NAME SOLUTIONS

RECITATION INSTRUCTOR ______________________

RECITATION TIME ______________________

LECTURER ______________________

INSTRUCTIONS

1. There are 11 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–11.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student 9 digit ID number (probably your social security number), and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. Which of the following statements are always true for any three–dimensional vectors $\vec{a}$ and $\vec{b}$?

(I) $|\vec{a}\cdot\vec{b}| \leq |\vec{a}||\vec{b}|$

(II) $|\vec{a}\times\vec{b}| \leq |\vec{a}||\vec{b}|$

(III) $\vec{a}\times\vec{a} = \vec{0}$

A. (I) only

B. (II) only

C. (I) and (II) only

D. (II) and (III) only

E. all

(I) $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore |\vec{a}\cdot\vec{b}| = |\vec{a}||\vec{b}||\cos\theta| \leq |\vec{a}||\vec{b}|$ true

(II) $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta \leq |\vec{a}||\vec{b}|$ true

III $\vec{a}\times\vec{a} = \vec{0}$ true

2. Let $\vec{v}_1 = \vec{i} + \vec{j}$ and $\vec{v}_2 = -\vec{i} + \vec{j} + \vec{k}$. Find the values of a and b such that the vector $\vec{v} = 2\vec{i} + a\vec{j} + b\vec{k}$ is perpendicular to both $\vec{v}_1$ and $\vec{v}_2$.

$\vec{v} \perp \vec{v}_1$: $\vec{v}\cdot\vec{v}_1 = 2 + a = 0$

$\vec{v} \perp \vec{v}_2$: $\vec{v}\cdot\vec{v}_2 = -2 + a + b = 0$

$a = -2, \ b = 4$

A. $a = -4,\ b = 4$

B. $a = 2,\ b = -1$

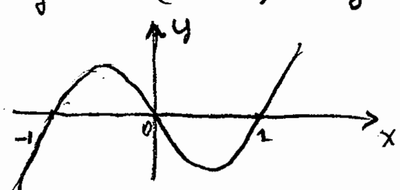
C. $a = -1,\ b = 2$

D. $a = -2,\ b = 4$

E. none of the above

3. Find the total area of the finite regions bounded by the curves $y = 4(x^3 - x)$ and $y = 0$.

$y = 4x(x^2 - 1)$ $y = 0$ when $x = 0, -1, 1$



$$A = \int_{-1}^{0} 4(x^3 - x)\,dx + \int_0^1 [0 - 4(x^3 - x)]\,dx$$

$$= \left[x^4 - 2x^2\right]_{-1}^{0} + \left[-x^4 + 2x^2\right]_0^1$$

$$= -(1-2) + (-1+2) = 2$$

A. 2

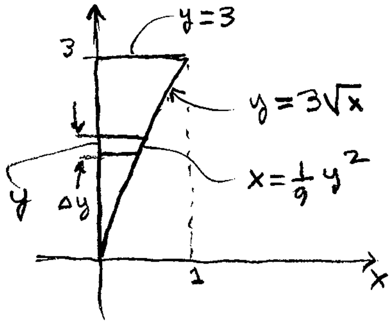
B. 4

C. 0

D. $\frac{1}{3}$

E. 1

4. Let R be the region in the first quadrant bounded by the curves $y = 3\sqrt{x}$, $y = 3$, and $x = 0$. Find the volume of the solid obtained by rotating R about the y–axis.



A. $\frac{\pi}{3}$

B. $\frac{2\pi}{9}$

C. $\frac{3\pi}{5}$

D. $\frac{\pi}{27}$

E. $\frac{2\pi}{15}$

Method of disks

Volume of typical disk:

$$\Delta V = \pi\left(\tfrac{1}{9}y^2\right)^2 \Delta y$$

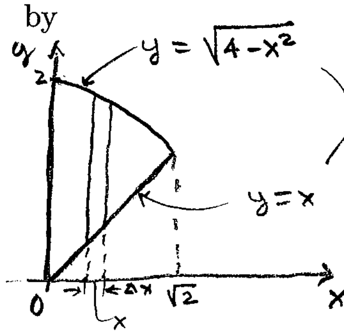
$$V = \int_0^3 \frac{\pi}{81} y^4\,dy = \frac{\pi}{81}\frac{y^5}{5}\Big|_0^3 = \frac{\pi\, 3^5}{81\cdot 5} = \frac{3\pi}{5}$$

Method of shells:

Volume of typical shell: $\Delta V = 2\pi x\left(3 - 3\sqrt{x}\right)\Delta x$

$$V = \int_0^1 2\pi x(3 - 3x^{1/2})\,dx = 6\pi\int_0^1 (x - x^{3/2})\,dx = 6\pi\left[\frac{x^2}{2} - \frac{2}{5}x^{5/2}\right]_0^1 = 6\pi(1/2 - 2/5) = \frac{3\pi}{5}$$

