NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4 .
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(6) 1. Find an equation of the sphere that passes through the origin and whose center is $(1,2,3)$.

radius $= \sqrt{1^2+2^2+3^2} = \sqrt{14}$

no credit if not a sphere $\boxed{\underbrace{(x-1)^2+(y-2)^2+(z-3)^2}_{3} = 14}$ 3

(6) 2. If $\vec{a} = \vec{i} - \vec{j} + \vec{k}$, find a vector $\vec{b}$ whose length is 5 and whose direction is the same as that of $\vec{a}$.

$|\vec{a}| = \sqrt{1^2+(-1)^2+1^2} = \sqrt{3}$ 2

unit vector in direction of $\vec{a}$ is $\frac{1}{|\vec{a}|}\vec{a} = \frac{1}{\sqrt{3}}(\vec{i} - \vec{j} + \vec{k}) = \vec{u}$

$\vec{b} = 5\vec{u} = \frac{5}{\sqrt{3}}\vec{a}$

$= \frac{5}{\sqrt{3}}\vec{i} - \frac{5}{\sqrt{3}}\vec{j} + \frac{5}{\sqrt{3}}\vec{k}$ 4

$\boxed{\vec{b} = \frac{5}{\sqrt{3}}\vec{i} - \frac{5}{\sqrt{3}}\vec{j} + \frac{5}{\sqrt{3}}\vec{k}}$

$= \left\langle \frac{5}{\sqrt{3}}, \frac{-5}{\sqrt{3}}, \frac{5}{\sqrt{3}} \right\rangle$

(9) 3. True or False. (Circle T or F)

(a) $\vec{a} = -\vec{i} + 2\vec{j} + 5\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} - \vec{k}$ are parallel. T F 3

(b) $\vec{a} = 2\vec{i} + 6\vec{j} - 4\vec{k}$ and $\vec{b} = -3\vec{i} - 9\vec{j} + 6\vec{k}$ are orthogonal. T F 3

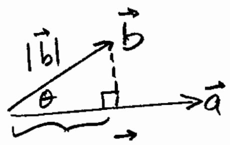
(c) $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} + 4\vec{j} - 10\vec{k}$ are neither orthogonal nor parallel. T F 3

(a) There is no scalar c such that $\vec{b} = c\vec{a}$ $\therefore$ not parallel

(b) $\vec{a} \cdot \vec{b} = -6 - 54 - 24 \neq 0$ $\therefore$ not orthogonal

(c) There is no scalar c such that $\vec{b} = c\vec{a}$ $\therefore$ not parallel.
$\vec{a} \cdot \vec{b} = 2 + 8 - 30 \neq 0$ $\therefore$ not orthogonal.

(7) 4. If $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 6\vec{j} - \vec{k}$, find the scalar projection of $\vec{b}$ onto $\vec{a}$, $\text{comp}_{\vec{a}}\, \vec{b}$.



$$\text{comp}_{\vec{a}}\, \vec{b} = |\vec{b}| \cos\theta = |\vec{b}| \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|} \right) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{2 - 18 - 1}{\sqrt{14}} = \frac{-17}{\sqrt{14}}$$ 2 2

OR 3

$\text{comp}_{\vec{a}}\, \vec{b} = \boxed{\dfrac{-17}{\sqrt{14}}}$

(12) 5. Let $\vec{a}, \vec{b}, \vec{c}$ be three–dimensional vectors. For each statement, circle T if the statement is always true, or F if it is not always true.

(i) $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ T F 2

(ii) $\vec{a} \cdot \vec{b} = -\vec{b} \cdot \vec{a}$ T F 2

(iii) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ T F 2

(iv) $(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$ T F 2

(v) $(\vec{a} \times \vec{b}) \times \vec{c}$ is a vector T F 2

(vi) $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is a real number T F 2

(8) 6. Find a vector orthogonal to the plane through points $P(1, 0, -1)$, $Q(2, 4, 5)$ and $R(3, 1, 7)$.

4 For crossing two non-parallel vectors formed by P, Q and R.

4 For correct answer. -2 if vector notation is incorrect

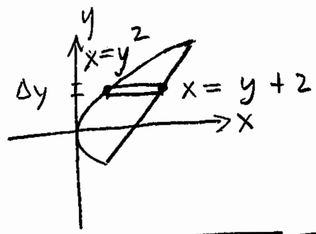
ex, $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 6 \\ 2 & 1 & 8 \end{vmatrix}$

$= \langle 26, -(-4), -7 \rangle$

Note: any nonzero multiple is correct.

