NAME SOLUTIONS

STUDENT ID # ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

LECTURER ______

INSTRUCTIONS

1. There are 10 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–10.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. Which of the following statements are always true for any three dimensional vectors $\vec{a}$ and $\vec{b}$?

(I) $(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$

(II) $(\vec{a} \times \vec{b}) \times \vec{a} = \vec{0}$

(III) $\vec{a} \cdot \vec{b} \geq 0$

(I) true : $\vec{a} \times \vec{b}$ is $\perp$ to both $\vec{a}$ and $\vec{b}$

(II) not true: ex $(\vec{i} \times \vec{j}) \times \vec{i} = \vec{k} \times \vec{i} = \vec{j}$

(III) not true ; $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta \leq 0$ for $\frac{\pi}{2} \leq \theta \leq \pi$

A. (I) only
B. (II) only
C. (I) and (II) only
D. (II) and (III) only
E. all

2. Find the area of the triangle with vertices $P(1,1,0)$, $Q(3,-2,2)$, and $R(4,-2,1)$.

Area of $\Delta PQR = \frac{1}{2}|\vec{PQ} \times \vec{PR}|$

$\vec{PQ} = 2\vec{i} - 3\vec{j} + 2\vec{k}$ $\quad \vec{PR} = 3\vec{i} - 3\vec{j} + \vec{k}$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 2 \\ 3 & -3 & 1 \end{vmatrix} = 3\vec{i} + 4\vec{j} + 3\vec{k}$$

$$|\vec{PQ} \times \vec{PR}| = \sqrt{9+16+9} = \sqrt{34}$$

Area of $\Delta PQR = \frac{1}{2}\sqrt{34} = \sqrt{\frac{34}{4}} = \sqrt{\frac{17}{2}}$

A. 12
B. $\sqrt{\frac{20}{3}}$
C. $\sqrt{\frac{31}{5}}$
D. $\frac{1}{2}\sqrt{39}$
E. $\sqrt{\frac{17}{2}}$

3. Suppose that $x^2 - 4ax + y^2 + 8y + z^2 = 0$, where a is a positive constant, is the equation of a sphere of radius 6. Find a.

Complete squares:

$$x^2 - 4ax + (2a)^2 + y^2 + 8y + 4^2 + z^2 = (2a)^2 + 4^2$$

$$(x-2a)^2 + (y+4)^2 + z^2 = 4a^2 + 16$$

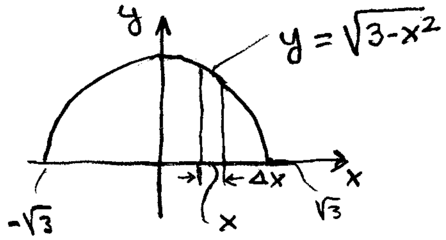
$$\therefore \quad R^2 = 4a^2 + 16 = 36$$

$$4a^2 = 20$$

$$a^2 = 5 \qquad a = \sqrt{5}$$

A. 5
B. $\sqrt{5}$
C. $\sqrt{3}$
D. $\frac{3}{2}$
E. $\sqrt{7}$

4. Let R be the region bounded by the curves $y = \sqrt{3 - x^2}$ and $y = 0$. Find the volume of the solid obtained by rotating R about the x-axis.



Volume of typical disk:

$$\Delta V = \pi\left(\sqrt{3-x^2}\right)^2 \Delta x$$

$$V = \int_{-\sqrt{3}}^{\sqrt{3}} \pi(3-x^2)\,dx = 2\pi\int_0^{\sqrt{3}} (3-x^2)\,dx$$

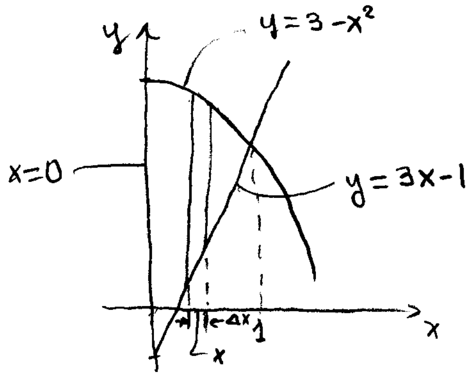
$$= 2\pi\left[3x - \frac{x^3}{3}\right]_0^{\sqrt{3}} = 2\pi(3\sqrt{3}-\sqrt{3})$$

$$= 4\sqrt{3}\pi$$

A. 2π
B. 3π
C. $4\sqrt{3}\pi$
D. $\sqrt{2}\pi$
E. $3\sqrt{2}\pi$

Or Notice that the solid is a solid sphere of radius $\sqrt{3}$

5. Let R be the region in the right half plane $x \geq 0$ bounded by the curves $y = 3 - x^2$, $y = 3x - 1$, and $x = 0$. The volume of the solid generated by rotating R about the y-axis is given by