5. Let R be the region in the first quadrant bounded by the curves $y = x$, $y = \sqrt{4 - x^2}$, and $x = 0$. The volume of the solid generated by rotating R about the y–axis is given by



intersect when

$\sqrt{4-x^2} = x$

$4 - x^2 = x^2$

$x = \sqrt{2}$

A. $\pi\int_0^{\sqrt{2}} [(4 - x^2) - x^2]\,dx$

B. $2\pi\int_0^{\sqrt{2}} x(\sqrt{4 - x^2} - x)\,dx$

C. $2\pi\int_0^{\sqrt{2}} (y^2 - y)dy$

D. $2\pi\int_{\sqrt{2}}^{2} (x - \sqrt{4 - x^2})\,dx$

E. $2\pi\int_{\sqrt{2}}^{2} (y - y^2)\,dy$

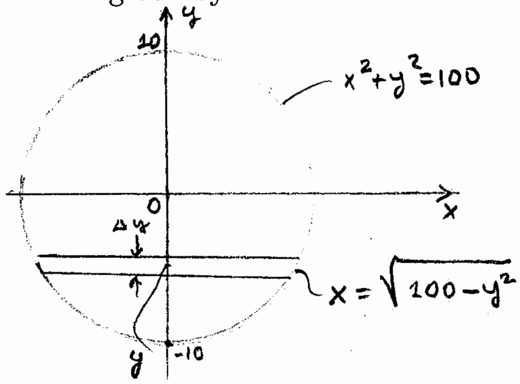
Method of shells

Volume of typical shell:

$$\Delta V = 2\pi x\left(\sqrt{4-x^2} - x\right)\Delta x$$

$$V = \int_0^{\sqrt{2}} 2\pi x\left(\sqrt{4-x^2} - x\right)dx$$

6. The bottom half of a spherical tank of radius 10 ft is filled with water. It is known that water weighs 62.5 lbs/ft^3. If the y–axis is vertically upwards with $y = 0$ at the center of the tank, the work required to pump all the water to the top of the tank is given by



A. $62.5\pi \int_{-10}^{0} (10-y)(100-y^2)\, dy$

B. $62.5\pi \int_{-10}^{0} y^2(100-y^2)\, dy$

C. $62.5 \int_{-10}^{0} y(100-y^2)\, dy$

D. $62.5\pi \int_{-10}^{0} (10-y)y^2\, dy$

E. $62.5\pi \int_{-10}^{0} (10-y)(10+y)\, dy$

Weight of typical layer of water:

$(62.5)\,\pi\left(\sqrt{100-y^2}\right)^2 \Delta y$

Work required to lift typical layer from height y to height 10:

$\Delta W = (10-y)\,62.5\pi\,(100-y^2)\,\Delta y$

$W = \int_{-10}^{0} (10-y)\,62.5\pi(100-y^2)\,dy$

7. $\int_1^2 x^2 \ln x\, dx =$

A. $4\ln 2 - 1$

B. $\frac{8\ln 2}{3}$

C. $\frac{4\ln 2}{3} + \frac{1}{3}$

D. $\frac{8\ln 2}{3} - \frac{7}{9}$

E. $\frac{3\ln 2}{4} - 1$

Integration by parts: $\int u\,dv = uv - \int v\,du$

$\int_1^2 x^2 \ln x\, dx = (\ln x)\frac{x^3}{3}\Big|_1^2 - \int_1^2 \frac{x^3}{3}\frac{1}{x}\,dx =$

$u = \ln x \quad dv = x^2 dx$

$du = \frac{1}{x}dx \quad v = \frac{x^3}{3}$

$= \left[(\ln x)\frac{x^3}{3} - \frac{x^3}{9}\right]_1^2$

$= \left[(\ln 2)\frac{8}{3} - \frac{8}{9}\right] - \left(-\frac{1}{9}\right)$

$= \frac{8}{3}\ln 2 - \frac{7}{9}$

8 $\displaystyle\int_0^{\frac{\pi}{4}} \tan^4 x \sec^4 x \, dx =$

$$= \int_0^{\frac{\pi}{4}} \tan^4 x \sec^2 x \sec^2 x \, dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^4 x \, (1+\tan^2 x) \sec^2 x \, dx =$$

$u = \tan x \quad du = \sec^2 x \, dx$

$x = 0 \longrightarrow u = 0 \quad x = \frac{\pi}{4} \rightarrow u = 1$

$$= \int_0^1 u^4 (1+u^2) \, du = \left[\frac{u^5}{5} + \frac{u^7}{7} \right]_0^1$$

$$= \frac{1}{5} + \frac{1}{7} = \frac{12}{35}$$

A. $\dfrac{7}{18}$

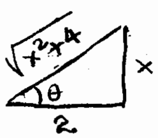
B. $\dfrac{12}{35}$

C. $\dfrac{3}{11}$

D. $\dfrac{16}{41}$

E. $\dfrac{5}{21}$

9. For the integral $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}} \, dx$, choose the right trigonometric substitution and find the resulting integral.



