NAME Solutions

STUDENT ID #

RECITATION INSTRUCTOR

RECITATION TIME

LECTURER

INSTRUCTIONS

1. There are 10 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–10.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. Find the value of c so that the vectors $\vec{a} = 2\vec{i} + c\vec{j} - \vec{k}$ and $\vec{b} = \vec{i} - 3\vec{j} + 2\vec{k}$ are perpendicular.

$\vec{a} \cdot \vec{b} = 0$

$2 - 3c - 2 = 0$

$c = 0$

A. $c = \frac{4}{3}$

B. $c = -1$

C. $c = 1$

✓D. $c = 0$

E. no value of c

2. Calculate the length of the projection of $\vec{b} = \vec{i} + \vec{j} + \vec{k}$ onto $\vec{a} = 2\vec{i} - \vec{k}$.

$\| pr_{\vec{a}} \vec{b} \| = \frac{\vec{a} \cdot \vec{b}}{\|a\|} = \frac{2-1}{\sqrt{5}} = \frac{1}{\sqrt{5}}$

A. $\frac{1}{\sqrt{3}}$

✓B. $\frac{1}{\sqrt{5}}$

C. $\frac{1}{3}$

D. $\frac{1}{5}$

E. 1

3. Find $\lim\limits_{x \to 0} \frac{x^3}{x - \sin x}$.

$= \lim\limits_{x \to 0} \frac{3x^2}{1 - \cos x} = \lim\limits_{x \to 0} \frac{6x}{\sin x} = 6$

A. 0

B. 3

✓C. 6

D. -6

E. -3

4. Find $\lim_{x\to\infty} (\ln x)^{\frac{1}{x}}$ $= \lim_{x\to\infty} e^{\frac{1}{x}\ln(\ln x)}$.

$\lim_{x\to\infty} \frac{\ln(\ln x)}{x} = \lim_{x\to\infty} \frac{\frac{1}{\ln x}\frac{1}{x}}{1} = 0$

$\lim_{x\to\infty} (\ln x)^{1/x} = e^0 = 1$

A. 0

B. ∞

C. e

D. e^{-1}

✓E. 1

5. $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx =$ $\int_0^1 u^2(1-u^2)\,du$

$u = \sin x \quad du = \cos x\,dx$

$x=0 \to u = 0$

$x = \frac{\pi}{2} \to u = 1.$

$= \left(\frac{u^3}{3} - \frac{u^5}{5}\right)\Big|_0^1 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

✓A. $\frac{2}{15}$

B. $\frac{1}{4}$

C. $\frac{2}{3}$

D. $\frac{1}{20}$

E. $\frac{1}{5}$

6. $\int_1^2 x \ln x dx =$ $\frac{x^2}{2}\ln x\Big|_1^2 - \int_1^2 \frac{x}{2}dx$

$u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$= \frac{4}{2}\ln 2 - \left[\frac{x^2}{4}\right]_1^2$

$= 2\ln 2 - [1 - \frac{1}{4})$

$= 2\ln 2 - \frac{3}{4}$

A. $2\ln 2 - 1$

B. $\frac{1}{2}(\ln 2)^2$

C. $2\ln 2$

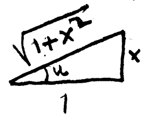
✓D. $2\ln 2 - \frac{3}{4}$

E. $1 - \ln x$

7. $\displaystyle\int_0^{\sqrt{3}} \frac{1}{(1+x^2)^{\frac{3}{2}}}\,dx =$ $\displaystyle\int_0^{\frac{\pi}{3}} \frac{1}{\sec^3 u}\sec^2 u\,du = \int_0^{\pi/3} \cos u\,du$

