MA166 — EXAM II — FALL 2018 — OCTOBER 19, 2018
TEST NUMBER 11

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. There are 10 problems and each one is worth is 10 points. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have doubts, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, given it to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, given it to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: SOLUTIONS

STUDENT SIGNATURE: ______________________________

STUDENT ID NUMBER: ______________________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________________

1. Compute the integral $\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}} =$ Set $x = \sec\theta$, $x^2 - 1 = \sec^2\theta - 1$

$= \tan^2\theta.$

$\boxed{\text{A. } \frac{\pi}{12}}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

E. $\frac{\pi}{9}$

$$= \int_{\pi/4}^{\pi/3} \frac{\sec\theta \tan\theta \, d\theta}{\sec\theta \tan\theta}$$

$dx = \sec\theta \tan\theta \, d\theta$

$x = \sqrt{2} = \sec\theta$, $\theta = \pi/4$

$x = 2 = \sec\theta$, $\theta = \pi/3$

$$= \int_{\pi/4}^{\pi/3} d\theta = \pi/3 - \pi/4 = \pi/12$$

2. Find the length of the curve $y = x^3 + \frac{1}{12}x^{-1}$ for $1 \le x \le 2$. L = Length of the curve

A. $8 + \frac{1}{24}$

B. $8 - \frac{1}{24}$

C. $8 - \frac{1}{12}$

$\boxed{\text{D. } 7 + \frac{1}{24}}$

E. $7 - \frac{1}{24}$

$$L = \int_1^2 \sqrt{1 + (y'(x))^2}\, dx \; ; \quad y'(x) = 3x^2 - \frac{1}{12}x^{-2}$$

$$1 + (y'(x))^2 = 1 + \left(3x^2 - \frac{1}{12}x^{-2}\right)^2 =$$

$$= 1 + 9x^4 - \frac{1}{2} + \frac{1}{144}x^{-2} = 9x^4 + 1/2 + 1/144\, x^{-2}$$

$$= \left(3x^2 + 1/12\, x^{-2}\right)^2. \text{ So}$$

$$L = \int_1^2 \left(3x^2 + \tfrac{1}{12}x^{-2}\right) dx = x^3 - \frac{1}{12}x^{-1} \Big|_1^2 =$$

$$= \left(8 - \frac{1}{24}\right) - \left(1 - \frac{1}{12}\right) = 7 + 1/12 - 1/24 = 7 + 1/24$$

3. Which of the following integrals converge?

I. $\int_0^{\infty} xe^{-x^2}dx$ II. $\int_1^2 \frac{1}{\sqrt{(x-1)(2+x)}}dx$ III. $\int_1^3 \frac{1}{x-2}dx$

$x^2 = u$, $du = 2x$

$\boxed{\text{A. I and II only}}$

B. II and III only

C. II only

D. I and III only

E. All of them

(I) $\int_0^{\infty} xe^{-x^2}dx = \frac{1}{2}\int_0^{\infty} e^{-u}du = \frac{1}{2}$

Converges.

(II) The problem is when $x=1$. But near $x=1$

$\frac{1}{\sqrt{(x-1)(x+2}} \sim \frac{1}{\sqrt{3(x-1)}}$ and thus is integrable.

(III) the problem when $x=2$, thus is not integrable.

4. Compute the integral $\int_0^1 \frac{1}{x^2+4x+3}dx$.

$x^2+4x+3 = (x+1)(x+3)$

$\frac{1}{(x+1)(x+3)} = \frac{1}{2}\left(\frac{1}{x+1} - \frac{1}{x+3}\right)$

A. $\frac{1}{2}\ln(\frac{5}{4})$

B. $\frac{1}{2}\ln(\frac{5}{2})$

$\boxed{\text{C. } \frac{1}{2}\ln(\frac{3}{2})}$

D. $\frac{1}{2}\ln(\frac{5}{3})$

E. $\frac{1}{2}\ln(\frac{8}{3})$

So $\int_0^1 \frac{dx}{x^2+4x+3} = \frac{1}{2}\int_0^1 \left(\frac{1}{x+1} - \frac{1}{x+3}\right)dx$