$x = 2\tan\theta \ , \ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$dx = 2\sec^2\theta \, d\theta$

$\sqrt{x^2+4} = 2\sec\theta$

$$\int \frac{1}{x^2\sqrt{x^2+4}} \, dx = \int \frac{1}{4\tan^2\theta \cdot 2\sec\theta} 2\sec^2\theta \, d\theta$$

$$= \frac{1}{4} \int \frac{\sec\theta}{\tan^2\theta} \, d\theta$$

$$= \frac{1}{4} \int \frac{\cos\theta}{\sin^2\theta} \, d\theta$$

A. $x = 2\tan\theta$; $\displaystyle\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta} \, d\theta$

B. $x = 2\tan\theta$; $\displaystyle\int \frac{\cos\theta}{\sin^2\theta} \, d\theta$

C. $x = 2\sec\theta$; $\displaystyle\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta} \, d\theta$

D. $x = 2\sin\theta$; $\displaystyle\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta} \, d\theta$

E. $x = \tan\theta$; $\displaystyle\int \frac{\cos\theta}{\sin^2\theta} \, d\theta$

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10. $\displaystyle\int_1^2 \frac{x^2+4}{x^3+2x} \, dx =$

$$\frac{x^2+4}{x^3+2x} = \frac{x^2+4}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$$

$x^2+4 = A(x^2+2) + (Bx+C)x$

$x^2+4 = (A+B)x^2 + Cx + 2A$

$A+B = 1 \ , \ C=0, \ 2A = 4$

$\therefore A = 2, \ B = -1, \ C = 0$

$$\int_1^2 \frac{x^2+4}{x^3+2x} \, dx = \int_1^2 \left(\frac{2}{x} - \frac{x}{x^2+2} \right) dx = \left[2\ln x - \frac{1}{2}\ln(x^2+2) \right]_1^2$$

$$= 2\ln 2 - \frac{1}{2}\ln 6 - \left(-\frac{1}{2}\ln 3\right)$$

$$= 2\ln 2 + \frac{1}{2}\ln\frac{3}{6} = 2\ln 2 - \frac{1}{2}\ln 2 = \frac{3}{2}\ln 2$$

A. $\ln 5$

B. $\tan^{-1}(\sqrt{3})$

C. $\dfrac{3}{2}\ln 2$

D. 4

E. $-3\ln 2$

11. $\displaystyle\int_{-2}^{3} \frac{dx}{x-1} = \int_{-2}^{1} \frac{1}{x-1}\,dx + \int_{1}^{3} \frac{1}{x-1}\,dx$

$$\int_{-2}^{1} \frac{1}{x-1}\,dx = \lim_{t\to 1^-} \int_{-2}^{t} \frac{1}{x-1}\,dx$$

$$= \lim_{t\to 1^-} \Big[\ln|x-1|\Big]_{-2}^{t}$$

$$= \lim_{t\to 1^-} \Big[\ln|t-1| - \ln 3\Big] = -\infty$$

$$\therefore \int_{-2}^{3} \frac{dx}{x-1} \text{ is divergent}$$

A. $\ln 2 - \ln 3$

B. $\ln 2 + \ln 3$

C. The integral is divergent

D. 0

E. ∞

12. The length of the curve $y = \frac{1}{3}(x^2+2)^{3/2}$, $0 \le x \le 1$, is

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = \frac{1}{3}\,\frac{3}{2}\,(x^2+2)^{1/2}\,2x = x(x^2+2)^{1/2}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^2(x^2+2) = x^4 + 2x^2 + 1$$

$$= (x^2+1)^2$$

$$L = \int_0^1 \sqrt{(x^2+1)^2}\,dx = \int_0^1 (x^2+1)\,dx =$$

$$= \left[\frac{x^3}{3} + x\right]_0^1 = \frac{4}{3}$$

A. $\sqrt{3}$

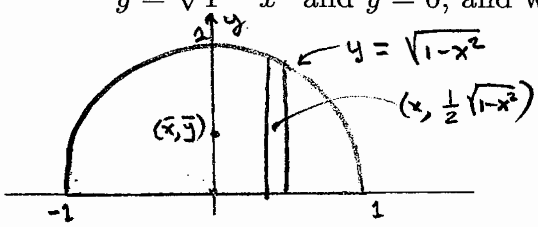
B. 4

C. ∞

D. $\tan^{-1}(3)$

E. $\frac{4}{3}$

13. Find the center of mass $(\overline{x}, \overline{y})$ of the semicircular lamina bounded by the curves $y = \sqrt{1-x^2}$ and $y = 0$, and with density $\rho = 1$.



