NAME SOLUTIONS

10–digit PUID # ____________________

RECITATION INSTRUCTOR ____________________

RECITATION TIME ____________________

LECTURER ____________________

INSTRUCTIONS

1. There are 8 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–8.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
4. No books, notes, calculators, or any electronic devices may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10–digit PUID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. $\lim_{x \to -2} \frac{|x| - 2}{x + 2} = \lim_{x \to -2} \frac{-x-2}{x+2} = -1$

$|x| = -x$ near $x = -2$

A. -1
B. 0
C. 1
D. ∞
E. Does not exist

2. $\lim_{x \to 2^+} e^{\frac{3}{2-x}} =$

$\lim_{x \to 2^+} \frac{3}{2-x} = -\infty$

$\therefore \lim_{x \to 2^+} e^{\frac{3}{2-x}} = 0$

A. 0
B. 1
C. e^{-3}
D. ∞
E. Does not exist

3. The domain of $f(x) = \ln\left(\frac{\pi x}{x-1}\right)$ is

$\frac{\pi x}{x-1} > 0$

$\frac{\pi x}{x-1}$ $+ + + + \; 0 \; - - - \; 1 \; + + + \;\; x$

A. $(1, \infty)$
B. $(0, 1)$
C. $(-\infty, 0)$
D. $(1, \infty)$ and $(-\infty, 0)$
E. $\left(\frac{1}{\pi}, \infty\right)$ and $(-\infty, 0)$

4. Find the value of c for which the function

$$f(x) = \begin{cases} 2x+1, & \text{if } x \le c \\ -x+2, & \text{if } x > c \end{cases}$$

is continuous for all x.

f is continuous for all $x \neq c$ (polynomials are continuous)

$f(c) = 2c+1$

$\lim_{x \to c^-} f(x) = 2c+1 \qquad \lim_{x \to c^+} f(x) = -c+2$

$\lim_{x \to c} f(x)$ exists if $2c+1 = -c+2$
or $c = \frac{1}{3}$

When $c = \frac{1}{3}$

$\lim_{x = \frac{1}{3}} f(x) = \frac{5}{3} = f\left(\frac{1}{3}\right)$ and f is cont at $\frac{1}{3}$

A. 0
B. $\frac{1}{2}$
C. $\frac{1}{3}$
D. $\frac{1}{4}$
E. for no value of c

5. $\lim_{x\to 0} \frac{\cos x - 1}{x^2} \overset{L'H}{=} \lim_{x\to 0} \frac{-\sin x}{2x} \overset{L'H}{=} \lim_{x\to 0} \frac{-\cos x}{2}$

$\frac{0}{0}$ $\frac{0}{0}$ $= -\frac{1}{2}$

A. -1

B. $-\frac{1}{2}$

C. 0

D. 1

E. Does not exist

6. If $f(x) = x^2 \ln x$, then $f''(x) =$

$f'(x) = x^2 \frac{1}{x} + 2x \ln x = x + 2x \ln x$

$f''(x) = 1 + 2x \cdot \frac{1}{x} + 2 \ln x$

$= 3 + 2 \ln x$

A. $x + 2x \ln x$

B. $3 + 2 \ln x$

C. $3x + 2 \ln x$

D. $3 + 2x \ln x$

E. $\frac{1}{x}$

7. The equation $y^2 \ln x + y = 2x$ defines y as a function of x. Compute $\frac{dy}{dx}$ at $(x, y) = (1, 2)$.

$y^2 \frac{1}{x} + 2y \frac{dy}{dx} \ln x + \frac{dy}{dx} = 2$

At $(x,y) = (1,2)$:

$4 + 2 \cdot 2 \frac{dy}{dx} \cdot 0 + \frac{dy}{dx} = 2$

$\frac{dy}{dx} = -2$

A. 0

B. 2

C. $-\frac{2}{3}$

D. -4

E. -2

8. Sand falling at the rate of 3 ft^3/min forms a conical pile whose radius is always twice the height. The rate at which the height is changing when the height is 10 feet is

$\frac{dV}{dt} = 3$ $V = \frac{1}{3}\pi r^2 h$ $r = 2h$

Find $\frac{dh}{dt}$ when $h = 10$

$V = \frac{1}{3}\pi (2h)^2 h$

$V = \frac{4}{3}\pi h^3$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

When $h = 10$: $3 = 4\pi\, 10^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{3}{400\pi}$ ft/min