$= \frac{1}{2}\left(\ln|x+1| - \ln|x+3|\right)\Big|_0^1 =$

$= \frac{1}{2}\ln\left|\frac{x+1}{x+3}\right|\Big|_0^1 = \frac{1}{2}\left(\ln\left(\frac{2}{4}\right) - \ln\left(\frac{1}{3}\right)\right)$

$= \frac{1}{2}\ln\left(\frac{2}{4}\times\frac{3}{1}\right) = \frac{1}{2}\ln\left(\frac{3}{2}\right).$

5. Compute $\displaystyle\int_1^2 \frac{1}{x(x^2+1)}\,dx$

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$= \frac{A(x^2+1) + Bx^2 + Cx}{x(x^2+1)}$$

$$= \frac{(A+B)x^2 + A + Cx}{x(x^2+1)}$$

$A+B=0$

$A=1.$

A. $\ln(4\sqrt{5})$

B. $\boxed{\ln(\frac{2\sqrt{2}}{\sqrt{5}})}$

C. $\ln(\frac{4}{\sqrt{5}})$

D. $\ln(\frac{8}{5})$

E. $\ln(20\sqrt{5})$

So $\displaystyle\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}.$

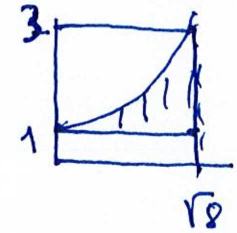
$$\int_1^2 \frac{dx}{x(x^2+1)} = \int_1^2 \left(\frac{1}{x} - \frac{x}{x^2+1}\right) dx =$$

$$= \left(\ln x - \tfrac{1}{2}\ln(1+x^2)\right)\Big|_1^2 = \left(\ln 2 - \tfrac{1}{2}\ln 5\right) + \tfrac{1}{2}\ln 2$$

$$= \tfrac{3}{2}\ln 2 - \tfrac{1}{2}\ln 5 = \ln 2^{3/2} - \ln 5^{1/2}$$

$$= \ln\left(\frac{2\sqrt{2}}{\sqrt{5}}\right).$$

6. The area of the region of the plane bounded by the curves $y = \sqrt{1+x^2}$, $y = 1$, $x = 0$ and $x = \sqrt{8}$ is equal to A. The x-coordinate of its centroid is equal to



A. $\frac{5}{A}$

B. $\frac{8}{3A}$

C. $\frac{12}{5A}$

D. $\frac{8}{3A}$

E. $\boxed{\frac{14}{3A}}$

$$\bar{x} = \frac{1}{A}\int_0^{\sqrt{8}} \left(x\sqrt{1+x^2} - x\right) dx$$

$$= \frac{1}{A}\int_0^{\sqrt{8}} x\sqrt{1+x^2}\,dx - 4/A.$$

Set $u = 1+x^2$; $du = 2x\,dx.$

$$\int_0^{\sqrt{8}} x\sqrt{1+x^2}\,dx = \frac{1}{2}\int_1^9 u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_1^9$$

$$= \frac{1}{3}\left(9^{3/2} - 1\right) = \frac{1}{3}(27-1) = \frac{26}{3}$$

$$\bar{x} = \frac{1}{A}\left(\frac{26}{3} - 4\right) = \frac{1}{A}\left(\frac{26-12}{3}\right) = \frac{14}{3A}$$

7. The curve $y = 1 + x^3$, $1 \le x \le 2$, is rotated about the line $x = 1$. The resulting surface has area given by

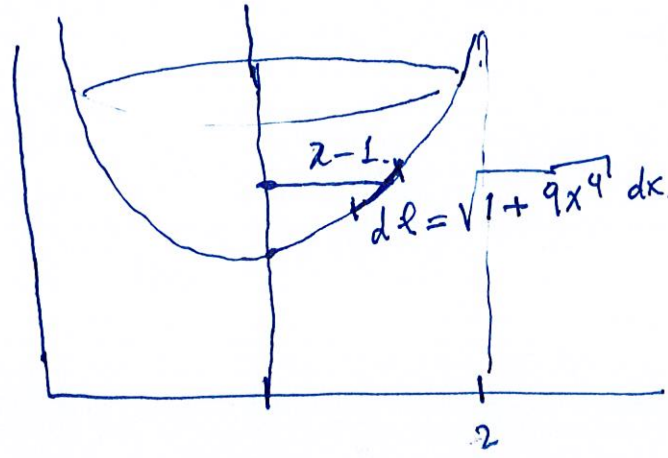
A. $2\pi \int_1^2 (1+x^3)\sqrt{1+9x^4}\,dx$

