NAME GRADING KEY

10–DIGIT PUID ______________

RECITATION INSTRUCTOR ______________

RECITATION TIME ______________

Page 1	/16
Page 2	/25
Page 3	/26
Page 4	/33
TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators, or any electronic devices may be used on this test.

---

(10) 1. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three–dimensional vectors. For each statement below, circle T if the statement is always true, or F if it is not always true.

(i) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then $\vec{a}\cdot\vec{b} = \cos\theta$ — 2pts each — T F

(ii) If $\vec{i}\cdot\vec{b} = \vec{i}\cdot\vec{c}$, then $\vec{b} = \vec{c}$ — $b_1 = c_1 \Rightarrow \vec{b} = \vec{c}$ X — T F

(iii) If $\vec{a}\cdot\vec{b} = 0$, then $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ — $|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \cdots$ — T F

(iv) The vector $\vec{a}\times(\vec{b}\times\vec{c})$ is always parallel to $\vec{b}\times\vec{c}$ — $\vec{a}\times(\vec{b}\times\vec{c}) \perp (\vec{b}\times\vec{c})$ — T F

(v) $(\vec{a}\times\vec{b})\times\vec{a} = \vec{0}$ — $(\vec{i}\times\vec{j})\times\vec{i} = \vec{k}\times\vec{i} = \vec{j}$ — T F

$\boxed{10}$

(6) 2. Find an equation of the sphere that passes through the origin and whose center is $(1, 2, 3)$.

$(x-1)^2 + (y-2)^2 + (z-3)^2 = r^2$ (3)

$(x,y,z) = (0,0,0)$ lies on the sphere.

$\therefore\ (-1)^2 + (-2)^2 + (-3)^2 = r^2$

$1 + 4 + 9 = r^2$

$r^2 = 14$ (3)

$$\boxed{(x-1)^2 + (y-2)^2 + (z-3)^2 = 14}$$

$\boxed{6}$

(4) 3. Find the values of c for which the vectors $\langle 0,2,3\rangle$ and $\langle 2,c,-2\rangle$ are orthogonal.

$\langle 0,2,3\rangle \cdot \langle 2,c,-2\rangle = 0$

$2c-6=0 \rightarrow c=3$

NPC

$\boxed{c = 3}$ $\boxed{4}$

(4) 4. If $\vec{v} = \langle 1,2,2\rangle$ and $\vec{w} = \langle 1,0,1\rangle$, find the angle θ between $\vec{v}$ and $\vec{w}$.

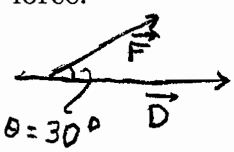
$\vec{v}\cdot\vec{w} = |\vec{v}||\vec{w}|\cos\theta$

$3 = \sqrt{9}\cdot\sqrt{2}\cos\theta \longrightarrow \cos\theta = \frac{1}{\sqrt{2}} \rightarrow \theta = \frac{\pi}{4}$

NPC

$\boxed{\theta = \frac{\pi}{4}}$ $\boxed{4}$

(4) 5. A sled is pulled along a level path through snow by a rope. A 30-lb force acting at an angle of 30° above the horizontal moves the sled 80 ft. Find the work done by the force.



$|\vec{F}| = 30 \text{ lbs} \quad |\vec{D}| = 80 \text{ ft}$

$W = \vec{F}\cdot\vec{D} = |\vec{F}||\vec{D}|\cos\theta$

$= 30\cdot 80\cos 30°$

$= 2400\cdot\frac{\sqrt{3}}{2}$

NPC

$\boxed{1200\sqrt{3} \text{ ft-lbs}}$ $\boxed{4}$

(13) 6. Consider the points $P(1,-2,1), Q(-1,3,2)$ and $R(2,1,1)$.

(a) Find $\overrightarrow{PQ}\times\overrightarrow{PR}$.

