NAME GRADING KEY

10–DIGIT PUID

RECITATION INSTRUCTOR

RECITATION TIME

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Page 2	/29
Page 3	/26
Page 4	/28
TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(10) 1. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three-dimensional vectors. For each statement below, circle T if the statement is always true, or F if it is not always true.

(i) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ always true 2pts each T F

(ii) $\vec{a}\times\vec{b}=\vec{b}\times\vec{a}$ not true. $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$ T F

(iii) $\vec{a}\cdot(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{b})\times(\vec{a}\cdot\vec{c})$ not true; $\vec{a}\cdot\vec{b}$ and $\vec{a}\cdot\vec{c}$ are not vectors and right side is meaningless T F

(iv) $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$ always true T F

(v) $(\vec{a}\times\vec{b})\times\vec{a}=\vec{0}$ not always true: $(\vec{i}\times\vec{j})\times\vec{i}=\vec{k}\times\vec{i}=\vec{j}$ T F

$\boxed{10}$

(7) 2. Find the center and radius of the sphere

$$x^2+y^2+z^2-2x+6z=15$$

$(x^2-2x+1)+y^2+(z^2+6z+9)=15+1+9$

$(x-1)^2+y^2+(z+3)^2=25$

center $(1,0,-3)$ 4

radius 5 3

−2pts if only one coord. is wrong

center:	$(1,0,-3)$
radius:	5

$\boxed{7}$

(15) 3. If $\vec{a} = \vec{\imath} + 2\vec{\jmath} - \vec{k}$ and $\vec{b} = 3\vec{\jmath} + \vec{k}$, find the following   3 pts each

(a) $\vec{a} \cdot \vec{b}$ $= 1\cdot 0 + 2\cdot 3 + (-1)1 = 5$

$\boxed{5}$

(b) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 0 & 3 & 1 \end{vmatrix} = \vec{i}(2+3) - \vec{j}(1) + \vec{k}(3)$

$\boxed{5\vec{i} - \vec{j} + 3\vec{k}}$

Grade consistently with (a) and (b)

(c) $\cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{5}{\sqrt{1+4+1}\,\sqrt{9+1}} = \dfrac{5}{\sqrt{6}\sqrt{10}}$

$\boxed{\dfrac{5}{\sqrt{60}}}$

(d) the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$

$|\vec{a} \times \vec{b}| = \sqrt{25+1+9} = \sqrt{35}$

$\boxed{\sqrt{35}}$

(e) a unit vector orthogonal to both $\vec{a}$ and $\vec{b}$

$\vec{u} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \dfrac{1}{\sqrt{35}}(5\vec{i} - \vec{j} + 3\vec{k})$

$\boxed{\dfrac{1}{\sqrt{35}}(5\vec{i} - \vec{j} + 3\vec{k})}$

$\boxed{15}$

(6) 4. Let $A(1,2)$ and $B(2,0)$ be two points in the plane. Find the coordinates (p,q) of the point $C(p,q)$ such that $\vec{AC} = 2\vec{AB}$.

$\vec{AB} = \langle 1, -2 \rangle$ $\vec{AC} = \langle p-1, q-2 \rangle$

$\vec{AC} = 2\vec{AB}$: $\langle p-1, q-2 \rangle = \langle 2, -4 \rangle$

$p - 1 = 2 \rightarrow p = 3$

$q - 2 = -4 \quad q = -2$

3 3

$\boxed{(p,q) = (3, -2)}$

$\boxed{6}$

(8) 5. Find the value of the number c such that the vectors $\langle 1, c, 2\rangle$ and $\langle -2, -1, -4\rangle$ are

(a) orthogonal

$\langle 1, c, 2 \rangle \cdot \langle -2, -1, -4 \rangle = 0$

$-2 - c - 8 = 0 \rightarrow c = -10$

4

$\boxed{c = -10}$

(b) parallel $\langle 1, c, 2 \rangle \times \langle -2, -1, -4 \rangle = \vec{0}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & c & 2 \\ -2 & -1 & -4 \end{vmatrix} = \vec{i}(-4c+2) - \vec{j}(-4+4) + \vec{k}(-1+2c)$

$-4c + 2 = 0, \ -1 + 2c = 0 \rightarrow c = \frac{1}{2}$

4

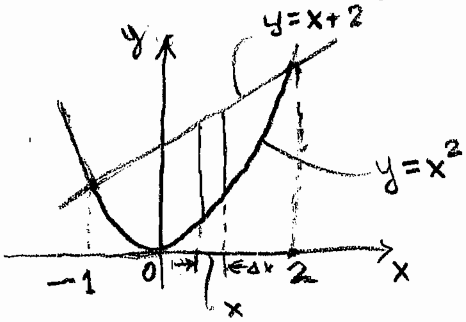
$\boxed{c = \frac{1}{2}}$

$\boxed{8}$

Or $\langle 1, c, 2 \rangle = k\langle -2, -1, -4 \rangle \rightarrow \dfrac{1}{-2} = \dfrac{c}{-1} = \dfrac{2}{-4} = k \rightarrow c = \frac{1}{2}$

where k is a constant

(10) 6. Find the area of the region enclosed by the curves
$y = x^2$ and $y = x + 2$.