$= \sin u\Big|_0^{\pi/3}$

$= \frac{\sqrt{3}}{2}$



$x = \tan u \qquad \sqrt{1+x^2} = \sec u$

$dx = \sec^2 u\,du$

$x = 0 \rightarrow u = 0$

$x = \sqrt{3} \rightarrow u = \frac{\pi}{3}$

A. $\frac{1}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{2}{3}$

✓ E. $\frac{\sqrt{3}}{2}$

8. Express $\dfrac{3x+7}{(x^2+1)(x^2-4)}$ as a sum of partial fractions.

A. $\dfrac{A}{x^2+1} + \dfrac{B}{x-2} + \dfrac{C}{x+2}$

B. $\dfrac{A}{x^2+1} + \dfrac{B}{x^2-4}$

C. $\dfrac{A}{x+1} + \dfrac{B}{x^2+1} + \dfrac{C}{x-2} + \dfrac{D}{x+2}$

D. $\dfrac{Ax+B}{x^2+1} + \dfrac{B}{x^2-4}$

✓ E. $\dfrac{Ax+B}{x^2+1} + \dfrac{C}{x-2} + \dfrac{D}{x+2}$

9. Determine whether the improper integral

$$\int_0^{\infty} xe^{-x^2}\,dx$$

converges. If it does, find its value.

$= \lim\limits_{b\to\infty} \displaystyle\int_0^b x e^{-x^2}\,dx$

$= \lim\limits_{b\to\infty} \left[-\frac{1}{2}e^{-x^2}\right]_0^b$

$= \lim\limits_{b\to\infty} \left(\frac{1}{2} - \frac{1}{2}e^{-b^2}\right) = \frac{1}{2}$

A. 0

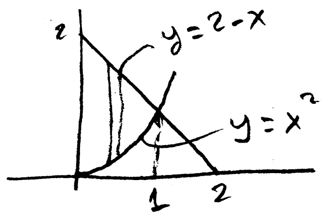
✓ B. $\frac{1}{2}$

C. 1

D. 2

E. diverges

10. Let R be the region in the first quadrant bounded by the graphs of $x+y=2$, $y=x^2$, and the y-axis. Find the volume of the solid generated by revolving R about the y axis.



$2-x=x^2$

x^2+x-2

$(x-1)(x+2)=0$

$$\int_0^1 2\pi x\left(2-x-x^2\right)dx$$

$$=2\pi\left(x^2-\frac{x^3}{3}-\frac{x^4}{4}\right)\Big|_0^1$$

$$=2\pi\left(1-\frac{1}{3}-\frac{1}{4}\right)$$

$$=2\pi\,\frac{12-4-3}{12}$$

$$=\frac{5\pi}{6}$$

✓A. $\frac{5\pi}{6}$

B. $\frac{47\pi}{15}$

C. $\frac{43\pi}{15}$

D. $\frac{5\pi}{12}$

E. $\frac{5\pi}{3}$

11. Find the length of the curve $y=\frac{2}{3}(x-1)^{\frac{3}{2}}$ for $1 \le x \le 4$.

$$L=\int_1^4 \sqrt{1+\left[(x-1)^{1/2}\right]^2}\,dx$$

$$=\int_1^4 \sqrt{x}\,dx=\frac{2}{3}x^{3/2}\Big|_1^4$$

$$=\frac{2}{3}(8-1)=\frac{14}{3}$$

A. 14

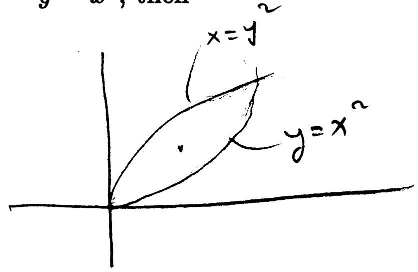
✓B. $\frac{14}{3}$

C. $\frac{7}{3}$

D. $\frac{16}{3}$

E. 5

12. If $(\bar{x},\bar{y})$ is the center of gravity of the region bounded by the graph of $y=x^{\frac{1}{2}}$ and $y=x^2$, then



region is symmetric about the line $y=x$

A. $\bar{x}<\bar{y}$

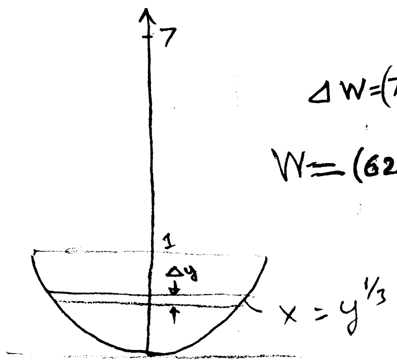
B. $\bar{x}>\bar{y}$

C. $\bar{x}\bar{y}=0$

D. $\bar{x}\bar{y}<0$

✓E. $\bar{x}=\bar{y}$

13. A tank has the shape of a surface generated by revolving the curve $y = x^3$, $0 \le x \le 1$ about the y-axis. The tank is full of water. Set up an integral (in y) for the work W required to pump all the water to a level 6 feet above the top of the tank. (Water weighs 62.5 lbs/ft^3).



