NAME GRADING KEY

10–digit PUID

RECITATION INSTRUCTOR

RECITATION TIME

Page 1	/17
Page 2	/39
Page 3	/18
Page 4	/26
TOTAL	/100

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit. Please write neatly. Remember, if we cannot read your work and follow it logically, you may receive no credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic device may be used on this exam.

---

(10) 1. Find the absolute maximum and absolute minimum values of the function $f(x) = \dfrac{x^2-4}{x^2+4}$ on the interval $[-2,4]$.

Critical number: $0 = f'(x) = \dfrac{2x(x^2+4)-(x^2-4)(2x)}{(x^2+4)^2} = \dfrac{16x}{(x^2+4)^2}$ 2

$\Rightarrow x=0$ 2

$f(0) = -1$

$f(-2) = 0$

$f(4) = \frac{12}{20} = \frac{3}{5}$

2 1

abs. max. $f(4) = \frac{3}{5}$

abs. min. $f(0) = -1$

2 1

$\boxed{10}$

(7) 2. Let $f(x) = \dfrac{x-4}{x^2+9}$. Explain why there is at least one number c in $(-1,4)$ such that $f'(c) = \dfrac{1}{10}$. Make sure to state the name of the theorem used.

Mean Value Theorem: f differentiable and hence cont. in $\mathbb{R}$,
So there exists $-1<c<4$: $f'(c) = \dfrac{f(4)-f(-1)}{4-(-1)}$ 2

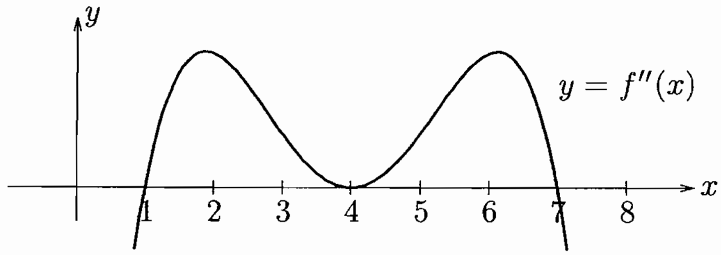
$\Rightarrow f'(c) = \dfrac{0 - \frac{-1-4}{1+9}}{5} = \dfrac{1}{10}$ 2

Name of theorem used: Mean Value Theorem 3

$\boxed{7}$

Name: ____________________

(5) 3. The graph of the second derivative f'' of a function f is shown. State the x–coordinates of all the inflection points of f.



$x =$ 1, 7

5

0 credit if answer is correct but gives no work

(24) 4. Find each of the following as a real number, $+\infty$, $-\infty$ or write DNE (does not exist).

(a) $\lim_{x\to 0}\left(\frac{\sin x - x}{x^3}\right) \overset{L'H}{=} \lim_{x\to 0}\frac{\cos x - 1}{3x^2} \overset{L'H}{=} \lim_{x\to 0}\frac{-\sin x}{6x}$ (2)

$\overset{L'H}{=} \lim_{x\to 0}\frac{-\cos x}{6} = -\frac{1}{6}$

$-\frac{1}{6}$ (4) 6

(b) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x} = \lim_{x\to\infty} e^{3x\ln(1+\frac{1}{x})}$ (2)

$= e^{\lim_{x\to\infty} 3\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}} \overset{L'H}{=} e^{3\lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}} = e^3$

e^3 (4) 6

(c) $\lim_{x\to 0^+}((\sin x)(\ln x)) = \lim_{x\to 0^+}\frac{\ln x}{\csc x} \overset{L'H}{=} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\cot x\csc x}$ (2)

$= -\lim_{x\to 0^+}\frac{\sin x}{x}\cdot\frac{\sin x}{\cos x} = -1\cdot 0 = 0$

0 (4) 6

(d) $\lim_{x\to(\pi/2)^-}(\tan x - \sec x)$

$= \lim_{x\to(\frac{\pi}{2})^-}\left(\frac{\sin x}{\cos x} - \frac{1}{\cos x}\right) = \lim_{x\to(\frac{\pi}{2})^-}\frac{\sin x - 1}{\cos x} \overset{L'H}{=} \lim_{x\to(\frac{\pi}{2})^-}\frac{\cos x}{-\sin x} = 0$ (2)

0 (4) 6

(10) 5. Find f if $f'(x) = \frac{4}{\sqrt{1-x^2}}$ and $f(\frac{1}{2}) = 1$.

$f'(x) = \frac{4}{\sqrt{1-x^2}} \Rightarrow f(x) = 4\sin^{-1}x + C$ (4)

(4)

$x=\frac{1}{2}: \quad 1 = 4\sin^{-1}\frac{1}{2} + C = 4\frac{\pi}{6} + C$

$\Rightarrow C = 1 - \frac{2\pi}{3} = \frac{3-2\pi}{3}$ (2)