A. $\frac{3}{100\pi}$ ft/min

B. $\frac{3}{200\pi}$ ft/min

C. $\frac{3}{400\pi}$ ft/min

D. $\frac{3}{800\pi}$ ft/min

E. $\frac{1}{50\pi}$ ft/min

9. The function $f(x) = x - \dfrac{4}{x^2}$ has a

A. relative max at $x = 2$

B. relative min at $x = 2$

C. relative max at $x = -2$

D. relative min at $x = -2$

E. none of the above

$f'(x) = 1 + \frac{8}{x^3}$

$f'(x) = 0$: $x^3 = -8 \rightarrow x = -2$

$f''(x) = -\frac{24}{x^4}$

$f''(-2) = -\frac{24}{16} < 0$

$\therefore f$ has rel. max. at $x = -2$

10. The graph of $y = \dfrac{1}{12}x^4 - \dfrac{1}{3}x^3 + \dfrac{1}{2}x^2$ has how many inflection points?

A. None

B. 1

C. 2

D. 3

E. 4

$\frac{dy}{dx} = \frac{1}{3}x^3 - x^2 + x$

$\frac{d^2y}{dx^2} = x^2 - 2x + 1 = (x-1)^2 \geq 0$

$f''(x)$ $+ + + + 0 + + + +$ $\rightarrow x$

CU   1   CU

11. The maximum slope of the curve $y = 6x^2 - x^3$ is

A. 16

B. 2

C. 6

D. 4

E. 12

$\frac{dy}{dx} = 12x - 3x^2$

Slope $S = 12x - 3x^2$

$\frac{dS}{dx} = 12 - 6x$

$\frac{dS}{dx} = 0$: $x = 2$

$\frac{dS}{dx}$: $+ + + + 0 - - -$ $\rightarrow x$

2

$\therefore S$ is max at $x = 2$

$S(2) = 12 \cdot 2 - 3 \cdot 2^2 = 12$

12. If the highest point on the curve $y = K - x^2 - 4x$ is on the x-axis, then $K =$

A. 0

B. -4

C. -2

D. 1

E. 3

$\frac{dy}{dx} = -2x - 4$, $\frac{dy}{dx} = 0$ when $x = -2$

When $x = -2$ $y = 0 \rightarrow 0 = K - (-2)^2 - 4(-2)$

$K = -4$

13. A linear approximation shows that $(16.2)^{\frac{1}{4}}$ is approximately

$f(x) \approx f(a) + f'(a)(x-a)$, for x near a

$f(x) = x^{1/4} \qquad a = 16$

$f'(x) = \frac{1}{4} x^{-3/4}$

$f(16) = 16^{1/4} = 2$

$f'(16) = \frac{1}{4}(16)^{-3/4} = \frac{1}{4}\,\frac{1}{8} = \frac{1}{32}$

$x^{1/4} \approx 2 + \frac{1}{32}(x-16)$, for x near 16

Let $x = 16.2$

$(16.2)^{1/4} \approx 2 + \frac{1}{32}(16.2-16) = 2 + \frac{0.2}{32} = 2 + \frac{1}{160}$

A. $2+\frac{1}{8}$

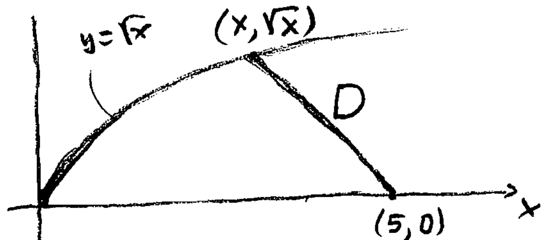
B. $2+\frac{1}{20}$

C. $2+\frac{1}{16}$

D. $2+\frac{1}{32}$

E. $2+\frac{1}{160}$

14. Let P be the point on the curve $y = \sqrt{x}$ that is closest to $(5,0)$. The x-coordinate of P is



$D = \sqrt{(x-5)^2 + x} = \sqrt{x^2 - 10x + 25 - x} = \sqrt{x^2 - 9x + 25}$

$\frac{dD}{dx} = \frac{1}{2\sqrt{x^2-9x+25}}(2x-9)$

$\frac{dD}{dx}$ $- - - \; 0 \; + + +$ at $\frac{9}{2}$ $\quad \therefore$ D is min when $x = \frac{9}{2}$