$\Delta W = (7-y)(62.5)\pi\, y^{2/3}\Delta y$

$W = (62.5)\pi \int_0^1 (7-y)\, y^{2/3}\, dy$

A. $(62.5)\pi \int_0^1 (7-y)y^{\frac{2}{3}}\,dy$

B. $(62.5)\pi \int_0^1 (7-y)y^2\,dy$

C. $(62.5)\pi \int_0^1 (6-y)y^{\frac{2}{3}}\,dy$

D. $(62.5)\pi \int_0^1 (5-y)y^3\,dy$

E. $(62.5)\pi \int_1^7 (1-y)y\,dy$

14. The fourth Taylor polynomial of $f(x) = e^{x^2}$ about 0 is

A. $1 + x + x^2 + x^4$

B. $1 + x^2 + \frac{x^4}{2!}$

C. $1 + \frac{x^2}{2!} + \frac{x^4}{4!}$

D. $1 + x + \frac{x^3}{3!} + \frac{x^4}{4!}$

E. $1 + \frac{x^4}{4!}$

15. Find $\lim_{n\to\infty} \left(\sqrt[n]{n} + \frac{(-1)^n\sqrt{n}}{n+1} \right)$.

A. 1

B. 2

C. -1

D. 0

E. does not exist

16. Of the two series

(i) $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^5+3n-1}}$, (ii) $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$

By lim. comp. test both conv. or both div.

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ conv. (p-series $p=\frac{3}{2}>1$)

A. both diverge

B. (i) converges, (ii) diverges

C. (i) diverges, (ii) converges

✓D. both converges

E. (i) converges conditionally and (ii) diverges

17. Which of the following series converge?

(i) $\sum_{n=1}^{\infty} \frac{1}{3^n}$, (ii) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$, (iii) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

(i) geom. ser. $r=\frac{1}{3}$ $\left|\frac{1}{3}\right|<1$ conv.

(ii) comp. with $\sum_{n=2}^{\infty} \frac{1}{n}$ div (p-ser $p=1$)

$\frac{\ln n}{n} \geq \frac{1}{n}$ for $n \geq 3$.

(iii) conv. (alt. ser. test)

A. Only (i)

B. Only (i) and (ii)

C. Only (ii) and (iii)

✓D. Only (i) and (iii)

E. all three

18. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^{2n}}{4^n}$ is

gen. rat. test

$$\lim_{n\to\infty} \left| \frac{x^{2n+2}}{4^{n+1}} \cdot \frac{4^n}{x^{2n}} \right|$$

$$= \lim_{n\to\infty} \frac{1}{4} x^2 = \frac{x^2}{4}$$

conv if $\frac{x^2}{4} < 1$ or $-2 < x < 2$

✓A. 2

B. 4

C. ∞

D. 1

E. 0

19. The interval of convergence of the power series $\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$ is

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{\ln(n+1)} \cdot \frac{\ln n}{x^n} \right| =$$

$= |x|$ $\quad |x|<1 \quad -1<x<1$

$x=1.$ $\quad \sum_{n=2}^{\infty} \frac{1}{\ln n}$ div (comp with $\sum \frac{1}{n}$)

$x=-1$ $\quad \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ conv (alt. ser. test)

$-1 \le x < 1$ or $[-1,1)$

A. $(-1,1)$

✓B. $[-1,1)$

C. $(-1,1]$

D. $[-1,1]$

E. (e,e)

20. Match the functions with their Taylor series about $a=0$.

(1)	e^{-x}	(a)	$x - \frac{x^3}{2!} + \frac{x^5}{4!} - \ldots, \ -\infty < x < \infty$
(2)	$\frac{1}{1-x}$	(b)	$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}, \ -\infty < x < \infty$
(3)	$\frac{1}{1+x^2}$	(c)	$1 - x^2 + x^4 - x^6 + \ldots, \ -1 < x < 1$
(4)	$\sin 2x$	(d)	$2x - \frac{2^3x^3}{3!} + \frac{2^5x^5}{5!} - \ldots, \ -\infty < x < \infty$
(5)	$x \cos x$	(e)	$1 + x + x^2 + x^3 + \ldots, \ -1 < x < 1$