A. $\int_0^1 2\pi(4x - x^3 - 3x^2)\,dx$

B. $\int_0^2 \pi[(3-x^2)^2 - (3x-1)^2]\,dx$

C. $\int_0^1 \pi[(3x-1)^2 - (3-x^2)^2]\,dx$

D. $\int_0^1 2\pi(6x - 3x^2 - 5)\,dx$

E. $\int_0^1 2\pi(9x - 6x^2 + x^3)\,dx$

Point of intersection in the right half plane $x \geq 0$:

$$3 - x^2 = 3x - 1 \longrightarrow x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0 \longrightarrow x = 1.$$

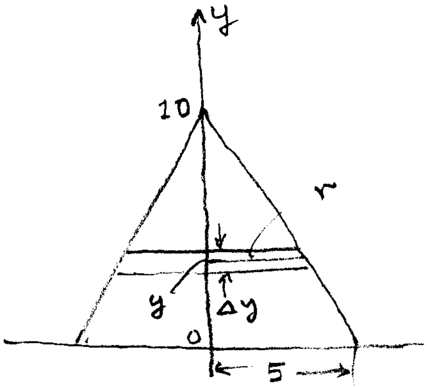
Method of cylindrical shells:

Volume of typical shell:

$$\Delta V = 2\pi x\left[(3-x^2) - (3x-1)\right]\Delta x$$

$$V = \int_0^1 2\pi\left(4x - x^3 - 3x^2\right)dx$$

6. The circular base of a conical tank is at ground level, its axis is vertical and the conical tip of the tank is 10 ft above the ground. The radius of the base is 5 ft and tank is full of water which weighs 62.5 lbs/ft^3. If the y–axis is upwards, along the axis of the tank, and $y = 0$ at the base, the work required to pump all the water to the top of the tank is given by



$\frac{r}{10-y} = \frac{5}{10}$ $\therefore r = \frac{1}{2}(10-y)$

Weight of typical layer of water:

$62.5\,\pi\left[\frac{1}{2}(10-y)\right]^2 \Delta y$

Work required to lift typical layer from height y to height 10:

$\Delta W = (10-y)(62.5)\pi\left[\frac{1}{2}(10-y)\right]^2 \Delta y$

$W = 62.5\,\pi \int_0^{10} \frac{1}{4}(10-y)^3\,dy$

A. $62.5\pi \int_0^{10} \frac{1}{4}(10-y)^2\,dy$

B. $62.5\pi \int_0^{10} \frac{1}{4}y(10-y)^2\,dy$

C. $62.5\pi \int_0^{10} \frac{1}{4}(10-y)^3\,dy$

D. $62.5\pi \int_0^{10} \frac{1}{2}y^2\,dy$

E. $62.5\pi \int_0^{10} \frac{1}{2}y(10-y)\,dy$

7. $\int_0^1 \sin^{-1} x\,dx =$

Integration by parts

$u = \sin^{-1}x \quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \quad v = x$

$\therefore \int_0^1 \sin^{-1}x\,dx = x\sin^{-1}x\Big]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$

$= \sin^{-1}1 - \left[-\sqrt{1-x^2}\right]_0^1$

$= \frac{\pi}{2} - \left[0+1\right] = \frac{\pi}{2} - 1$

A. $\frac{\pi}{2} + 2$

B. π

C. $\pi + 1$

D. 2π

E. $\frac{\pi}{2} - 1$

8 By a suitable trigonometric substitution, the integral $\int_5^{10/\sqrt{3}} \frac{\sqrt{x^2-25}}{5x}\,dx$ is transformed to the integral.