$f(x) = 4\sin^{-1}x + \frac{3-2\pi}{3}$

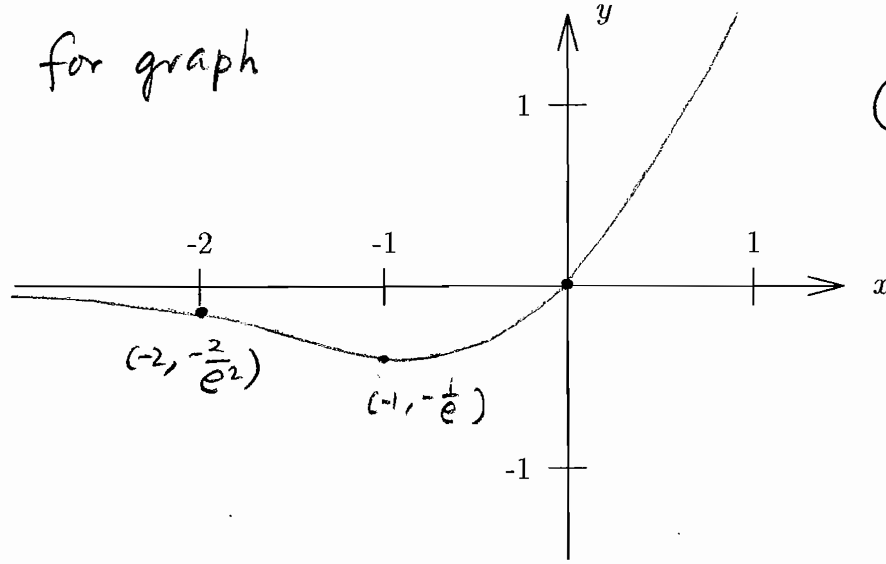
10

Name: ______________________________

(18) 6. Let $f(x) = xe^x$. Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation for each asymptote. Write NONE where appropriate.

NPC for graph

(4)



$y = xe^x$

Domain: all $x \in \mathbb{R}$

$y = 0 \iff x = 0$

$f(-x) = -xe^{-x}$, neither odd nor even

$\lim_{x\to-\infty} xe^x = \lim_{x\to-\infty} \frac{x}{e^{-x}} \overset{L'H}{=} \lim_{x\to-\infty} \frac{1}{-e^{-x}} = 0$

$\therefore y = 0$ H.A.

$\lim_{x\to\infty} xe^x = \infty$

f(x) finite for all x. no V.A.

$f' = e^x + xe^x = (1+x)e^x$

$f' > 0 \iff x > -1$: increasing

$f' < 0 \iff x < -1$: decreasing

Critical number : $x = -1$

1st der test : local min

$f(-1) = -e^{-1}$

$f'' = 2e^x + xe^x = (2+x)e^x$

$f'' > 0 \iff x > -2$: CU

$f'' < 0 \iff x < -2$: CD

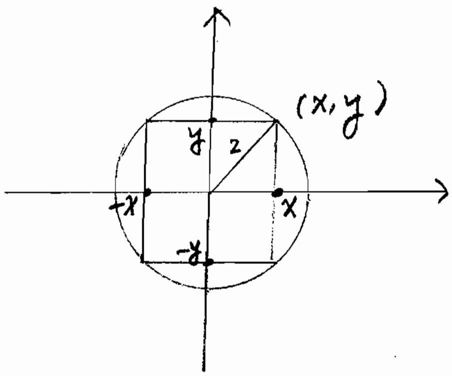
$f''(-2) = 0$ point of inflection

$x = -2$, $y = -2e^{-2}$

domain	$(-\infty, \infty)$	1
intercepts	$(0, 0)$	1
symmetry	NONE	1
horizontal asymptotes	$y = 0$	2
vertical asymptotes	NONE	1
intervals of increase	$(-1, \infty)$	1
intervals of decrease	$(-\infty, -1)$	1
local maxima	NONE	1
local minima	$(-1, -e^{-1})$	1
intervals of concave down	$(-\infty, -2)$	1
intervals of concave up	$(-2, \infty)$	1
points of inflection	$(-2, -2e^{-2})$	2

Name: ______________________________

(14) 7. Use calculus to find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius 2.



$x^2+y^2=2^2$ 3

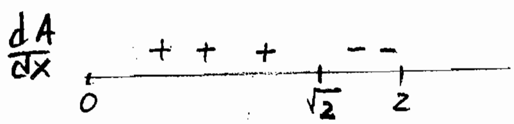
Area $A=(2x)(2y)$ 3

$=4x\sqrt{4-x^2}$ 2 $0\le x\le 2$

$\frac{dA}{dx}$ 2 $= 4\sqrt{4-x^2}+4x\cdot\frac{1}{2}\frac{-2x}{\sqrt{4-x^2}} = \frac{4}{\sqrt{4-x^2}}(4-2x^2)$

$\frac{dA}{dx}=0$: $4-2x^2=0 \Rightarrow x=\sqrt{2}$ 2

then $y=\sqrt{4-2}=\sqrt{2}$



max $A=(2\sqrt{2})(2\sqrt{2})=8$

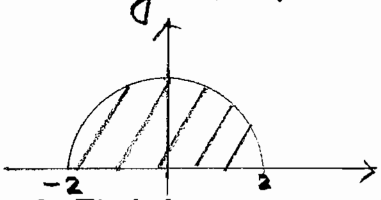
1 1

dimensions: | $2\sqrt{2}$ | $2\sqrt{2}$ |

14

(6) 8. Evaluate the intergral $\int_{-2}^{2}\sqrt{4-x^2}\,dx$ by interpreting it in terms of an area.

$y=\sqrt{4-x^2} \Rightarrow x^2+y^2=2^2$: circle (top half)



$\int_{-2}^{2}\sqrt{4-x^2}\,dx$ 3 $=$ shaded area

$=\frac{1}{2}\pi(2)^2$

3

$\boxed{2\pi}$

6

(6) 9. Find the most general antiderivative of $f(x)=\sin x+5e^x$.

$\boxed{-\cos x+5e^x+C}$

3 3

6

-1 pt for missing C