B. $2\pi \int_1^2 x\sqrt{1+9x^4}\,dx$

C. $\boxed{2\pi \int_1^2 (x-1)\sqrt{1+9x^4}\,dx}$

D. $2\pi \int_1^2 (x+1)\sqrt{1+9x^4}\,dx$

E. $2\pi \int_1^2 (x+1)^3\sqrt{1+9x^4}\,dx$



$$A = 2\pi \int_1^2 (x-1)\sqrt{1+9x^4}\,dx.$$

8. Compute the limit $\lim_{n\to\infty} \frac{2n^3+8n^2+2n+1}{n^3+2n+2}$ $= \lim_{n\to\infty} \frac{2 + 8/n + 2/n^2 + 1/n^3}{1 + 2/n^2 + 2/n^3}$

$= 2$

A. 1

$\boxed{\text{B. } 2}$

C. 3

D. 0

E. 4

9. The sum of the series $S = \sum_{n=1}^{\infty} \frac{1}{(\sqrt{2})^n}$ is equal to

$$\sum_{n=1}^{\infty} r^n = \frac{r}{1-r} \text{ if } |r|<1$$

A. $S = \frac{1}{2}(\sqrt{2}+1)$

B. $\boxed{S = \sqrt{2}+1}$

C. $S = \frac{1}{\sqrt{2}+1}$

D. $S = \frac{2}{\sqrt{2}+1}$

E. $S = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

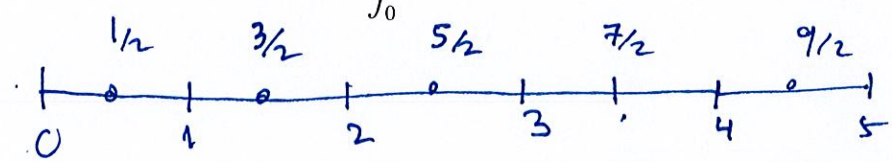
$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^n = \frac{1/\sqrt{2}}{1 - 1/\sqrt{2}} = \frac{1}{\sqrt{2}-1}$$

$$= \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} =$$

$$= \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1.$$

10. If we use the midpoint rule to approximate the integral $\int_0^5 2^{-x}dx$ with $N = 5$ we obtain the following:

$\Delta x = 1$



A. $64(2^{-\frac{9}{2}})$

B. $15(2^{-\frac{7}{2}})$

C. $\boxed{31(2^{-\frac{9}{2}})}$

D. $41(2^{-\frac{7}{2}})$

E. $21(2^{-\frac{9}{2}})$

$$\int_0^5 2^{-x}dx \sim 2^{-1/2} + 2^{-3/2} + 2^{-5/2} + 2^{-7/2} + 2^{-9/2}$$

$$= 2^{-9/2}(1 + 2 + 2^2 + 2^3 + 2^4)$$

$$= 2^{-9/2}(1+3+4+8+16) = 31\, 2^{-9/2}$$

ANSWER KEYS:
TEST 11: 1- A, 2- D, 3-A, 4-C, 5-B, 6-E, 7-C, 8-B, 9-B, 10-C
TEST 22: 1-C, 2-B, 3-D, 4-B, 5-D, 6- D, 7- D, 8- A, 9- A, 10-E
TEST 33: 1-B, 2- C, 3- E, 4-A, 5-A, 6-C, 7-B, 8-E, 9-C, 10-D
TEST 44: 1-D, 2-E, 3-C, 4-E, 5-E, 6-A, 7-E, 8- C, 9-D, 10-A