NAME GRADING KEY

10-digit PUID ______________________

RECITATION INSTRUCTOR ______________________

RECITATION TIME ______________________

Page 1	/20
Page 2	/36
Page 3	/18
Page 4	/26
TOTAL	/100

DIRECTIONS

1. Write your name, 10-digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit. Please write neatly. Remember, if we cannot read your work and follow it logically, you may receive no credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators, or any electronic devices may be used on this exam.

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(10) 1. Find the absolute maximum and absolute minimum values of the function $f(x) = \frac{x^2-4}{x^2+4}$ on the interval $[-2, 4]$.

Critical numbers:

$$f'(x) = \frac{(x^2+4)2x - (x^2-4)2x}{(x^2+4)^2} = \frac{16x}{(x^2+4)^2}$$ ③

$f'(x)=0$: $\frac{16x}{(x^2+4)^2} = 0 \longrightarrow x = 0$ ①

$f(-2) = 0$

$f(0) = -1$

$f(4) = \frac{16-4}{16+4} = \frac{12}{20} = \frac{3}{5}$

③

abs. max. $f(4) = \frac{3}{5}$

abs. min. $f(0) = -1$

③

10

(10) 2. Suppose that the function f is continuous on $[-3, 5]$ and differentiable on $(-3, 5)$. If $f(5) = 12$ and $-1 \le f'(x) \le 2$ for $x \in (-3, 5)$, what is the smallest possible value of $f(-3)$?

From Mean Value Theorem: $\frac{f(5)-f(-3)}{5-(-3)} = f'(c)$ ⑤ for some $c \in (-3, 5)$

Since $-1 \le f'(c) \le 2$, $-1 \le \frac{12 - f(-3)}{8} \le 2$

$-8 \le 12 - f(-3) \le 16$

$-20 \le -f(-3) \le 4$

$20 \ge f(-3) \ge -4$

⑤

$f(-3) \ge -4$

10

Name: ______________________________

(6) 3. Circle the correct answer in each of the boxes. Suppose that f'' is continuous near 3, $f''(3) = 0$, and f' changes sign from positive to negative at 3. Using the

NPC

[✓ first derivative test]
second derivative test

we conclude that f has

$\boxed{6}$

[✓ a local maximum at 3]
a local minimum at 3
no local maximum or minimum at 3

(6) 4. Find the x–coordinates of the inflection points of the graph of $f(x) = 3x^5 - 5x^4 + 2x + 1$.

$f'(x) = 15x^4 - 20x^3 + 2$

$f''(x) = 60x^3 - 60x^2 = 60x^2(x-1)$

NPC

$f''(x)$: $\xrightarrow{\;------\underset{0}{0}---\underset{1}{0}++++\;} x$

$x = 1$

$\boxed{6}$

For #4, 0 credit if answer is correct but there is no work.

(24) 5. Find each of the following as a real number, $+\infty$, $-\infty$, or write DNE (does not exist).

(a) $\lim\limits_{x\to\infty} \frac{(\ln x)^2}{x}$ $\frac{\infty}{\infty}$ $\overset{L'H}{=} \lim\limits_{x\to\infty} \frac{2(\ln x)\frac{1}{x}}{1}$ ② $=$

$= 2\lim\limits_{x\to\infty} \frac{\ln x}{x}$ $\frac{\infty}{\infty}$ $\overset{L'H}{=} 2\lim \frac{\frac{1}{x}}{1} = 0$ ②

② 0

$\boxed{6}$

(b) $\lim\limits_{x\to 0^+} \frac{\ln x}{x}$ $\lim\limits_{x\to 0^+} \ln x = -\infty$ ②, $\lim\limits_{x\to 0^+} x = 0$ ②

$\therefore \lim\limits_{x\to 0^+} \frac{\ln x}{x} = -\infty$

② $-\infty$

$\boxed{6}$

(c) $\lim\limits_{x\to\frac{\pi}{2}} \frac{1-\sin x}{(x-\frac{\pi}{2})^2}$ $\frac{0}{0}$ $\overset{L'H}{=} \lim\limits_{x\to\frac{\pi}{2}} \frac{-\cos x}{2(x-\frac{\pi}{2})}$ ② $\frac{0}{0}$ $\overset{L'H}{=} \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x}{2}$ ② $= \frac{1}{2}$

② $\frac{1}{2}$

$\boxed{6}$

(d) $\lim\limits_{x\to 0^+} (1-2x)^{\frac{1}{x}}$

$\lim\limits_{x\to 0^+} (1-2x)^{\frac{1}{x}} = \lim\limits_{x\to 0^+} e^{\ln(1-2x)^{1/x}} = \lim\limits_{x\to 0^+} e^{\frac{1}{x}\ln(1-2x)}$ ②

$= e^{\lim_{x\to 0^+} \frac{1}{x}\ln(1-2x)}$

$\lim\limits_{x\to 0^+} \frac{\ln(1-2x)}{x}$ $\frac{0}{0}$ $\overset{L'H}{=} \lim\limits_{x\to 0^+} \frac{\frac{1}{1-2x}(-2)}{1} = -2$ ②

$\therefore \lim\limits_{x\to 0^+} (1-2x)^{\frac{1}{x}} = e^{-2}$

② e^{-2}

$\boxed{6}$

Name: ______________________________

(18) 6. Let $f(x) = \frac{x-1}{x^2}$. Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation for each asymptote. Write NONE where appropriate.

