NAME SOLUTIONS

STUDENT ID # ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

LECTURER ____________

INSTRUCTIONS

1. There are 9 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–9.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. $\lim_{x\to 3} \dfrac{x^2-x-6}{x-3} = \lim_{x\to 3} \dfrac{(x-3)(x+2)}{x-3}$

$= \lim_{x\to 3} (x+2) = 5$

A. Does not exist
B. 5
C. 3
D. 1
E. 0

2. The domain of $f(x) = \sqrt{1-\ln(2-x)}$ is

$\ln(2-x)$ is defined for $2-x>0$ : $\boxed{x<2}$

$\sqrt{1-\ln(2-x)}$ is defined for

$1-\ln(2-x) \geq 0$

$\ln(2-x) \leq 1$

$2-x \leq e \rightarrow \boxed{2-e \leq x}$

$\therefore\ 2-e \leq x < 2$

A. $x<2$
B. $-2 \leq x < 2$
C. $2<x$
D. $2-e<x$
E. $2-e \leq x < 2$

3. If $f(x) = |x|$, choose the correct statement.

A. $f$ is continuous but not differentiable at $x=0$
B. $f$ is differentiable but not continuous at $x=0$
C. $f$ is differentiable and continuous at $x=0$
D. $f$ is continuous but not differentiable at $x=1$
E. $f$ is differentiable but not continuous at $x=-1$

4. Assume that $y$ is defined implicitly as a differentiable function of $x$ by the equation $x^3 - x^2y + y^3 = 7$. Find $\dfrac{dy}{dx}$ at $(x,y) = (1,2)$.

$3x^2 - x^2\dfrac{dy}{dx} - 2xy + 3y^2\dfrac{dy}{dx} = 0$

At $(x,y)=(1,2)$: $3\cdot 1 \ -1\dfrac{dy}{dx} - 2\cdot 1\cdot 2 + 3\cdot 4\dfrac{dy}{dx} = 0$

$11\dfrac{dy}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{1}{11}$

A. $\frac{1}{11}$
B. $\frac{3}{7}$
C. 2
D. $\frac{4}{3}$
E. $\frac{1}{21}$

5. The equation $x^3 + 10x + 1 = 0$ has a root in the interval

Let $f(x) = x^3+10x+1$ and use Intermediate Value Theorem

$f(-3) = -27 - 30 + 1 = -56$

$f(-1) = -1 - 10 + 1 = -10 < 0$

$f(0) = 1 > 0$ $\therefore$ root in $(-1,0)$

A. $(-3,-1)$
B. $(-1,0)$
C. $(0,1)$
D. $(1,2)$
E. $(2,3)$

6. The equation of the tangent line to the graph of $y = \sin^{-1}(2x)$ at the point where $x = \frac{1}{4}$ is

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$$

At $x=\frac{1}{4}$: $\frac{dy}{dx} = \frac{2}{\sqrt{1-\frac{1}{4}}} = \frac{4}{\sqrt{3}}$

and $y = \sin^{-1}(2\cdot\frac{1}{4}) = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

equation of tangent line:

$$y - \frac{\pi}{6} = \frac{4}{\sqrt{3}}\left(x - \frac{1}{4}\right)$$

or $$y = \frac{4}{\sqrt{3}}x + \frac{\pi}{6} - \frac{1}{\sqrt{3}}$$

A. $y = \frac{1}{\sqrt{3}}x + \frac{\pi}{4} - \frac{1}{4\sqrt{3}}$
B. $y = \frac{2}{\sqrt{3}}x + \frac{\pi}{6} - \frac{1}{2\sqrt{3}}$
C. $y = \frac{4}{\sqrt{3}}x + \frac{\pi}{6} - \frac{1}{\sqrt{3}}$
D. $y = x + \frac{\pi}{3} - \frac{1}{4}$
E. $y = \sqrt{3}x + \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