A. 3

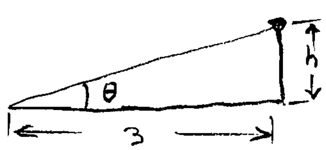
B. $\frac{7}{2}$

C. $\frac{9}{2}$

D. $\frac{\sqrt{3}}{2}$

E. 4

15. An observer 3 miles from the launch pad watches the shuttle go straight up. He measures the angle between the horizontal and his line of sight of the shuttle. When that angle is $\frac{\pi}{4}$, it is increasing at the rate of $\frac{1}{4}$ radians/sec. How fast is the shuttle rising at that instant (in miles/sec)?



When $\theta = \frac{\pi}{4}$ $\quad \frac{d\theta}{dt} = \frac{1}{4}$

Find $\frac{dh}{dt}$ when $\theta = \frac{\pi}{4}$

$\tan\theta = \frac{h}{3}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{3}\frac{dh}{dt}$

When $\theta = \frac{\pi}{4}$: $\quad \sec^2\left(\frac{\pi}{4}\right) \cdot \frac{1}{4} = \frac{1}{3}\frac{dh}{dt}$

$(\sqrt{2})^2 \cdot \frac{1}{4} = \frac{1}{3}\frac{dh}{dt} \longrightarrow \frac{dh}{dt} = \frac{3}{2}$

A. 2

B. 1.5

C. 1

D. 1.2

E. 1.75

16. $\int_0^1 xe^{x^2}\,dx = \frac{1}{2}\int_0^1 e^u\,du = \frac{1}{2}e^u\Big|_0^1 = \frac{1}{2}e - \frac{1}{2}$

$= \frac{e-1}{2}$

$u = x^2$

$du = 2x\,dx$

$x=0 \to u=0$

$x=1 \to u=1$

A. $\frac{e^2}{2}$

B. $\frac{e-1}{2}$

C. $\frac{e+1}{2}$

D. $e-2$

E. $\frac{e^2-1}{2}$

17. $\int_1^e \frac{(\ln x)^3}{x}\,dx = \int_0^1 u^3\,du = \frac{u^4}{4}\Big|_0^1 = \frac{1}{4}$

$u = \ln x$

$du = \frac{1}{x}dx$

$x=1 \to u=0$

$x=e \to u=1$

A. $\frac{1}{3}$

B. $\frac{e}{4}$

C. e

D. e^2

E. $\frac{1}{4}$

18. $\int_{-1}^0 x\sqrt{x+1}\,dx = \int_0^1 (u-1)\sqrt{u}\,du = \int_0^1 (u^{3/2} - u^{1/2})\,du$

$= \left[\frac{u^{5/2}}{\frac{5}{2}} - \frac{u^{3/2}}{\frac{3}{2}}\right]_0^1$

$= \frac{2}{5} - \frac{2}{3} = \frac{-4}{15}$

$u = x+1$

$du = dx$

$x = u-1$

$x=-1 \to u=0$

$x=0 \to u=1$

A. $-\frac{4}{3}$

B. $-\frac{4}{5}$

C. $-\frac{4}{9}$

D. $-\frac{4}{15}$

E. $-\frac{2}{15}$

19. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on the interval $[0, 2]$.

$f'(x) = 6x^2 + 6x - 12 = 6(x^2+x-2) = 6(x+2)(x-1)$

$f'(x) = 0$ when $x=-2$ and $x=1$.