Domain: all $x \neq 0$

x-int. $y=0 \to x=1$

y-int. none

Symmetry none

$\lim_{x\to\infty} \frac{x-1}{x^2} = 0$

$\lim_{x\to-\infty} \frac{x-1}{x^2} = 0$

∴ Hor. as. $y=0$

$\lim_{x\to 0^+} \frac{x-1}{x^2} = -\infty$

$\lim_{x\to 0^-} \frac{x-1}{x^2} = -\infty$

∴ Ver. as. $x=0$

$f'(x) = \frac{x^2\cdot 1-(x-1)2x}{x^4} = \frac{-x^2+2x}{x^4} = -\frac{x-2}{x^3}$

$f'(x)$ - - - - | + + + + 0 - - - - x

0 2

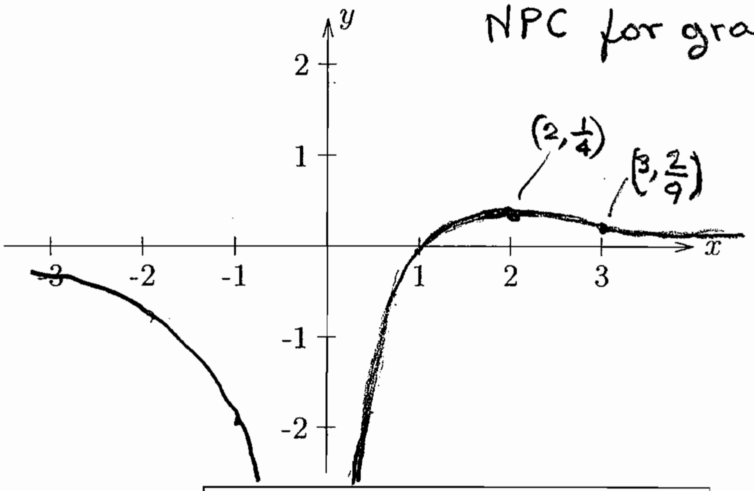
↑ loc max

$f''(x) = -\frac{x^3\cdot 1-(x-2)3x^2}{x^6} = -\frac{-2x^3+6x^2}{x^6} = 2\frac{x-3}{x^4}$

$f''(x)$ - - - | - - - - 0 + + + x

0 3

pt of infl. $(3, \frac{2}{9})$



NPC for graph

④

domain	$(-\infty, 0) \cup (0, \infty)$	①
intercepts	$(1, 0)$	①
symmetry	NONE	①
horizontal asymptotes	$y=0$	①
vertical asymptotes	$x=0$	①
intervals of increase	$(0, 2)$	①
intervals of decrease	$(-\infty, 0), (2, \infty)$	②
local maxima	$(2, \frac{1}{4})$	①
local minima	NONE	①
intervals of concave down	$(-\infty, 0), (0, 3)$	②
intervals of concave up	$(3, \infty)$	①
points of inflection	$(3, \frac{2}{9})$	①

Name: ______________________________

(14) 7. A box with square base must have a volume of 20 ft^3. The material for the base costs 30 cents/ft^2, the material for the sides costs 10 cents/ft^2, and the material for the top costs 20 cents/ft^2. Determine the dimensions of the box that can be constructed at minimum cost. (Let x denote the length of a side of the base and y denote the height of the box).

$x^2y = 20$ ②

Cost $C = x^2(30) + 4xy(10) + x^2(20)$ ②

$y = \frac{20}{x^2}$

$C = 50x^2 + \frac{800}{x}$ ② , $0 < x$

$\frac{dC}{dx} = 100x - \frac{800}{x^2}$ ②

$\frac{dC}{dx} = 0$: $100x - \frac{800}{x^2} = 0 \rightarrow x^3 = 8 \rightarrow x = 2$

$\frac{dC}{dx}$ − − − 0 + + +, on the line marked 0 and 2; ↑ min at 2

$y = \frac{20}{4} = 5$

④ ②

$x =$ 2 ft, $y =$ 5 ft

$\boxed{14}$

(6) 8. Find the most general antiderivative of

$$f(x) = \sin x + 3e^x - \frac{2}{\sqrt{1-x^2}}$$

② ② ②

$-\cos x + 3e^x - 2\sin^{-1}x + C$

−1 pt for missing $+C$

$\boxed{6}$

(6) 9. Find $f(t)$ if $f'(t) = 2\cos t + \sec^2 t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$, and $f(\frac{\pi}{3}) = 4$.

$f(t) = 2\sin t + \tan t + C$

$t = \frac{\pi}{3}$: $4 = 2\sin\frac{\pi}{3} + \tan\frac{\pi}{3} + C$

$4 = 2\frac{\sqrt{3}}{2} + \sqrt{3} + C$

$C = 4 - 2\sqrt{3}$

② ② ②

$f(t) = 2\sin t + \tan t + 4 - 2\sqrt{3}$

$\boxed{6}$