NAME Grading Key

STUDENT ID

RECITATION INSTRUCTOR

RECITATION TIME

Page 1	/16
Page 2	/32
Page 3	/28
Page 4	/24
TOTAL	/100

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4 .
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(8) 1. Find an equation of the sphere that passes through the point $(4,3,-1)$ and has center $(3,8,1)$.

$$(x-3)^2+(y-8)^2+(z-1)^2=a^2 \quad 4$$

$$a=\sqrt{(4-3)^2+(3-8)^2+(-1-1)^2}$$

$$=\sqrt{1+25+4}=\sqrt{30} \quad 4$$

-2pts if $\sqrt{30}$

$\boxed{(x-3)^2+(y-8)^2+(z-1)^2=30}$ $\boxed{8}$

(8) 2. Find the positive number c for which the angle between the vectors $\vec{a}=\langle 0,1,1\rangle$ and $\vec{b}=\langle 1,0,c\rangle$ is $\frac{\pi}{3}$.

$$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta \quad 2$$

$$c=\sqrt{2}\sqrt{1+c^2}\,\frac{1}{2} \quad 4$$

$$2c=\sqrt{2}\sqrt{1+c^2}$$

$$4c^2=2+2c^2$$

$$c^2=1 \rightarrow c=1 \quad 2$$

-1pt if ± 1

$\boxed{c=1}$ $\boxed{8}$

(8) 3. If $\vec{a} = \vec{i} - \vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + \vec{j} + x\vec{k}$, find the value of x so that $\vec{a}$ is perpendicular to $2\vec{a} - \vec{b}$.

$$2\vec{a} - \vec{b} = \vec{i} - 3\vec{j} + (2-x)\vec{k}$$

$$(2\vec{a} - \vec{b}) \cdot \vec{a} = 0 \quad 4$$

$$1 + 3 + 2 - x = 0 \rightarrow x = 6 \quad 4$$

$\boxed{x = 6}$ $\boxed{8}$

(8) 4. If $\vec{a} = \langle 4, 2, 0 \rangle$ and $\vec{b} = \langle 1, 1, 1 \rangle$, find the vector projection of $\vec{b}$ onto $\vec{a}$, $\text{proj}_{\vec{a}}\vec{b}$.

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\vec{a} \quad 4$$

$$= \frac{4+2}{16+4}\langle 4, 2, 0 \rangle \quad \text{or } 4$$

$$= \frac{6}{20}\langle 4, 2, 0 \rangle = \frac{3}{5}\langle 2, 1, 0 \rangle$$

-1 pt for minor numerical error in simplifying

$\boxed{\langle \frac{6}{5}, \frac{3}{5}, 0 \rangle}$ $\boxed{8}$

(16) 5. Let $\vec{a} = 4\vec{i} + 3\vec{j} - 5\vec{k}$ and $\vec{b} = 2\vec{i} + \vec{j} - 2\vec{k}$.

(a) Find $\vec{a} \times \vec{b}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & -5 \\ 2 & 1 & -2 \end{vmatrix} = \vec{i}(-6+5) - \vec{j}(-8+10) + \vec{k}(4-6)$$

$$= -\vec{i} - 2\vec{j} - 2\vec{k}$$

-2 pts for only one wrong coefficient

$\boxed{\vec{a} \times \vec{b} = -\vec{i} - 2\vec{j} - 2\vec{k}}$ $\boxed{5}$

(b) Find the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$.

$$A = |\vec{a} \times \vec{b}| \quad 3$$

$$= \sqrt{1+4+4} = 3 \quad 2$$

OK if consistent with wrong (a)

$\boxed{3}$ $\boxed{5}$

(c) Find a unit vector $\vec{u}$ orthogonal to both $\vec{a}$ and $\vec{b}$ and having a positive $\vec{k}$ component.

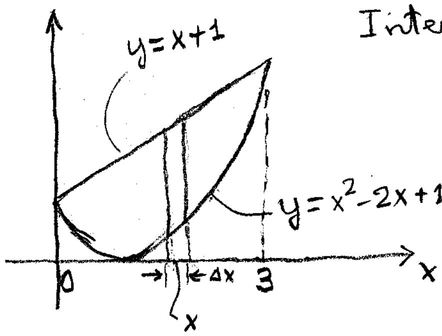
$$\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = -\frac{1}{3}\vec{i} - \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}$$

OK if consistent with wrong (a)

$$\vec{u} = \frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$$

$\boxed{\vec{u} = \frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}}$ $\boxed{6}$

(10) 6. Find the area of the region enclosed by the curves $y = x^2 - 2x + 1$ and $y = x + 1$.



