MA 165 EXAM 3 Fall 2010

NAME GRADING KEY

10-digit PUID

RECITATION INSTRUCTOR

RECITATION TIME

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Page 2	/36
Page 3	/16
Page 4	/30
TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

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(9) 1. Find the absolute maximum and absolute minimum values of the function $f(x) = \frac{x}{x^2+4}$ on the interval $[0, 4]$.

$f'(x) = \frac{(x^2+4)1 - x2x}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$ ②

$f'(x) = 0 : \frac{4-x^2}{(x^2+4)^2} = 0 \rightarrow 4-x^2=0 \rightarrow x = 2, \cancel{x=-2}$ ③ ← not in $(0,4)$

$f(0) = 0$ ← abs. min.

$f(2) = \frac{2}{8} = \frac{1}{4}$ ← abs. max.

$f(4) = \frac{4}{20} = \frac{1}{5}$

abs. max. $f(2) = \frac{1}{4}$ ②

abs. min. $f(0) = 0$ ②

9

(9) 2. Let $f(x) = 5 - x^{\frac{2}{3}}$ and note that $f(-1) = f(1) = 4$

(a) Can you apply Rolle's theorem?

circle one YES [✓ NO]

(b) If your answer is YES, find a numer $c \in (-1, 1)$ such that $f'(c) = 0$.
If your answer is NO, explain.

$f'(x) = -\frac{2}{3}x^{-\frac{1}{3}}$ f is not differentiable at $x=0$

or f is not differentiable in $(-1, 1)$

NPC : Both (a) and (b) must be correct

9

(30) 3. Find each of the following as a real number, $+\infty$, $-\infty$, or write DNE (does not exist).

(a) $\underset{\frac{0}{0}}{\lim_{x\to 0} \frac{e^x-1-x}{x^2}}. \overset{L'H}{=} \underset{\frac{0}{0}}{\lim_{x\to 0} \frac{e^x-1}{2x}} \overset{L'H}{=} \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$

5 pts each NPC

$\boxed{\frac{1}{2}}$

(b) $\underset{\infty-\infty}{\lim_{x\to\infty} (x\, e^{\frac{1}{x}} - x)} = \underset{\infty\cdot 0}{\lim_{x\to\infty} x(e^{\frac{1}{x}}-1)} = \underset{\frac{0}{0}}{\lim_{x\to\infty} \frac{e^{\frac{1}{x}}-1}{\frac{1}{x}}}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{e^{\frac{1}{x}}(-\frac{1}{x^2})}{-\frac{1}{x^2}} = \lim_{x\to\infty} e^{\frac{1}{x}} = e^0 = 1$

$\boxed{1}$

(c) $\underset{\frac{0}{0}}{\lim_{x\to 0} \frac{\sin 5x}{\tan 3x}} \overset{L'H}{=} \lim_{x\to 0} \frac{5\cos 5x}{3\sec^2 x} = \frac{5}{3}$

$\boxed{\frac{5}{3}}$

(d) $\underset{\frac{0}{0}}{\lim_{x\to 0} \frac{\sin x - x}{x^3}}. \overset{L'H}{=} \underset{\frac{0}{0}}{\lim_{x\to 0} \frac{\cos x - 1}{3x^2}} \overset{L'H}{=} \lim_{x\to 0} \frac{-\sin x}{6x}$

$= -\frac{1}{6} \lim_{x\to 0} \frac{\sin x}{x} = -\frac{1}{6}\, 1 = -\frac{1}{6}$

$\boxed{-\frac{1}{6}}$

(e) $\lim_{x\to 0^+} (\csc x - \cot x) = \lim_{x\to 0^+} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$

$= \underset{\frac{0}{0}}{\lim_{x\to 0^+} \frac{1-\cos x}{\sin x}} \overset{L'H}{=} \lim_{x\to 0} \frac{\sin x}{\cos x} = \frac{0}{1} = 0$

$\boxed{0}$

(f) $\lim_{x\to 0} (1-3x)^{\frac{5}{x}} = \lim_{x\to 0} e^{[\ln(1-3x)]\frac{5}{x}} = e^{\lim_{x\to 0} \frac{5}{x}\ln(1-3x)} = e^{-15}$

$\underset{\frac{0}{0}}{\lim_{x\to 0} \frac{5\ln(1-3x)}{x}} \overset{L'H}{=} \lim_{x\to 0} \frac{5\frac{1}{1-3x}(-3)}{1} = -15$

$\boxed{e^{-15}}$

$\boxed{30}$

(6) 4. If f' is continuous, $f(2) = 0$ and $f'(2) = 7$, find $\lim_{x\to 0} \frac{f(2+3x)+f(2+5x)}{x}$

