NAME GRADING KEY

10–DIGIT PUID

RECITATION INSTRUCTOR

RECITATION TIME

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Page 2	/30
Page 3	/28
Page 4	/21
TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators, or any electronic devices may be used on this test.

-1pt for missing +C (one time for test)

-1pt for missing dx, du, dθ, etc (one time for each problem)

(9) 1. $\int \tan^2 x \sec^4 x \, dx$

$= \int \tan^2 x \sec^2 x \sec^2 x \, dx$

$= \int \tan^2 x (1+\tan^2 x) \sec^2 x \, dx$ (5)

$u = \tan x \quad du = \sec^2 x \, dx$

$= \int u^2(1+u^2)\, du = \frac{u^3}{3} + \frac{u^5}{5} + C$

$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$

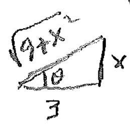
(2) (2)

$\boxed{\frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C}$

9

(12) 2. $\int \frac{1}{(9+x^2)^2} dx$ [Hint: Use trigonometric substitution].

$= \int \frac{1}{(\sqrt{9+x^2})^4} = \int \frac{1}{(3\sec\theta)^4} 3\sec^2\theta \, d\theta$ (5) $= \frac{1}{27}\int \frac{1}{\sec^2\theta} d\theta = \frac{1}{27}\int \cos^2\theta \, d\theta$



$x = 3\tan\theta, \; -\frac{\pi}{2} < x < \frac{\pi}{2}$

$dx = 3\sec^2\theta \, d\theta$

$\sqrt{9+x^2} = 3\sec\theta$

$= \frac{1}{54}\int (1+\cos 2\theta)\, d\theta$ (3)

$= \frac{1}{54}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$

$= \frac{1}{54}\left(\theta + \sin\theta\cos\theta\right) + C$

$= \frac{1}{54}\left(\tan^{-1}\frac{x}{3} + \frac{x}{\sqrt{9+x^2}}\frac{3}{\sqrt{9+x^2}}\right) + C$

or

-1pt for each wrong coefficient

(2) (2)

$\boxed{\frac{1}{54}\tan^{-1}\frac{x}{3} + \frac{1}{18}\frac{x}{9+x^2} + C}$

12

(8) 3. $\int \frac{tan(\frac{1}{x})}{x^2}\, dx.$ $= -\int \tan u\, du = \int \frac{-\sin u}{\cos u} du = \int \frac{1}{v} dv$ (4)

$u = \frac{1}{x}$ $du = -\frac{1}{x^2} dx$ $v = \cos u$ $dv = -\sin u\, du$

$= \ln|v| + C = \ln|\cos u| + C$

$= \ln|\cos(\frac{1}{x})| + C$

−1pt for missing abs. value

(4) $\boxed{\ln|\cos(\frac{1}{x})| + C}$ $\boxed{8}$

(12) 4. Write out the form of the partial fraction decomposition of the following functions. Do not determine the numerical values of the coefficients.

(a) $\frac{x^3}{x^2+4x+3} = \boxed{\underbrace{x-4}_{3} + \underbrace{\frac{A}{x+1} + \frac{B}{x+3}}_{3}}$ $\boxed{6}$

$$\begin{array}{r} x-4 \\ x^2+4x+3 \overline{) x^3} \\ \underline{x^3+4x^2+3x} \\ -4x^2-3x \\ \underline{-4x^2-16x-12} \\ 13x+12 \end{array}$$

$\frac{x^3}{x^2+4x+3} = x-4 + \frac{13x+12}{(x+1)(x+3)}$

(b) $\frac{2x+1}{(x+1)^3(x^2+4)^2} = \boxed{\underbrace{\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}}_{3} + \underbrace{\frac{Dx+E}{x^2+4} + \frac{Fx+G}{(x^2+4)^2}}_{3}}$ $\boxed{6}$

(10) 5. $\int \frac{1}{x^2(x-1)}\, dx$ $= \int \left(-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x-1}\right) dx =$ (3) (3) (3)

$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$ *

$= -\ln|x| + \frac{1}{x} + \ln|x-1| + C$

0 pts for problem if ↑ is wrong

$1 = Ax(x-1) + B(x-1) + Cx^2$

$1 = Ax^2 - Ax + Bx - B + Cx^2$

$\left.\begin{array}{r} A+C=0 \\ -A+B=0 \\ -B=1 \end{array}\right) \rightarrow \begin{array}{l} B=-1 \\ A=-1 \\ C=1 \end{array}$

* 0 pts for problem if anything is wrong here

−1 pt for missing abs. values

(1) $\boxed{-\ln|x| + \frac{1}{x} + \ln|x-1| + C}$ $\boxed{10}$

(18) 6. Determine whether each integral is convergent or divergent and find its value if it is convergent. Important: You must use the definition of improper integrals in terms of limits.