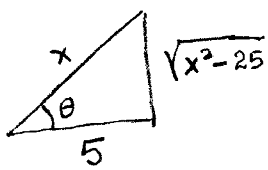
A. $\int_0^{\frac{\pi}{3}} \sin\theta d\theta$

B. $\int_0^{\frac{\pi}{6}} \sin\theta d\theta$

C. $\int_0^{\frac{\pi}{3}} \cos\theta d\theta$

D. $\int_0^{\frac{\pi}{6}} \tan^2\theta d\theta$

E. $\int_0^{\frac{\pi}{3}} \tan^2\theta d\theta$



$x = 5\sec\theta$

$dx = 5\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-25} = 5\tan\theta$

$x = 5 \rightarrow \sec\theta = 1 \rightarrow \theta = 0$

$x = \frac{10}{\sqrt{3}} \rightarrow \frac{10}{\sqrt{3}} = 5\sec\theta \rightarrow \cos\theta = \frac{\sqrt{3}}{2} \rightarrow \theta = \frac{\pi}{6}$

$$\int_5^{\frac{10}{\sqrt{3}}} \frac{\sqrt{x^2-25}}{5x}\,dx = \int_0^{\frac{\pi}{6}} \frac{5\tan\theta\, 5\sec\theta\tan\theta\,d\theta}{25\sec\theta}$$

$$= \int_0^{\frac{\pi}{6}} \tan^2\theta\,d\theta$$

9. $\int_3^4 \frac{3}{x^2-x-2}\,dx =$

A. $\ln\frac{3}{4}$

B. $\ln\frac{8}{5}$

C. $\ln\frac{2}{3}$

D. $3\ln 2$

E. $\frac{1}{2}\ln 5$

$$\frac{3}{x^2-x-2} = \frac{3}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$3 = Ax + A + Bx - 2B \rightarrow \begin{bmatrix} A+B=0 \\ A-2B=3 \end{bmatrix} \rightarrow \begin{matrix} A=1 \\ B=-1 \end{matrix}$$

$$\int_3^4 \frac{3}{x^2-x-2}\,dx = \int_3^4 \left(\frac{1}{x-2} - \frac{1}{x+1}\right)dx$$

$$= \Big[\ln|x-2| - \ln|x+1|\Big]_3^4 = \ln\frac{x-2}{x+1}\Big|_3^4$$

$$= \ln\frac{2}{5} - \ln\frac{1}{4} = \ln\frac{8}{5}$$

10. $\int_0^{\frac{\pi}{4}} \tan^3 x \sec^4 x\,dx =$

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{5}{12}$

E. $\frac{7}{6}$

$$= \int_0^{\frac{\pi}{4}} \tan^3 x\,(1+\tan^2 x)\sec^2 x\,dx =$$

$u = \tan x \quad du = \sec^2 x\,dx$

$x = 0 \rightarrow u = 0, \quad x = \frac{\pi}{4} \rightarrow u = 1$

$$= \int_0^1 (u^3 + u^5)\,du = \left[\frac{u^4}{4} + \frac{u^6}{6}\right]_0^1$$

$$= \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$$

11. Decide whether each improper integral conveges or diverges.

(I) $\int_1^2 \frac{1}{x-1}\,dx$ (II) $\int_0^2 \frac{1}{(x-1)^2}\,dx$

A. (I) converges, (II) converges

B. (I) converges, (II) diverges

C. (I) diverges, (II) converges

D. (I) diverges, (II) diverges

(I) $\int_1^2 \frac{1}{x-1}\,dx = \lim_{t\to 1^+} \int_t^2 \frac{1}{x-1}\,dx$

$= \lim_{t\to 1^+} \ln|x-1|\Big|_t^2 =$

$= \lim_{t\to 1^+} \left[\ln 1 - \ln|t-1|\right] = \infty$

$\therefore$ integral diverges

(II) $\int_0^2 \frac{1}{(x-1)^2}\,dx = \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx$

$\int_0^1 \frac{1}{(x-1)^2}\,dx = \lim_{t\to 1^-} \int_0^t \frac{1}{(x-1)^2}\,dx = \lim_{t\to 1^-} \left[-\frac{1}{x-1}\right]_0^t = \lim_{t\to 1^-} \left[-\frac{1}{t-1} - 1\right] = \infty$