$\underset{\frac{0}{0}}{\lim_{x\to 0} \frac{f(2+3x)+f(2+5x)}{x}} \overset{L'H}{=} \lim_{x\to 0} \frac{f'(2+3x)\cdot 3 + f'(2+5x)\cdot 5}{1}$

③ for attempting to use L'Hospital's rule

$= 3\lim_{x\to 0} f'(2+3x) + 5\lim_{x\to 0} f'(2+5x)$

because f' is continuous

$= 3\cdot f'(2) + 5f'(2)$ ②

$= 3\cdot 7 + 5\cdot 7 = 56$ ①

$\boxed{56}$

$\boxed{6}$

(16) 5. Let $f(x) = x - \ln x$. Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation for each asymptote. Write NONE where appropriate.

$f(x) = x - \ln x$

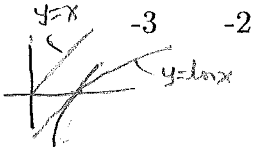
Domain $x > 0$

y-intercept none

x-intercepts : $y = 0$

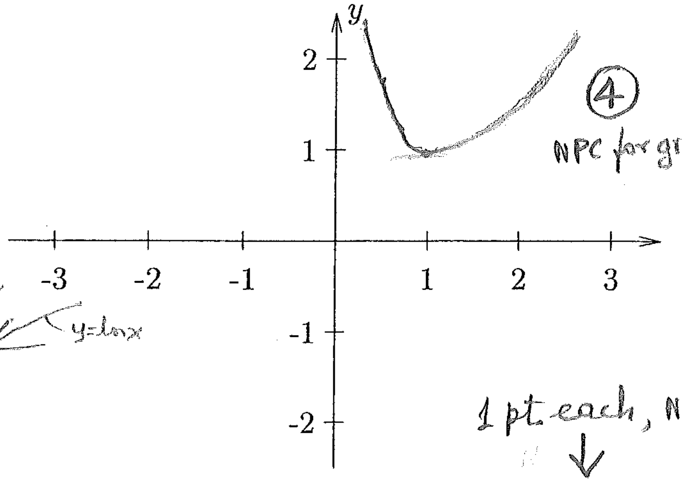
$x - \ln x = 0$

$x = \ln x$



none

Symmetry: None



④ NPC for graph

H.A. $\lim_{x\to\infty}(x - \ln x) = \lim_{x\to\infty} x\left[1 - \underbrace{\left(\frac{\ln x}{x}\right)}_{0}\right] = \infty$

NONE

V.A. $\lim_{x\to 0^+}(x - \ln x) = \infty$

$f'(x) = 1 - \frac{1}{x}$

$f'(x) = 0$: $x = 1$

$f'(x)$: 0 | − − − 0 + + + → x ; 1 ↑ loc. min.

$f(1) = 1$

$f''(x) = \frac{1}{x^2} > 0$

$f''(x)$: 0 | + + + + + → x

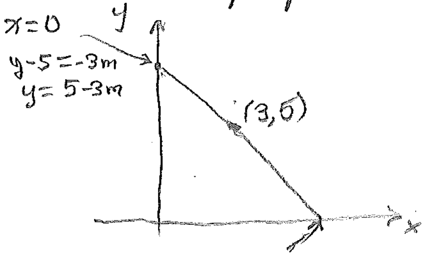
1 pt each, NPC

domain	(0, ∞)
intercepts	NONE
symmetry	NONE
horizontal asymptotes	NONE
vertical asymptotes	$x = 0$
intervals of increase	(1, ∞)
intervals of decrease	(0, 1)
local maxima	NONE
local minima	(1, 1)
intervals of concave down	NONE
intervals of concave up	(0, ∞)
points of inflection	NONE

$\boxed{16}$

(12) 6. Find the slope m of the line through the point $(3, 5)$ that cuts the least area from the first quadrant.

eq. of line through $(3,5)$: $y-5=m(x-3)$ $m<0$



$y=0$; $-5=mx-3m$
$x=\frac{3m-5}{m}$

Let A = area of region cut off from the first quadrant

$$A=\frac{1}{2}\,\frac{3m-5}{m}(5-3m)=\frac{15m-25-9m^2+15m}{2m}$$

$$A=15-\frac{9}{2}m-\frac{25}{2m}\ \text{⑤}\ ,\ -\infty<m<0$$

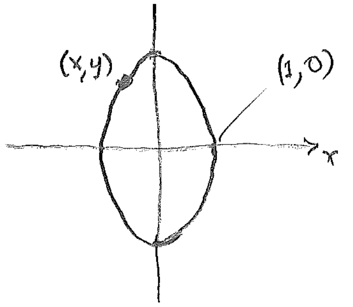
$$\frac{dA}{dm}=-\frac{9}{2}+\frac{25}{2m^2}\underset{\text{or}}{=}\frac{25-9m^2}{2m^2}\ \text{⑤}$$

$$\frac{dA}{dm}=0\ :\ 25-9m^2=0\rightarrow m^2=\frac{25}{9}\rightarrow m=-\frac{5}{3}$$

