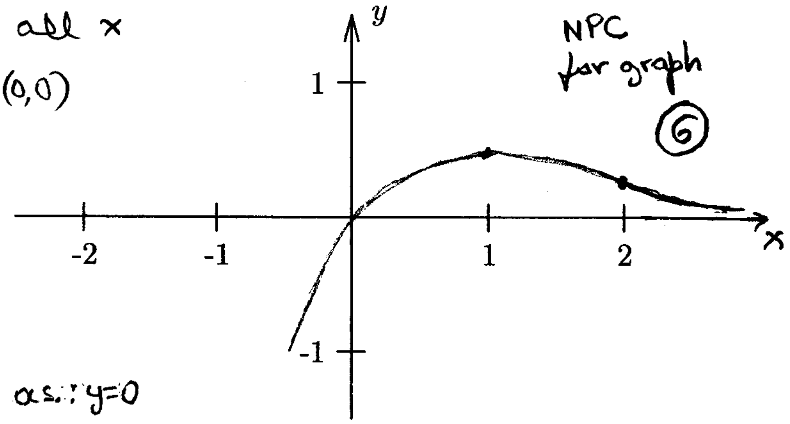
$f(-x) = -xe^{x}$ no symmetry

$\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = \overset{\frac{\infty}{\infty}}{}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{1}{e^x} = 0$ ∴ hor. as.: $y=0$

$\lim_{x\to-\infty} xe^{-x} = -\infty$

ver. as.: none



NPC for graph ⑥

$f'(x) = -xe^{-x} + e^{-x} = e^{-x}(1-x)$

$f'(x)$ $\underset{1}{+ + + + + 0 - - - - - -}\rightarrow x$

$f(1) = 1e^{-1} = \frac{1}{e}$ loc. max.

$f''(x) = -e^{-x} - e^{-x}(1-x)$

$= -e^{-x}(2-x)$

$f''(x)$ $\underset{2}{- - - - - - 0 + + + +}\rightarrow x$

$f(2) = 2e^{-2} = \frac{2}{e^2}$

domain	$(-\infty, \infty)$	①
intercepts	(0,0)	①
symmetry	NONE	①
horizontal asymptotes	$y=0$	①
vertical asymptotes	NONE	①
intervals of increase	$(-\infty, 1)$	①
intervals of decrease	$(1, \infty)$	①
local maxima	$f(1) = \frac{1}{e}$ or $(1, \frac{1}{e})$	①
local minima	NONE	①
intervals of concave down	$(-\infty, 2)$	①
intervals of concave up	$(2, \infty)$	①
points of inflection	$(2, \frac{2}{e^2})$	①

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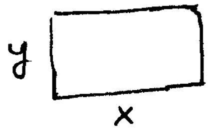
(8) 8. Among all the rectangles with base x, height y, and perimeter 20 in, find the dimensions of the one with largest area. You must use Calculus.



$A = xy$

$2x + 2y = 20 \rightarrow y = 10 - x$

$A = x(10 - x)$ ④ $\quad 0 \le x \le 10$

$\frac{dA}{dx} = 10 - 2x \qquad$ When $x = 0$ or $x = 10$: $A = 0$

$\frac{dA}{dx} = 0$: $10 - 2x = 0 \quad x = 5 \qquad$ When $x = 5$ $A = 25$

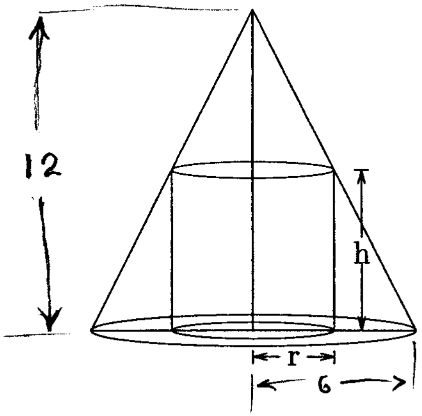
② ②

$x = 5$, $y = 5$

$\boxed{8}$

(12) 9. A right circular cylinder is inscribed in a cone of height 12 in and base radius 6 in. Find the radius of such a cylinder with largest possible volume. (Let h denote the height and r the radius of the cylinder).

Let V denote the volume of the cylinder



$V = \pi r^2 h$ ②

$\frac{h}{6 - r} = \frac{12}{6} \rightarrow h = 12 - 2r$ ④

$V = \pi r^2(12 - 2r) = \underline{12\pi r^2 - 2\pi r^3} \qquad 0 \le r \le 6$

← or — ②

$\frac{dV}{dr} = 24\pi r - 6\pi r^2$ ②

$\frac{dV}{dr} = 0$: $24\pi r - 6\pi r^2 = 0$

$6\pi r(4 - r) = 0$

$r = 0$, $r = 4$

When $r = 0$: $V = 0$

" $r = 6$: $V = 0$

" $r = 4$ $V = \pi\, 16(12 - 8) = 64\pi$ max

②

$r = 4$ in

$\boxed{12}$

(12) 10. A particle is moving with acceleration $a(t) = t^2 + 3\cos\, t$. Its initial position is $s(0) = 2$ and its initial velocity is $v(0) = 3$. Find the position function $s(t)$.

$a(t) = t^2 + 3\cos t$

$v(t) = \frac{t^3}{3} + 3\sin t + C$

When $t=0$: $3 = 0 + 0 + C \rightarrow C = 3$

$v(t) = \frac{t^3}{3} + 3\sin t + 3$

$s(t) = \frac{t^4}{12} - 3\cos t + 3t + D$

When $t=0$: $2 = 0 - 3 + 0 + D \rightarrow D = 5$

③ ③ ③ ③

$s(t) = \frac{t^4}{12} - 3\cos t + 3t + 5$

$\boxed{12}$