$\therefore$ integral diverges

12. The length of the curve $y = \sin x$, $0 \le x \le \pi$ is given by

A. $\int_0^\pi \sqrt{1+\cos^2 x}\,dx$

B. $\int_0^\pi 2\pi\sqrt{1+\cos^2 x}\,dx$

C. $\int_0^\pi \sqrt{1+\sin^2 x}\,dx$

D. $\int_0^\pi 2\pi \sin x\sqrt{1+\cos^2 x}\,dx$

E. $\int_0^\pi 2\pi x\sqrt{1+\sin^2 x}\,dx$

$L = \int_0^\pi \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$= \int_0^\pi \sqrt{1+\cos^2 x}\,dx$

13. Evaluate $\lim_{n\to\infty}\left[\frac{(-1)^n}{\sqrt{n}} + \frac{(2n-1)(3n-2)}{3n^2+1}\right] =$, if it exists.

A. 0

B. 2

C. 3

D. 1

E. does not exist

$= \lim_{n\to\infty} \frac{(-1)^n}{\sqrt{n}} + \lim_{n\to\infty} \frac{6n^2-7n+2}{3n^2+1}$

$= 0 + 2 = 2$

14. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3}{4^n}-\frac{2}{5^{n-1}}\right)$ if it is convergent.

A. $-\frac{3}{2}$

B. $\frac{3}{2}$

C. $\frac{7}{2}$

D. $\frac{1}{2}$

E. divergent

$$\sum_{n=1}^{\infty}\frac{3}{4^n} = \frac{3}{4}+\frac{3}{4^2}+\frac{3}{4^3}+\cdots = \frac{3}{4}\left[1+\frac{1}{4}+\left(\frac{1}{4}\right)^2+\cdots\right] = \frac{3}{4}\,\frac{1}{1-\frac{1}{4}} = 1$$

$$\sum_{n=1}^{\infty}\frac{2}{5^{n-1}} = 2+\frac{2}{5}+\frac{2}{5^2}+\cdots = 2\left[1+\frac{1}{5}+\left(\frac{1}{5}\right)^2+\cdots\right] = 2\,\frac{1}{1-\frac{1}{5}} = \frac{5}{2}$$

$$\therefore \sum_{n=1}^{\infty}\left(\frac{3}{4^n}-\frac{2}{5^{n-1}}\right) = 1-\frac{5}{2} = -\frac{3}{2}$$

15. Which of the following statements are always true?

(I) If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=1}^{\infty}\frac{1}{a_n}$ converges.

(II) If the sequence $\{a_n\}_{n=1}^{\infty}$ is increasing and $0 \le a_n \le 1$, for all $n \ge 1$, then the sequence $\{a_n\}_{n=1}^{\infty}$ is convergent.

(III) If the sequence $\{a_n\}_{n=1}^{\infty}$ is convergent, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

A. all

B. (I) and (II) only

C. (II) and (III) only

D. (II) only

E. (III) only

(I) not true: $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges but $\sum_{n=1}^{\infty} n^2$ diverges

(II) true: by theorem: a bounded and increasing sequence is convergent

(III) not true: $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is convergent but $\sum_{n=1}^{\infty}\frac{1}{n}$ is divergent

16. Which of the following series converge?

(I) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$ (II) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2+\sqrt{n}}$ (III) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{n!}$

A. (I) only

B. (II) only

C. (III) only

D. (I) and (III) only

E. (II) and (III) only

(I) Integral test: $\displaystyle\int_2^{\infty}\frac{1}{x\ln x}\,dx = \lim_{t\to\infty}\int_2^{t}\frac{1}{x\ln x}\,dx = \lim_{t\to\infty}\ln|\ln x|\Big|_2^t = \lim_{t\to\infty}\left[\ln|\ln t| - \ln|\ln 2|\right] = \infty$

$\therefore$ series diverges

(II) Compare with $\sum_{n=1}^{\infty}\frac{1}{n^2}$ which conv. (p-ser. $p=2>1$)

Comp. test

$\frac{1}{n^2+\sqrt{n}} \le \frac{1}{n^2}$ for all $n \ge 1$ $\therefore$ ser. conv.