$\frac{dA}{dm}$ $- - - - - - 0 + + + +$ at $-\frac{5}{3}$, 0 abs. min.

$$\boxed{m=-\frac{5}{3}\quad ②}\qquad \boxed{12}$$

(12) 7. Find the x–coordinate of the points on the ellipse $4x^2+y^2=4$ that are farthest from the point $(1, 0)$.



Let D = distance of a point (x,y) on the ellipse from $(1,0)$

$$D=\sqrt{(x-1)^2+y^2}$$

$$D=\sqrt{(x-1)^2+4-4x^2}\ \text{⑤}\qquad -1\le x\le 1$$

$$\frac{dD}{dx}=\frac{1}{2\sqrt{(x-1)^2+4-4x^2}}\left[2(x-1)-8x\right]$$

$$\frac{dD}{dx}=0\ :\ 2(x-1)-8x=0\ \text{⑤}\longrightarrow -6x=2\rightarrow x=-\frac{1}{3}$$

$D(-1)=\sqrt{4}=2$

$D(1)=0$

$D(-\frac{1}{3})=\sqrt{(-\frac{4}{3})^2+4-4(-\frac{1}{3})^2}=\sqrt{\frac{16}{9}+4-\frac{4}{9}}$

$=\sqrt{4+\frac{12}{9}}$ ← abs. max.

$$\boxed{x=-\frac{1}{3}\quad ②}\qquad \boxed{12}$$

(6) 8. Find the function f such that $f'(x)=\dfrac{1}{x^2+1}$ and $f(-\sqrt{3})=1$.

$f(x)=\tan^{-1}x+C$

$x=-\sqrt{3}$: $1=\tan^{-1}(-\sqrt{3})+C$

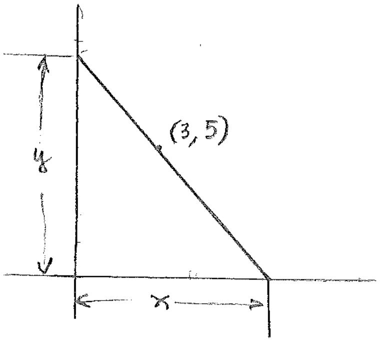
$1=-\frac{\pi}{3}+C$

$C=1+\frac{\pi}{3}$

$$\boxed{f(x)=\overset{③}{\tan^{-1}x}\ +\overbrace{1+\frac{\pi}{3}}^{③}}\qquad \boxed{6}$$

Alternate solution of problem 6

(12) 6. Find the slope m of the line through the point $(3,5)$ that cuts the least area from the first quadrant.



Let A be the area of the region cut off from the first quadrant

$A = \frac{1}{2}xy$

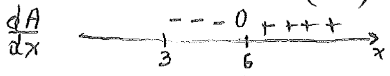
From similar triangles:

$\frac{y}{x} = \frac{y-5}{3} \rightarrow 3y = xy - 5x \rightarrow y = \frac{5x}{x-3}$

$A = \frac{1}{2} x \frac{5x}{x-3} = \frac{5}{2}\frac{x^2}{x-3}$ ⑤ $3 < x < \infty$

$\frac{dA}{dx} = \frac{5}{2}\frac{(x-3)2x - x^2 \cdot 1}{(x-3)^2} = \frac{5}{2}\frac{x^2-6x}{(x-3)^2}$ ⑤

$\frac{dA}{dx} = 0 : \frac{5}{2}\frac{x(x-6)}{(x-3)^2} = 0 \rightarrow x = 6$



abs min of A.

From the points (3,5) and (6,0)

$m = \frac{0-5}{6-3} = -\frac{5}{3}$

$m = -\frac{5}{3}$ ②

$\boxed{12}$

(12) 7. Find the x–coordinate of the points on the ellipse $4x^2 + y^2 = 4$ that are farthest from the point $(1,0)$.

$x =$

(6) 8. Find the function f such that $f'(x) = \frac{1}{x^2+1}$ and $f(-\sqrt{3}) = 1$.

$f(x) =$