7. If $f(x) = x + \frac{1}{x}$, then $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = f'(2)$ by definition

$$f'(x) = 1 - \frac{1}{x^2}$$

$$f'(2) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \frac{3}{4}$$

A. $\frac{1}{2}$
B. $\frac{5}{2}$
C. $\frac{1}{3}$
D. $\frac{3}{4}$
E. Does not exist

8. $\frac{d}{dx}\left(x^2 e^{\sin(3x)}\right) = x^2 e^{\sin(3x)} \cos(3x)\cdot 3$
$+ 2x e^{\sin(3x)}$

A. $3x^2 e^{\sin(3x)} + 2xe^{\sin(3x)}$

B. $x^2 e^{\sin(3x)} \cos(3x) + 3x^2 e^{\sin(3x)}$

C. $x^2 e^{\sin(3x)} + 2xe^{\sin(3x)} \cos(3x)$

D. $3x^2 \cos(3x) e^{\sin(3x)} + 2xe^{\sin(3x)}$

E. $x^2 e^{\sin(3x)} + 2xe^{\sin(3x)}$

9. The position of a particle is given by the equation $s(t) = 3t^2 + e^{2t} + \cos(\pi t)$. The acceleration of the particle at $t = 1$ is

$v(t) = \frac{ds(t)}{dt} = 6t + 2e^{2t} - \pi \sin(\pi t)$

$a(t) = \frac{dv(t)}{dt} = 6 + 4e^{2t} - \pi^2 \cos(\pi t)$

$a(1) = 6 + 4e^2 - \pi^2 \cos\pi$

$= 6 + 4e^2 + \pi^2$

A. $4 + e^2 - \pi^2$

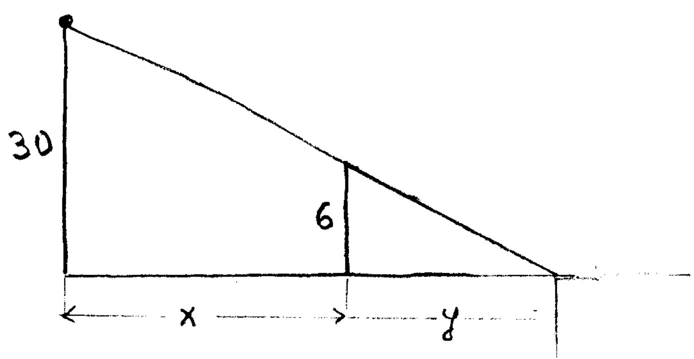
B. $6 + 2e^2 + \pi$

C. $1 + e^2 - 4\pi$

D. $3 + 4e^2 + \pi^2$

E. $6 + 4e^2 + \pi^2$

10. The lamp of a street light is 30 ft above ground. A man 6 ft tall walks towards the street light. If the length of the man's shadow is decreasing at the rate of 2 ft/sec, how fast is he walking?