(III) Ratio test: $\displaystyle\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{(n+1)^2}{(n+1)!}}{\frac{n^2}{n!}} = \frac{(n+1)^2\, n!}{(n+1)!\, n^2} = \left(\frac{n+1}{n}\right)^2\frac{1}{n+1} \xrightarrow[\text{as } n\to\infty]{} 0 < 1$

$\therefore$ ser. conv.

17. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{n3^n}{2n-1} x^n$ is

Ratio test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{(n+1)3^{n+1}x^{n+1}}{2n+1}}{\frac{n3^n x^n}{2n-1}}\right|$$

$$= \left|\frac{(n+1)3^{n+1}x^{n+1}(2n-1)}{(2n+1)\ n3^n x^n}\right| = \frac{n+1}{n}\, 3\, \frac{2n-1}{2n+1}|x| \longrightarrow 3|x| \text{ as } n\to\infty$$

$\therefore$ series conv. if $3|x|<1$, or $|x|<\frac{1}{3}$, or $-\frac{1}{3}<x<\frac{1}{3}$

When $x=-\frac{1}{3}$: $\sum_{n=1}^{\infty} \frac{n3^n}{2n-1}\left(-\frac{1}{3}\right)^n = \sum_{n=1}^{\infty} (-1)^n \frac{n}{2n-1}$

diverges because $\lim_{n\to\infty} (-1)^n \frac{n}{2n-1}$ DNE

When $x=\frac{1}{3}$: $\sum_{n=1}^{\infty} \frac{n3^n}{2n-1}\left(\frac{1}{3}\right)^n = \sum_{n=1}^{\infty} \frac{n}{2n-1}$ diverges because $\lim_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2} \neq 0$

A. $\left(-\frac{1}{3}, \frac{1}{3}\right)$

B. $\left(-\frac{1}{3}, \frac{1}{3}\right]$

C. $\left[-\frac{1}{3}, \frac{1}{3}\right)$

D. $(-\infty, \infty)$

E. series converges for $x=0$ only

18. In the Taylor series for $f(x) = \frac{1}{x^2}$ about $a=1$, the coefficient of $(x-1)^4$ is

The coefficient of $(x-1)^4$ is $\frac{f^{(4)}(1)}{4!}$

$f(x) = x^{-2}$

$f^{(1)}(x) = -2x^{-3}$

$f^{(2)}(x) = 6x^{-4}$

$f^{(3)}(x) = -24x^{-5}$

$f^{(4)}(x) = 120x^{-6}$

$$\frac{f^{(4)}(1)}{4!} = \frac{120}{24} = 5$$

A. $\frac{1}{24}$

B. $-\frac{1}{24}$

C. 5

D. 120

E. 1

19. Use the Maclaurin series for $\cos(x^2)$ to approximate $\int_0^1 \cos(x^2)\,dx$. The smallest number of terms needed to approximate the integral with error < 0.01 is

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\int_0^1 \cos(x^2)\,dx = \int_0^1 \left(1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots\right)dx$$

$$= \left[x - \frac{x^5}{5\cdot 2} + \frac{x^9}{9\cdot 24} - \frac{x^{13}}{13\cdot 6!} + \cdots\right]_0^1$$

$$= 1 - \frac{1}{10} + \frac{1}{216} - \frac{1}{13\cdot 6!} + \cdots$$

$\frac{1}{10} > 0.01$ and $\frac{1}{216} < 0.01$ $\therefore$ the smallest number of terms needed is 2.

A. 1

B. 2

C. 3

D. 4

E. 5

20. An equation of the tangent line to the parametric curve $x = t^3 + 3t^2 - t$, $y = t^4 + t$ at the point corresponding to $t = 1$ is