1b, 2e

3c 4d 5a

A. 1a,2e,3c,4b,5d

B. 1c,2a,3d,4b,5e

C. 1b,2c,3e,4d,5a

D. 1e,2b,3e,4c,5a

✓E. 1b,2e,3c,4d,5a

21. The parametric equations of a curve C are:

$$x = t,\ \ y = -\sqrt{4-t^2};\ \ -2 \le t \le 2$$

The curve C is

$x^2+y^2=4 \qquad y \le 0$

A. a half circle

B. a circle

C. an ellipse

D. a straight line

E. a quarter circle

22. The length L of the curve C in problem 21 is given by

$$L=\int_{-2}^{2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$
$$=\int_{-2}^{2}\sqrt{1^2+\left[-\frac{1}{2}\frac{1}{\sqrt{4-t^2}}(-2t)\right]^2}\,dt$$
$$=\int_{-2}^{2}\sqrt{1^2+\frac{t^2}{4-t^2}}\,dt$$
$$=\int_{-2}^{2}\frac{2}{\sqrt{4-t^2}}\,dt$$

A. $\displaystyle\int_{-2}^{2}\frac{t}{\sqrt{4-t^2}}\,dt$

B. $\displaystyle\int_{-2}^{2}\frac{1}{\sqrt{4-t^2}}\,dt$

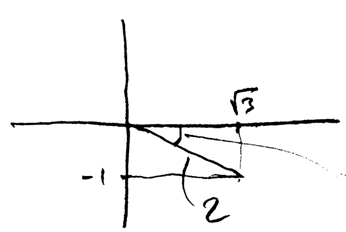
C. $\displaystyle\int_{-2}^{2}\frac{1}{4-t^2}\,dt$

D. $\displaystyle\int_{-2}^{2}\frac{2}{\sqrt{4-t^2}}\,dt$

E. $\displaystyle\int_{-2}^{2}\sqrt{5-t^2}\,dt$

23. A point P has cartesian coordinates $(\sqrt{3},-1)$. Which of the following are polar coordinates of P?

(i) $(-2,\frac{\pi}{6})$ (ii) $(-2,\frac{5\pi}{6})$ (iii) $(2,-\frac{\pi}{6})$ (iv) $(2,\frac{11\pi}{6})$



$\tan\theta = \frac{y}{x} = -\frac{1}{\sqrt{3}}$
$= -\frac{\sqrt{3}}{3}$
$\theta = -\frac{\pi}{6}$

A. (ii) and (iii) only

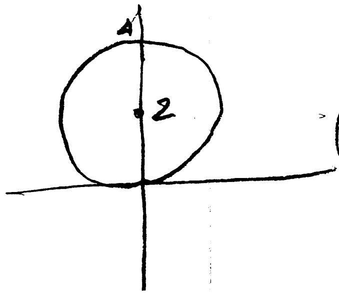
B. (i), (ii), and (iv) only

C. (ii), (iii) and (iv) only

D. (iii) and (iv) only

E. all of them

24. In rectangular coordinates, the center of the circle with polar equation $r = 4\sin\theta$ is



$(0, 2)$

A. $(\overset{2}{\cancel{4}}, 0)$

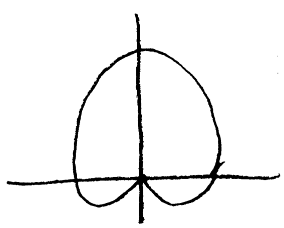
B. $(-2, 0)$

C. $(4, 0)$

✓D. $(0, 2)$

E. $(0, -2)$

25. The area of the region inside the cardioid $r = 1 + \sin\theta$ is



$$A = \int_0^{2\pi} \frac{1}{2} r^2 \, d\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{1}{2}\int_0^{2\pi} (1 + 2\sin\theta + \sin^2\theta)\, d\theta$$

$$= \frac{1}{2}\left[\theta - 2\cos\theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{2\pi}$$

$$= \frac{1}{2}\left[2\pi - 2 + \pi + 2\right] = \underline{\frac{3\pi}{2}}$$

✓A. $\frac{3\pi}{2}$

B. 3π

C. $\frac{3\pi}{2} - 1$

D. $\frac{3\pi}{2} + 1$

E. $3\pi - 1$