A. 8 ft/sec

B. 2 ft/sec

C. $\frac{1}{5}$ ft/sec

D. 5 ft/sec

E. 3 ft/sec

$\frac{dy}{dt} = -2$

$\frac{y}{6} = \frac{x+y}{30}$

$30y = 6x + 6y$

$x = 4y$

$\frac{dx}{dt} = 4\frac{dy}{dt} = 4(-2)$

11. The linear approximation of $f(x) = \sqrt{x+1}$ at $a = 3$ is

$f(x) \approx f(3) + f'(3)(x-3)$, for $x$ near 3

$f(3) = 2$

$f'(x) = \dfrac{1}{2\sqrt{x+1}}$

$f'(3) = \dfrac{1}{4}$

$f(x) \approx 2 + \frac{1}{4}(x-3)$, for $x$ near 3

or $f(x) \approx \frac{1}{4}x + \frac{5}{4}$, for $x$ near 3

A. $\sqrt{x+1} \approx \frac{1}{2}x - \frac{1}{2}$, for $x$ near 3

B. $\sqrt{x+1} \approx \frac{1}{2}x + \frac{1}{2}$, for $x$ near 3

C. $\sqrt{x+1} \approx \frac{1}{4}x + \frac{5}{4}$, for $x$ near 3

D. $\sqrt{x+1} \approx \frac{1}{4}x + \frac{1}{4}$, for $x$ near 3

E. $\sqrt{x+1} \approx \frac{1}{4}x - \frac{1}{4}$, for $x$ near 3

12. The absolute minimum value of $f(x) = 3x^2 - 12x + 5$ on the interval $[0, 3]$ is

$f'(x) = 6x - 12$

$f'(x) = 0$ : $6(x-2) = 0 \rightarrow x = 2$ critical number in $(0,3)$

$f(0) = 5$

$f(2) = 3 \cdot 4 - 24 + 5 = -7$ ← abs. min. value

$f(3) = 3 \cdot 9 - 12 \cdot 3 + 5 = -4$

A. 7

B. −7

C. −4

D. 5

E. 3

13. For what value(s) of $x$ does $f(x) = \dfrac{e^x}{x}$ have a local maximum?

$f'(x) = \dfrac{xe^x - e^x}{x^2}$

$f'(x) = 0$ : $\dfrac{e^x(x-1)}{x^2} = 0$

$x = 1$ is the only critical number

$f'(x) < 0$ for $x < 1$

$f'(x) > 0$ for $x > 1$

$\therefore$ $f(1) = e$ is a local minimum

There is no local maximum

A. $x = 0$

B. $x = \frac{1}{2}$

C. $x = 2$

D. $x = 1$

E. There is no local maximum

14. Find all intervals on which the graph of

$$f(x) = \cos x - \sin x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$$

is concave down.

$f'(x) = -\sin x - \cos x$

$f''(x) = -\cos x + \sin x$

$f''(x) = 0$ : $\sin x = \cos x \rightarrow \tan x = 1$

$x = \frac{\pi}{4}$

$f''(x) < 0$ for $-\frac{\pi}{2} < x < \frac{\pi}{4}$ ←

$f''(x) > 0$ for $\frac{\pi}{4} < x < \frac{\pi}{2}$

conc. down in $(-\frac{\pi}{2}, \frac{\pi}{4})$

A. $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$
B. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
C. $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
D. $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$
E. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

15. $\lim_{x\to\infty}\left(1 - \frac{1}{x}\right)^x = \lim_{x\to\infty} e^{x\ln(1-\frac{1}{x})} = e^{\lim_{x\to\infty}[x\ln(1-\frac{1}{x})]}$

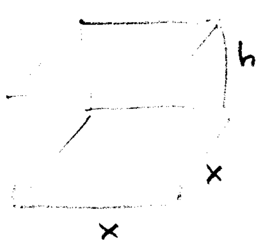
$\lim_{x\to\infty}[x\ln(1-\frac{1}{x})] = \lim_{x\to\infty}\frac{\ln(1-\frac{1}{x})}{\frac{1}{x}} \overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{1}{1-\frac{1}{x}}\left(\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$ $\quad \frac{0}{0}$

$= \lim_{x\to\infty} -\frac{1}{1-\frac{1}{x}} = -1$

$\therefore \lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x = e^{-1}$

A. 1
B. $e$
C. $\frac{1}{e}$
D. $\infty$
E. $-e$

16. A crate with square base and open top must have a volume of 4 cubic meters. Find the height of the crate that has the smallest possible surface area.




Volume = 4

$x^2h = 4$

Surface area:

$A = x^2 + 4xh$,

$A = x^2 + 4x\frac{4}{x^2}$

$A = x^2 + \frac{16}{x}$

$\frac{dA}{dx} = 2x - \frac{16}{x^2} = \frac{2}{x^2}(x^3 - 8)$

$\frac{dA}{dx} = 0$ : $\frac{2}{x^2}(x^3-8) = 0 \longrightarrow x = 2$

$\frac{dA}{dx} < 0$ for $x < 2$ $\quad \frac{dA}{dx} > 0$ for $x > 0$ $\quad$ min.