$\overrightarrow{PQ} = -2\vec{i}+5\vec{j}+\vec{k}$ (1) $\quad \overrightarrow{PR} = \vec{i}+3\vec{j}$ (1)

If wrong, grade the rest of the problem using consistency with student's answers here

$$\overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 5 & 1 \\ 1 & 3 & 0 \end{vmatrix}$$

$= -3\vec{i}+\vec{j}-11\vec{k}$ (3)

$\boxed{-3\vec{i}+\vec{j}-11\vec{k}}$ $\boxed{5}$

(b) Find the area of the triangle with vertices P, Q, R.

$|\overrightarrow{PQ}\times\overrightarrow{PR}| = \sqrt{9+1+121} = \sqrt{131}$

Area of $\triangle PQR = \frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$

$= \frac{1}{2}\sqrt{131}$ (4)

$\boxed{\frac{1}{2}\sqrt{131}}$ $\boxed{4}$

(c) Find two unit vectors orthogonal to the plane through the points P, Q, and R.

$\pm\frac{1}{|\overrightarrow{PQ}\times\overrightarrow{PR}|}\overrightarrow{PQ}\times\overrightarrow{PR}$

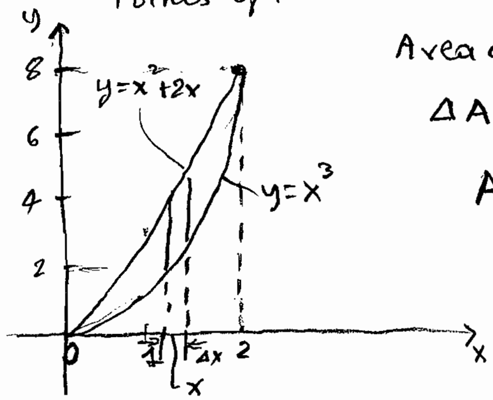
$\pm\frac{1}{\sqrt{131}}(-3\vec{i}+\vec{j}-11\vec{k})$ (4)

−2pts for one vector only in the box

$\boxed{\pm\frac{1}{\sqrt{131}}(-3\vec{i}+\vec{j}-11\vec{k})}$ $\boxed{4}$

(10) 7. Find the area of the region in the first quadrant bounded by the curves $y = x^3$ and $y = x^2 + 2x$.

Points of intersection: $x^3 = x^2 + 2x \to x(x^2 - x - 2) = 0 \to x(x-2)(x+1) = 0 \to x = 0, 2$



Area of typical approximating rectangle:

$$\Delta A = [(x^2+2x) - x^3]\Delta x \quad 1$$

$$A = \int_0^2 \underbrace{(x^2+2x-x^3)}_{5}\underbrace{dx}_{1} \quad 1 \leftarrow$$

Rule *: 0 pts for problem if more than 1 item is wrong. Limits count as 1 item in this rule

$$= \left[\frac{x^3}{3} + x^2 - \frac{x^4}{4}\right]_0^2$$

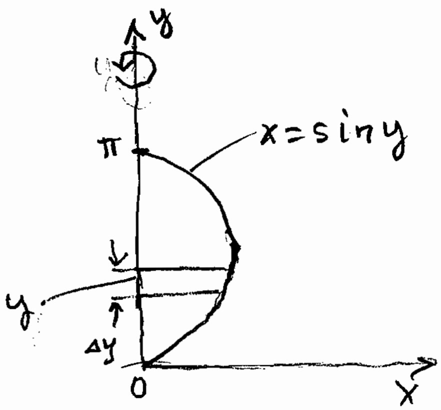
$$= \frac{8}{3} + 4 - \frac{16}{4} = \frac{8}{3} \quad 2$$

$$\boxed{\frac{8}{3}}$$

10

(8) 8. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating about the y–axis, the region bounded by the curves

$$x = \sin y,\ 0 \le y \le \pi,\ \text{and } x = 0.$$



Volume of typical approximating disk:

$$\Delta V = \pi \sin^2 y\, \Delta y$$

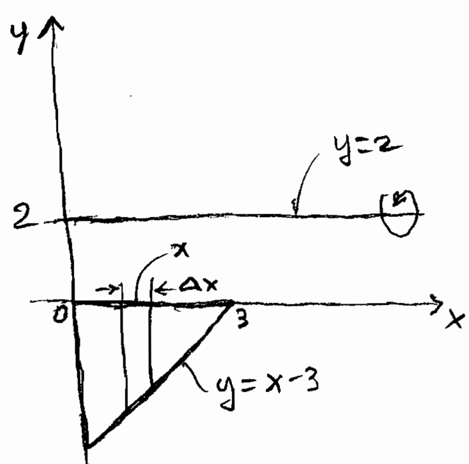
Rule *

$$\boxed{V = \int_{0}^{\pi} \underbrace{\pi \sin^2 y}_{5}\ \underbrace{dy}_{1}}$$