$\boxed{\langle 26, 4, -7 \rangle}$

(8) 7. Set up an integral for the area of the region enclosed by the curves $x = y^2$ and $y = x - 2$. Do not evaluate the integral.



intersection: $y = y^2 - 2$

$\rightarrow 0 = y^2 - y - 2 = (y-2)(y+1)$

$\rightarrow y = -1, 2$

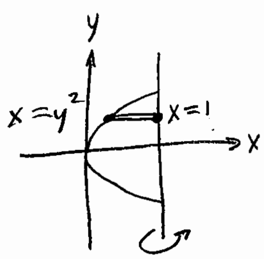
alt. answer $\boxed{\int_0^1 2\sqrt{x}\,dx + \int_1^4 (\sqrt{x} - (x-2))\,dx}$

* 0 credit for problem if more than 1 item is wrong (limits count as one item in this rule)

$$\boxed{area = \int_{-1}^{2} \underbrace{((y+2) - (y^2))}_{5}\, dy}$$

1 1 1

(10) 8. Find the volume of the solid obtained by rotating the region bounded by $x = y^2$ and $x = 1$ about the line $x = 1$.

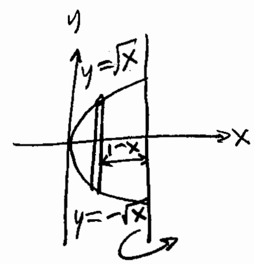


DISC METHOD

$$V = \int_{-1}^{1} \underbrace{\pi(1-y^2)^2}_{5}\, dy \quad 1$$

1 for each limit

Rule *



SHELL METHOD

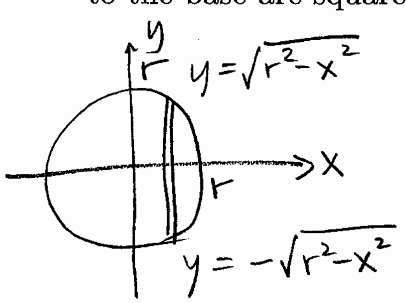
$$V = \int_0^1 \underbrace{2\pi(1-x)(2\sqrt{x})}_{5}\, dx \quad 1$$

1 for each limit

Rule *

2 $\boxed{\frac{16\pi}{15}}$

(10) 9. The base of a solid S is a circular disk of radius r. Parallel cross–sections perpendicular to the base are squares. Find the volume of S.



$$V = \int_{-r}^{r} \underbrace{\left(2\sqrt{r^2-x^2}\right)^2}_{5}\, dx \quad 1$$

1 1

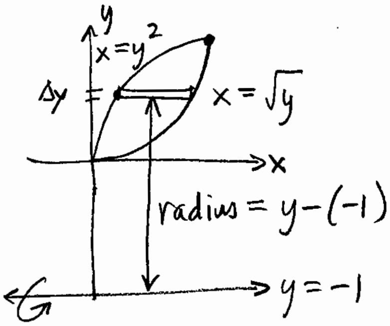
Rule *

$$= 4\left(r^2x - \tfrac{1}{3}x^3\right)\Big|_{-r}^{r}$$

$$= \frac{16}{3}r^3$$

2 $\boxed{\frac{16}{3}r^3}$

(8) 10. Using the method of cylindrical shells, set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $x = y^2$ about the axis $y = -1$. Do not evaluate the integral.

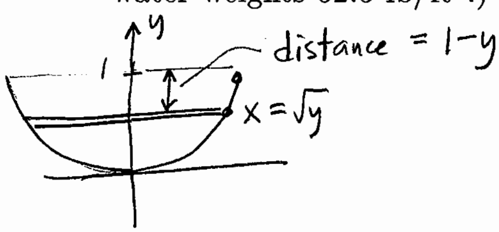


Rule *

$$\boxed{\text{volume} = \int_{0}^{1} \overbrace{2\pi(y+1)(\sqrt{y}-y^2)}^{5}\, dy}$$

1 1 1

(8) 11. A tank in the shape of the curve $y = x^2$, $0 \le x \le 1$ ft, rotated about the y–axis is full of water. Set up an integral for the work required to empty the tank by pumping all of the water to the top of the tank. Do not evaluate the integral. (Use the fact that water weights 62.5 lb/ft^3.)



$$\underbrace{\Delta W}_{\text{WORK TO LIFT SLICE}} \approx \underbrace{(1-y)}_{\text{DIST.}} \underbrace{\underbrace{(62.5)}_{\text{DENSITY}}\ \underbrace{\pi(\sqrt{y})^2\,\Delta y}_{\text{VOLUME}}}_{\text{FORCE}}$$

Rule *

$$\boxed{\text{work} = \int_{0}^{1} \overbrace{(1-y)}^{3}\ \overbrace{(62.5)\,\pi(\sqrt{y})^2}^{3}\, dy \ \textit{ft} - \textit{lbs}}$$

1 1

(8) 12. Find $\int \tan^{-1} x\, dx$.

Let $u = \tan^{-1} x$, $dv = dx$.

Then $du = \frac{1}{1+x^2}dx$, $v = x$

$$\int \tan^{-1} x\, dx = x\tan^{-1} x - \int \frac{x}{1+x^2}dx$$

$$= x\tan^{-1} x - \tfrac{1}{2}\ln|1+x^2| + C$$

-1pt if missing

$$\boxed{\overbrace{x\tan^{-1} x}^{4} - \overbrace{\tfrac{1}{2}\ln|1+x^2|}^{4} + C}$$