$x = 2 \longrightarrow h = \frac{4}{4} = 1$

A. $h = 2$
B. $h = 1$
C. $h = 4$
D. $h = \frac{1}{2}$
E. $h = \frac{1}{4}$

17. Find $f(x)$ if $f''(x) = e^x - \sin x$, and $f'(0) = 1$, $f(0) = -1$.

$f'(x) = e^x + \cos x + C_1$

$x=0$: $1 = 1 + 1 + C_1 \rightarrow C_1 = -1$

$\therefore f'(x) = e^x + \cos x - 1$

$f(x) = e^x + \sin x - x + C_2$

$x=0$: $-1 = 1 + 0 - 0 + C_2 \rightarrow C_2 = -2$

$f(x) = e^x + \sin x - x - 2$

A. $f(x) = e^x + \sin x - x - 2$
B. $f(x) = e^x + \sin x + x - 2$
C. $f(x) = e^x - \sin x - 2$
D. $f(x) = e^x + \cos x - 3$
E. $f(x) = e^x + \cos x - x$

18. $\frac{d}{dx} \int_1^{2x} e^{3t^2}\, dt = e^{3(2x)^2} \cdot 2$

$= 2e^{12x^2}$

A. $e^{3x^2}$
B. $2e^{6x^2}$
C. $e^{12x^2}$
D. $e^{6x^2} - e^3$
E. $2e^{12x^2}$

19. $\int_0^{\frac{\pi}{4}} \frac{\cos^3 x - 1}{\cos^2 x}\, dx = \int_0^{\frac{\pi}{4}} (\cos x - \sec^2 x)\, dx$

$= (\sin x - \tan x)\Big|_0^{\frac{\pi}{4}}$

$= \left(\frac{1}{\sqrt{2}} - 1\right) - (0 - 0)$

A. $-1 + \sqrt{2}$
B. $\sqrt{2} + 1$
C. $-1 - \frac{1}{\sqrt{2}}$
D. $\frac{1}{\sqrt{2}} - 1$
E. $1 + \frac{1}{\sqrt{2}}$

20. Find the area of the region between the graph of $y = \dfrac{1}{x \ln x}$ and the $x$-axis from $x = e$ to $x = e^2$.

$A = \int_e^{e^2} \frac{1}{x \ln x} dx = \int_1^2 \frac{1}{u} du = \ln u \Big|_1^2$

$u = \ln x \quad du = \frac{1}{x} dx$

$x = e \rightarrow u = 1$

$x = e^2 \rightarrow u = \ln e^2 = 2$

$= \ln 2 - \ln 1$

$= \ln 2$

A. $e - 1$

B. $(\ln 2) + 1$

C. $\ln 2$

D. $\ln(2 + e)$

E. $\ln(e^2 - e)$

21. $\int_0^1 \frac{x}{(x+1)^3} dx = \int_1^2 \frac{u-1}{u^3} du = \int_1^2 (u^{-2} - u^{-3}) du =$

$u = x + 1 \quad du = dx$

$x = u - 1$

$x = 0 \longrightarrow u = 1$

$x = 1 \longrightarrow u = 2$

$= \left( \frac{u^{-1}}{-1} - \frac{u^{-2}}{-2} \right) \Big|_1^2$

$= \left( -\frac{1}{u} + \frac{1}{2u^2} \right) \Big|_1^2$

$= \left( -\frac{1}{2} + \frac{1}{8} \right) - \left( -1 + \frac{1}{2} \right) = \frac{1}{8}$