Intersection points: $x^2 - 2x + 1 = x + 1$
$x^2 - 3x = 0 \to x = 0, 3$

$$\Delta A = \left[(x+1) - (x^2-2x+1)\right]\Delta x$$

$$A = \int_0^3 \underbrace{\left[(x+1) - (x^2-2x+1)\right]}_{5} dx$$

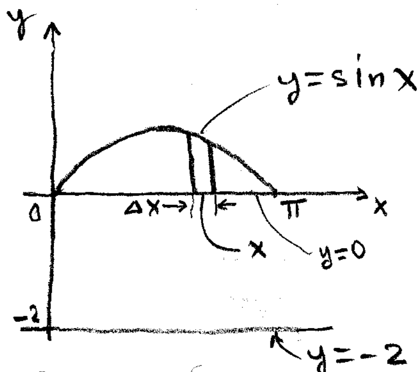
1 1 1 * see below

$$= \int_0^3 (-x^2 + 3x)\,dx = \left[-\frac{x^3}{3} + \frac{3}{2}x^2\right]_0^3$$

$$= -9 + \frac{27}{2} = \frac{27-18}{2} = \frac{9}{2}$$ 2

$\boxed{\frac{9}{2}}$ $\boxed{10}$

(8) 7. Using the method of washers, set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = 0$ and $y = \sin x$, $0 \le x \le \pi$, about the line $y = -2$. Do not evaluate the integral.



$$\Delta V = \left[\pi(\sin x + 2)^2 - \pi 2^2\right]\Delta x$$

$$V = \int_0^\pi \pi\left[(\sin x + 2)^2 - 4\right] dx$$

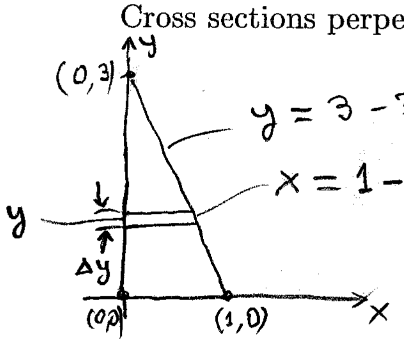
or

*

* 0 credit for problem if more than 1 item is wrong (Limits count as 1 item in this rule)

$\boxed{V = \int_0^\pi \overbrace{\pi\,(\sin^2 x + 4\sin x)}^{5}\,dx}$ 1 1 1 $\boxed{8}$

(10) 8. The base of a solid S is the triangular region with vertices $(0,0)$, $(1,0)$, and $(0,3)$. Cross sections perpendicular to the y–axis are squares. Find the volume of S.



$$\Delta V = A(y)\,\Delta y$$
$$= \left(1 - \tfrac{1}{3}y\right)^2 \Delta y$$

$$V = \int_0^3 \underbrace{\left(1 - \tfrac{1}{3}y\right)^2}_{5} dy$$ *

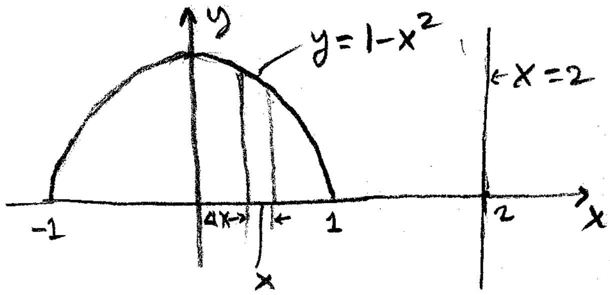
1 1 1

$$= \int_0^3 \left(1 - \tfrac{2}{3}y + \tfrac{1}{9}y^2\right) dy$$

$$= \left[y - \tfrac{1}{3}y^2 + \tfrac{1}{27}y^3\right]_0^3 = 3 - 3 + 1 = 1$$ 2

$\boxed{1}$ $\boxed{10}$

(8) 9. Using the method of cylindrical shells, set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = 1 - x^2$ and $y = 0$, about the line $x = 2$. Do not evaluate the integral.



$$\Delta V = 2\pi(2-x)(1-x^2)\Delta x$$

* (1) (5) (1)

$$\boxed{V = \int_{-1}^{1} 2\pi(2-x)(1-x^2)\,dx}$$

(1) [8]

(8) 10. In a certain city the temperature (in $°F$) t hours after 9 AM was approximated by the function

$$T(t) = 50 + 14\sin\frac{\pi t}{12}.$$

Find the average temperature during the period from 9 AM to 9 PM.

$$T_{ave} = \frac{1}{12}\int_0^{12}\left(50 + 14\sin\frac{\pi t}{12}\right)dt$$

(2) (1) (1) (1) (1) *

$$= \frac{1}{12}\left[50t - 14\cdot\frac{12}{\pi}\cos\frac{\pi t}{12}\right]_0^{12}$$

$$= \frac{1}{12}\left[\left(50\cdot 12 + \frac{14\cdot 12}{\pi}\right) - \left(0 - \frac{14\cdot 12}{\pi}\right)\right]$$

$$= 50 + \frac{28}{\pi} \quad (2)$$

$$\boxed{50 + \frac{28}{\pi}}$$

[8]

(8) 11. Find $\int \sin^{-1} x\,dx$.

$$\int u\,dv = uv - \int v\,du$$

$$\int \sin^{-1}x\,dx = x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx =$$

$u = \sin^{-1}x \quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \quad v = x$

$$= x\sin^{-1}x + \sqrt{1-x^2} + C$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int\frac{1}{\sqrt{u}}\,du = -\frac{1}{2}\frac{\sqrt{u}}{\frac{1}{2}} + C = -\sqrt{1-x^2} + C$$

$u = 1 - x^2$

$du = -2x\,dx$

(4) (4) −1 pt for missing +C

$$\boxed{x\sin^{-1}x + \sqrt{1-x^2} + C}$$

[8]