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3+1}{3t^2+6t-1}$$

When $t=1$; $x = 1+3-1=3$, $y=1+1=2$

$$\frac{dy}{dx} = \frac{4\cdot 1+1}{3\cdot 1+6\cdot 1-1} = \frac{5}{8}$$

Eq. of tangent line: $y-2 = \frac{5}{8}(x-3)$

$8y-16 = 5x-15$

$5x-8y+1=0$

A. $5x + 8y - 31 = 0$

B. $5x - 8y + 1 = 0$

C. $8x - 5y - 9 = 0$

D. $8x + 5y - 39 = 0$

E. $3x + 2y - 13 = 0$

21. The graph of the parametric curve $x = 2\sin^2 t$, $y = 3\cos^2 t$ is

$\frac{x}{2} + \frac{y}{3} = 1$; $0 \le x \le 2$, $0 \le y \le 3$

line segment

A. a circle

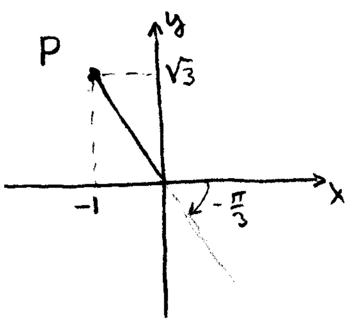
B. an ellipse

C. a parabola

D. a line

E. a line segment

22. A point P has Cartesian coordinates $(x, y) = (-1, \sqrt{3})$. Polar coordinates of P are



$r^2 = (-1)^2 + (\sqrt{3})^2 = 4$

$\therefore r = \pm 2$

$\tan\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

$\therefore \theta = -\frac{\pi}{3}, \frac{2\pi}{3}$, etc

$\theta = -\frac{\pi}{3}$ $r=-2$

A. $\left(-2, \frac{2\pi}{3}\right)$

B. $\left(2, \frac{\pi}{3}\right)$

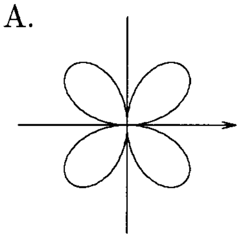
C. $\left(-2, \frac{\pi}{3}\right)$

D. $\left(-2, -\frac{\pi}{3}\right)$

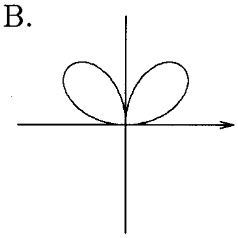
E. $\left(2, -\frac{2\pi}{3}\right)$

23. The graph of $r = \sin 2\theta$, $0 \le \theta \le \pi$, is

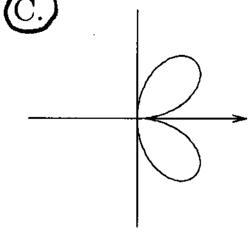
A.



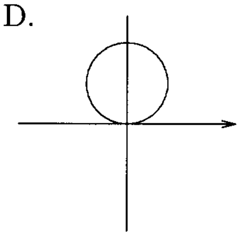
B.



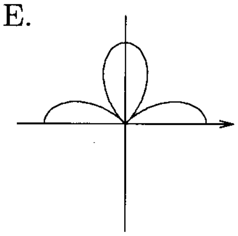
C.



D.



E.



θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
2θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	0	1	0	-1	0

24. If $z = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$ and $w = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ then $\frac{z}{w} =$

$z = 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \sqrt{3} + i$, $w = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 1 + i\sqrt{3}$

$$\frac{z}{w} = \frac{\sqrt{3}+i}{1+i\sqrt{3}} = \frac{\sqrt{3}+i}{1+i\sqrt{3}}\,\frac{1-i\sqrt{3}}{1-i\sqrt{3}} = \frac{\sqrt{3}-3i+i+\sqrt{3}}{1+(\sqrt{3})^2} = \frac{2\sqrt{3}-2i}{4}$$
$$= \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

or

$$\frac{z}{w} = \frac{2}{2}\left(\cos\left(\frac{\pi}{6}-\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{6}-\frac{\pi}{3}\right)\right)$$
$$= 1\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right) = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

A. $\frac{\sqrt{3}}{2} - \frac{1}{2}i$

B. $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$

C. $1 + i$

D. $-\sqrt{3} + i$

E. $1 + \sqrt{2}i$

25. $4e^{2+i\frac{3\pi}{4}} = 4e^2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$$= 4e^2\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$
$$= 2e^2\sqrt{2}\,(-1+i)$$

A. $4e^2 i(\sqrt{3} - 1)$

B. $2e^2(-1 + i)$

C. $4e^2(1 - i)$

D. $2\sqrt{2}e^2(-1 + i)$

E. $2\sqrt{2}e^2(1 - \sqrt{3}i)$