A. $\frac{5}{8}$

B. $\frac{1}{2}$

C. $\frac{3}{8}$

D. $\frac{1}{8}$

E. $\frac{1}{4}$

22. $\int_0^1 \frac{\tan^{-1} x}{1 + x^2} dx = \int_0^{\frac{\pi}{4}} u \, du = \frac{u^2}{2} \Big|_0^{\frac{\pi}{4}} = \frac{\pi^2}{32}$

$u = \tan^{-1} x \quad du = \frac{1}{1 + x^2} dx$

$x = 0 \rightarrow u = 0$

$x = 1 \rightarrow u = \frac{\pi}{4}$

A. $\frac{\pi}{4}$

B. $\frac{\pi^2}{32}$

C. $\frac{\pi^2}{8}$

D. $\frac{\pi}{2}$

E. $\frac{\pi^2}{16}$

23. A radioactive substance loses $\frac{7}{8}$ of its mass in 30 days. Its half-life (in days) is

$f(t) = f(0)e^{kt}$ $\qquad f(30) = \frac{1}{8} f(0)$

$t = 30$ $\qquad \frac{1}{8}\cancel{f(0)} = \cancel{f(0)}e^{k30}$

$-\ln 8 = 30k \rightarrow k = -\frac{\ln 8}{30}$

$\therefore f(t) = f(0)e^{-\frac{\ln 8}{30}t}$

$t?$ $\qquad f(t) = \frac{1}{2} f(0)$

$\frac{1}{2}\cancel{f(0)} = \cancel{f(0)}e^{-\frac{\ln 8}{30}t}$

$-\ln 2 = -\frac{\ln 8}{30}t \rightarrow t = 30\frac{\ln 2}{\ln 8}$

A. 15

B. $30\frac{\ln 2}{\ln 8}$

C. $\frac{105}{8}$

D. $-30\frac{\ln 7}{\ln 2}$

E. $-30\frac{\ln\frac{7}{8}}{\ln 2}$

24. The focus of the parabola

$$y^2 - 2y - 8x - 23 = 0$$

is at the point

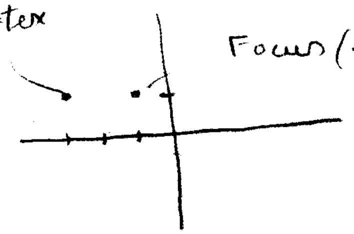
$y^2 - 2y + 1 = 8x + 23 + 1$

$(y-1)^2 = 8(x+3)$

Vertex $(-3,1)$ $\qquad 4p = 8 \rightarrow p = 2$

vertex

Focus $(-1,1)$



A. $(-1,-3)$

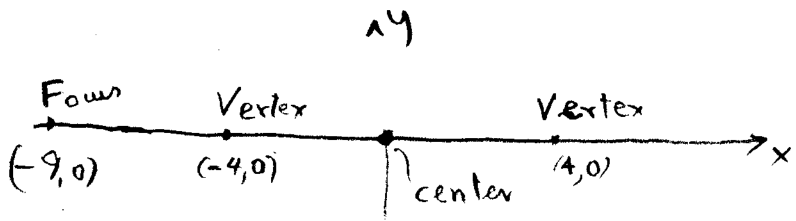
B. $(1,-1)$

C. $(-1,1)$

D. $(3,1)$

E. $(1,2)$

25. Find an equation for the hyperbola with vertices at $(4,0)$ and $(-4,0)$ and a focus at $(-9,0)$.




$a = 4$

$c = 9$

$a^2 + b^2 = c^2$ $\qquad b^2 = c^2 - a^2 = 81 - 16 = 65$

$\frac{x^2}{16} - \frac{y^2}{65} = 1$

A. $\frac{x^2}{16} - \frac{y^2}{25} = 1$

B. $\frac{x^2}{65} - \frac{y^2}{81} = 1$

C. $\frac{x^2}{81} - \frac{y^2}{16} = 1$

D. $\frac{x^2}{16} - \frac{y^2}{81} = 1$

E. $\frac{x^2}{16} - \frac{y^2}{65} = 1$