1 1

8

(8) 9. Let R be the region bounded by the curves $y = x - 3$, $y = 0$, and $x = 0$. Use the method of disks or washers to set up an integral for the volume of the solid obtained by rotating R about the line $y = 2$. Do not evaluate the integral.



Volume of typical approximating washer:

$$\Delta V = \left\{\pi[2-(x-3)]^2 - \pi 2^2\right\}\Delta y$$

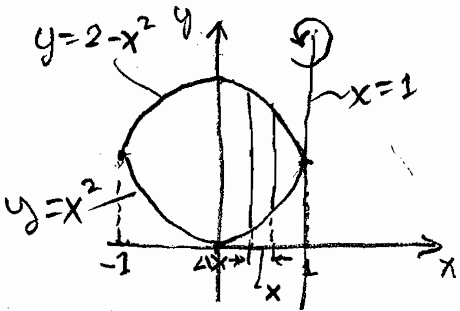
Rule *

$$\boxed{V = \int_0^3 \overbrace{\left\{\pi[2-(x-3)]^2 - 4\pi\right\}}^{5} \underbrace{dx}_{1}}$$

1 1

8

(8) 10. Let R be the region bounded by the curves $y = x^2$ and $y = 2 - x^2$. Use the method of cylindrical shells to set up an integral for the volume of the solid obtained by rotating R about the line $x = 1$. Do not evaluate the integral.



Volume of typical approximating cylindrical shell:

$\Delta V = 2\pi(1-x)[(2-x^2)-x^2]\Delta x$ 1

Rule * $\rightarrow$ $\boxed{V = \int_{-1}^{1} 2\pi(1-x)(2-2x^2)\,dx}$ 1 5 1

8

(8) 11. If the work required to stretch a spring 1 ft beyond its natural length is 12 ft-lbs, how much work is needed to stretch it 9 in beyond its natural length?

$W = \int_a^b kx\,dx$: $\int_0^1 kx\,dx = 12 \rightarrow k\frac{x^2}{2}\Big|_0^1 = 12 \rightarrow k\frac{1}{2} = 12 \rightarrow k = 24$ 4

$W = \int_0^{\frac{3}{4}} 24x\,dx = 12x^2\Big|_0^{3/4} = 12\left(\frac{3}{4}\right)^2 = \frac{27}{4}$ ft-lbs 4

$\boxed{\frac{27}{4} \text{ ft-lbs}}$

8

(7) 12. $\int x^3 \ln x\,dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4}\frac{1}{x}\,dx = \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3\,dx$

$u = \ln x \quad dv = x^3 dx$

$du = \frac{1}{x}dx \quad v = \frac{x^4}{4}$

$= \frac{x^4}{4}\ln x - \frac{1}{4}\cdot\frac{x^4}{4} + C$

3 3 1

$\boxed{\frac{x^4}{4}\ln x - \frac{x^4}{16} + C}$

7

(10) 13. First make a substitution and then use integration by parts to evaluate the integral

$\int \cos\sqrt{x}\,dx. = 2\int y\cos y\,dy$ 3

$y = \sqrt{x} \quad dy = \frac{1}{2\sqrt{x}}dx, \quad 2y\,dy = dx$

$2\int y\cos y\,dy = 2y\sin y - 2\int \sin y\,dy$

$u = y \quad dv = \cos y\,dy$

$du = dy \quad v = \sin y$

$= 2y\sin y + 2\cos y + C$

$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

-3pts if answer is left in terms of y

3 3 1

$\boxed{2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C}$

10