But $-2 \notin [0,2]$

$f(0) = 0$

$f(1) = 2+3-12 = -7$

$f(2) = 2\cdot 8 + 3\cdot 4 - 12\cdot 2 = 4$

A. max 4, min -3

B. max 3, min 1

C. max 2, min 0

D. max 0, min -7

E. max 4, min -7

20. If $F(x) = \int_0^{\sqrt{x}} e^{t^2}\,dt$, then $F'(4) =$

$$F'(x) = e^{(\sqrt{x})^2} \frac{1}{2\sqrt{x}} = \frac{e^x}{2\sqrt{x}}$$

$$F'(4) = \frac{e^4}{2 \cdot 2} = \frac{e^4}{4}$$

A. e^2

B. e^4

C. $\frac{e^4}{4}$

D. $\frac{e^2}{4}$

E. $\frac{e^{16}}{4}$

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21. If $f(x) = (\ln x)^x$, then $f'(e) =$

$$f(x) = e^{\ln(\ln x)^x} = e^{x \ln(\ln x)}$$

$$f'(x) = e^{x \ln(\ln x)} \left[x \frac{1}{\ln x} \frac{1}{x} + \ln(\ln x) \right]$$

$$f'(e) = e^{e \ln(\ln e)} \left[\frac{1}{\ln e} + \ln(\ln e) \right]$$

$$= e^0 (1 + 0) = 1$$

A. 1

B. e

C. $\frac{1}{e}$

D. e^e

E. 2

22. The area of the region between the graph of $f(x) = \frac{1}{x^2+1}$ and the x-axis, from $x = 1$ to $x = \sqrt{3}$ is

$$A = \int_1^{\sqrt{3}} \frac{1}{x^2+1}\,dx = \tan^{-1} x \Big|_1^{\sqrt{3}}$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1} 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

C. 1

D. $\frac{3}{2}$

E. $\frac{\pi}{3}$

23. The half–life of a certain radioactive substance is 10 years. How long will it take for 18 gms of the substance to decay to 6 gms?

$m(t) = m_0 e^{kt}$ $\qquad \frac{1}{2} m_0 = m_0 e^{k 10}$

$-\ln 2 = k 10 \rightarrow k = -\frac{\ln 2}{10}$

$m(t) = m_0 e^{-\frac{\ln 2}{10} t}$

Find t so that $6 = 18 e^{-\frac{\ln 2}{10} t}$

$\frac{1}{3} = e^{-\frac{\ln 2}{10} t}$

$-\ln 3 = -\frac{\ln 2}{10} t$

$t = 10 \frac{\ln 3}{\ln 2}$

A. $6 \ln 10$ years

B. $10 \ln 6$ years

C. $10 \frac{\ln 3}{\ln 6}$ years

D. $18 \ln 10$ years

E. $10 \frac{\ln 3}{\ln 2}$ years

24. The focus of the parabola $x^2 + 2x - y + 3 = 0$ is at

A. $(-1, 2)$

B. $(-1, \frac{9}{4})$

C. $(\frac{9}{4}, -1)$

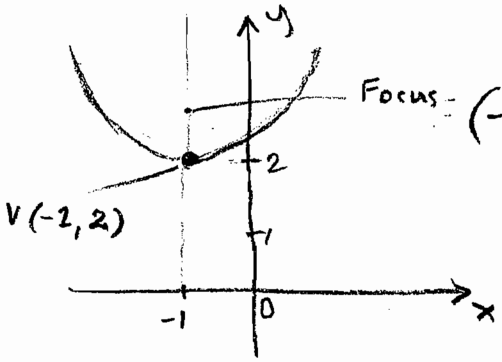
D. $(1, -\frac{9}{4})$

E. $(-1, \frac{7}{4})$

$$x^2 + 2x = y - 3$$
$$x^2 + 2x + 1 = y - 3 + 1$$
$$(x+1)^2 = (y-2)$$

vertex $(-1, 2)$

$$4p = 1 \qquad p = \frac{1}{4}$$



Focus: $\left(-1, 2+\frac{1}{4}\right) = \left(-1, \frac{9}{4}\right)$

25. The ellipse $9x^2 + 4y^2 - 36x + 8y + 4 = 0$ has vertices at the points

A. $(2, -\sqrt{5})$ and $(2, \sqrt{3})$

B. $(-2, -4)$ and $(-2, 2)$

C. $(-2, 1)$ and $(-2, 6)$

D. $(-4, 2)$ and $(2, 2)$

E. $(2, -4)$ and $(2, 2)$

$$9(x^2 - 4x + \quad) + 4(y^2 + 2y + \quad) = -4$$
$$9(x^2 - 4x + 4) + 4(y^2 + 2y + 1) = -4 + 36 + 4$$
$$9(x-2)^2 + 4(y+1)^2 = 36$$
$$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1$$

center $(2, -1)$, $a = 3$, $b = 2$