−1 pt for early omission of lim in (= lim)

(a) $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{t\to\infty} \int_1^t \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ 3

2

$= \lim_{t\to\infty} \left[-2e^{-\sqrt{x}}\right]_1^t = \lim_{t\to\infty} \left[-2e^{-\sqrt{t}} + 2e^{-1}\right]$

$= \frac{2}{e}$

1

$\boxed{\frac{2}{e}}$

6

(b) $\int_2^3 \frac{1}{\sqrt{3-x}}\,dx = \lim_{t\to 3^-} \int_2^t \frac{1}{\sqrt{3-x}}\,dx$ 3

2

$= \lim_{t\to 3^-} \left[-2\sqrt{3-x}\right]_2^t = \lim_{t\to 3^-} \left[-2\sqrt{3-t} + 2\right] = 2$

1

$\boxed{2}$

6

(c) $\int_{-\infty}^0 e^{-x}\,dx = \lim_{t\to -\infty} \int_t^0 e^{-x}\,dx$ 3

2

$= \lim_{t\to -\infty} \left[-e^{-x}\right]_t^0 = \lim_{t\to -\infty} \left[-1 + e^{-t}\right] = \infty$

1

$\boxed{\text{divergent}}$

6

(10) 7. Find the length of the curve $y = \ln(\cos x)$, $0 \le x \le \frac{\pi}{3}$.

[Hint: $\int \sec x dx = \ln|\sec x + \tan x| + C$].

$L = \int_0^{\frac{\pi}{3}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$ 4

$= \int_0^{\frac{\pi}{3}} \sqrt{1 + \left(\frac{-\sin x}{\cos x}\right)^2}\,dx = \int_0^{\frac{\pi}{3}} \sqrt{1 + \tan^2 x}\,dx = \int_0^{\frac{\pi}{3}} \sec x\,dx$ 3

$= \ln|\sec x + \tan x|\,\Big|_0^{\frac{\pi}{3}}$

$= \ln|2 + \sqrt{3}| - \ln 1$

$= \ln(2 + \sqrt{3})$

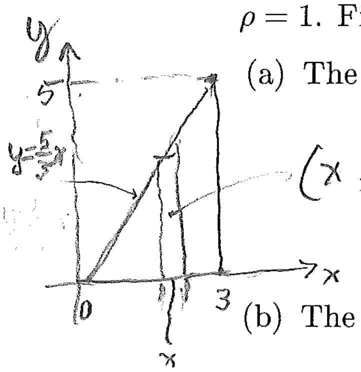
3

$\boxed{L = \ln(2+\sqrt{3})}$

10

(13) 8. Consider the triangular lamina with vertices at $(0,0)$, $(3,0)$ and $(3,5)$, and with density $\rho = 1$. Find the following:

NPC



(a) The mass m of the lamina.

$(x, \frac{5}{6}x)$ $\quad m = \int_0^3 \frac{5}{3}x\,dx = \frac{5}{3}\frac{x^2}{2}\Big|_0^3$

3

$m = \frac{15}{2}$

(b) The moment M_y of the lamina about the y–axis.

$M_y = \int_0^3 x\,\frac{5}{3}x\,dx = \frac{5}{3}\frac{x^3}{3}\Big|_0^3 = 15$

3

$M_y = 15$

(c) The moment M_x of the lamina about the x–axis.

$M_x = \int_0^3 \frac{1}{2}\left(\frac{5}{3}x\right)\frac{5}{3}x\,dx = \frac{1}{2}\cdot\frac{25}{9}\frac{x^3}{3}\Big|_0^3 = \frac{25}{2}$

3

$M_x = \frac{25}{2}$

(d) The center of mass $(\overline{x}, \overline{y})$ of the lamina.

$m\overline{x} = M_y \to \frac{15}{2}\overline{x} = 15 \to \overline{x} = 2$

$m\overline{y} = M_x \to \frac{15}{2}\overline{y} = \frac{25}{2} \to \overline{y} = \frac{5}{3}$

4 OK if correct consistently with above

$(\overline{x}, \overline{y}) = (2, \frac{5}{3})$

(8) 9. Determine whether each sequence below converges or diverges and if it converges find its limit. (You need not show work for this problem).

NPC

(a) $a_n = \frac{3^{n+2}}{5^n}$ $\quad \lim_{n\to\infty} \frac{3^{n+2}}{5^n} = \lim_{n\to\infty} 9\left(\frac{3}{5}\right)^n = 9\cdot 0 = 0$

2

0

(b) $\left\{\frac{(2n-1)!}{(2n+1)!}\right\}$ $\quad \lim_{n\to\infty} \frac{(2n-1)!}{(2n+1)!} = \lim_{n\to\infty} \frac{1}{2n(2n+1)} = 0$

2

0

(c) $a_n = \frac{e^n}{n^3}$ $\quad \lim_{n\to\infty} \frac{e^n}{n^3} = \lim_{x\to\infty} \frac{e^x}{x^3} \overset{L'H}{=} \infty$

2

diverges

(d) $a_n = \frac{n}{\sqrt{3n^2+1}}$ $\quad \lim_{n\to\infty} \frac{n}{\sqrt{3n^2+1}} = \lim_{x\to\infty} \frac{x}{\sqrt{3x^2+1}}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{3+\frac{1}{x^2}}} = \frac{1}{\sqrt{3}}$

2

$\frac{1}{\sqrt{3}}$