Points of intersection: $x^2 = x + 2 \to x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$

Area of typical approximating rectangle:

$\Delta A = [(x+2) - x^2]\Delta x$

$$A = \int_{-1}^{2} (x + 2 - x^2)\, dx \quad \text{or} \quad A = \int_{-1}^{2}(x+2)\,dx - \int_{-1}^{2} x^2\,dx$$

① ① ⑤ ①

Rule* 0 credit for problem if more than 1 item is wrong (limits count as 1 item in this rule)

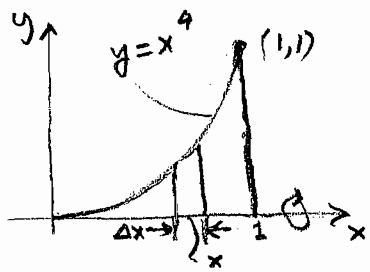
$$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Big|_{-1}^{2}$$
$$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$
$$= 2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$$
$$= 8 - \frac{9}{3} - \frac{1}{2} = 5 - \frac{1}{2} = \frac{9}{2} \quad ②$$

$\boxed{\frac{9}{2}}$

10

(16) 7. Set up, but do not evaluate, an integral for the volume V of the solid obtained by rotating the region bounded by the curves $y = x^4$, $y = 0$, and $x = 1$, about the x–axis,

(a) using the method of disks/washers



Volume of typical disk:

$\Delta V = \pi (x^4)^2 \Delta x$

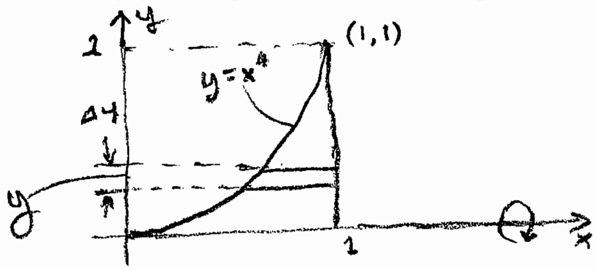
Rule *

$$V = \int_0^1 \underbrace{\pi x^8}_{⑤}\, dx$$

① ① ①

8

(b) using the method of cylindrical shells



Volume of typical cylindrical shell

$\Delta V = 2\pi y (1 - \sqrt[4]{y})\Delta y$

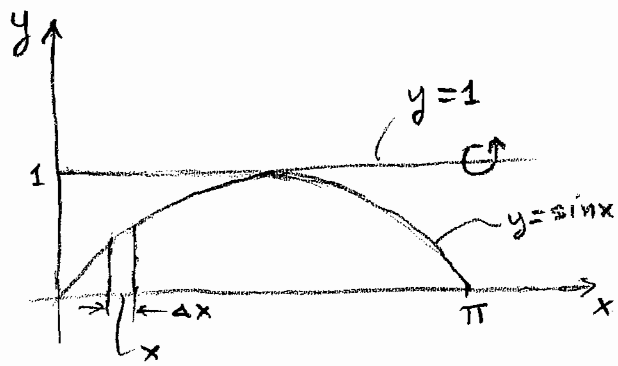
Rule *

$$V = \int_0^1 \underbrace{2\pi y (1 - \sqrt[4]{y})}_{⑤}\, dy$$

① ① ①

8

(8) 8. Using the method of disks/washers, set up, but do not evaluate, an integral for the volume V of the solid obtained by rotating the region bounded by the curves $y = 0$, $y = \sin x$, $0 \le x \le \pi$ about the line $y = 1$.



Volume of typical washer

$\Delta V = [\pi 1^2 - \pi(1-\sin x)^2]\Delta x$

Rule *

5

1

$$V = \int_0^{\pi} [\pi - \pi(1-\sin x)^2]\,dx$$

1 1

8

(8) 9. A force of 10 lb. is required to hold a spring stretched 4 in. beyond its natural length. How much work is done in stretching it from its natural length to 6 in. beyond its natural length?

$F = kx \rightarrow 10 = k\frac{1}{3} \longrightarrow k = 30$ (lb/ft) 4

$\left(k = \frac{5}{2} \text{ lb/in}\right)$

$\therefore F = 30x$

$$W = \int_0^{1/2} 30x\,dx$$

$$= 15x^2\Big|_0^{1/2} = \frac{15}{4} \text{ ft-lb}$$ 4

−1 pt for not changing in. to ft. and getting 45 (in-lb)

$\frac{15}{4}$ ft-lb

8

(12) 10. Find $\int (\ln x)^2\,dx = x(\ln x)^2 - \int x\, 2(\ln x)\frac{1}{x}\,dx =$

3 3

By parts $u = (\ln x)^2$, $dv = dx$

$du = 2(\ln x)\frac{1}{x}dx$, $v = x$

$= x(\ln x)^2 - 2\int \ln x\,dx = x(\ln x)^2 - 2\left[x\ln x - \int x\frac{1}{x}dx\right] =$

$u = \ln x$ $dv = dx$

$du = \frac{1}{x}dx$ $v = x$

$= x(\ln x)^2 - 2x\ln x + 2x + C$

2 2 2

−1 pt if +C is missing

$x(\ln x)^2 - 2x(\ln x) + 2x + C$