From symmetry $\overline{x} = 0$

$$m\overline{y} = M_x$$

$$m = \rho A = 1 \cdot \tfrac{1}{2}(\pi 1^2) = \frac{\pi}{2}$$

The moment of a typical strip about the x-axis is $\Delta M_x = \frac{1}{2}\sqrt{1-x^2}\,\rho\sqrt{1-x^2}\,\Delta x$

$$= \tfrac{1}{2}(1-x^2)\,\Delta x$$

$$M_x = \int_{-1}^{1} \tfrac{1}{2}(1-x^2)\,dx = \int_0^1 (1-x^2)\,dx = \left[x - \frac{x^3}{3}\right]_0^1 = \frac{2}{3}$$

$$\frac{\pi}{2}\overline{y} = \frac{2}{3} \longrightarrow \overline{y} = \frac{4}{3\pi}$$

A. $(0, 0)$

B. $(0, \frac{4}{3\pi})$

C. $(0, \frac{1}{2})$

D. $(0, \frac{3}{4\pi})$

E. $(0, \frac{1}{3})$

14. Find the sum of the series $\sum_{n=1}^{\infty}\left[\frac{5}{2^{n-1}}+\frac{2^{n+1}}{3^{n-1}}\right]$, if it is convergent.

A. $\frac{7}{6}$

B. 5

C. 7

D. 22

E. The series is divergent

$$\sum_{n=1}^{\infty}\left[\frac{5}{2^{n-1}}+\frac{2^{n+1}}{3^{n-1}}\right]$$

$$= 5\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} + 4\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n-1}$$

$$= 5\,\frac{1}{1-\frac{1}{2}} + 4\,\frac{1}{1-\frac{2}{3}}$$

$$= 10 + 4\cdot 3 = 22$$

15. The series $\sum_{n=1}^{\infty}\frac{n}{(1+n^2)^p}$ is convergent if

A. $p<-1$

B. $p<-\frac{1}{2}$

C. $p>\frac{1}{2}$

D. $p>1$

E. $p=1$

Compare with $\sum_{n=1}^{\infty}\frac{n}{n^{2p}} = \sum_{n=1}^{\infty}\frac{1}{n^{2p-1}}$

which converges if $2p-1>1$ or $p>1$

$$\frac{n}{(1+n^2)^p} \le \frac{n}{n^{2p}} = \frac{1}{n^{2p-1}}$$

$\therefore$ By the comparison test, $\sum_{n=1}^{\infty}\frac{n}{(1+n^2)^p}$ converges if $p>1$

16. Which of the following series converge?

(I) $\sum_{n=2}^{\infty}\frac{5^{n+1}}{3^{n-1}}$ (II) $\sum_{n=1}^{\infty}\frac{n^2+2}{\sqrt{n^5+n}}$ (III) $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$

A. None

B. All

C. (II) only

D. (II) and (III) only

E. (III) only

(I) diverges because $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{5^{n+1}}{3^{n-1}} = \infty$

(II) diverges: compare with $\sum_{n=1}^{\infty}\frac{n^2}{n^{5/2}} = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ which diverges, and use limit comparison test

(III) Ratio test: $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{2^{n+1}}{(n+1)^2}\frac{n^2}{2^n} = \lim_{n\to\infty} 2\left(\frac{n}{n+1}\right)^2 = 2>1$

$\therefore$ series diverges

17. Which of the following series are absolutely convergent?

(I) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ (II) $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^3+n}$ (III) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$

A. None

B. All

C. (III) only

D. (I) only

E. (I) and (II) only

(I) $\sum_{n=1}^{\infty} \left|\frac{\sin n}{n^2}\right| = \sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$ is convergent by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^2}$ which is convergent.

$\therefore$ (I) is abs. conv.

(II) $\sum_{n=1}^{\infty}\left|(-1)^n \frac{n}{n^3+n}\right| = \sum_{n=1}^{\infty} \frac{n}{n^3+n}$ is convergent by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^2}$ wich is convergent.

$\therefore$ (II) is abs. conv.

(III) $\sum_{n=1}^{\infty}\left|(-1)^n \frac{1}{n}\right| = \sum_{n=1}^{\infty} \frac{1}{n}$ is divergent

$\therefore$ (III) is not abs. conv.

18. The interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{2^n}{5^n n} x^n$$

is

A. $\left(-\frac{5}{2}, \frac{5}{2}\right)$

B. $\left[-\frac{5}{2}, \frac{5}{2}\right)$

C. $\left(-\frac{5}{2}, \frac{5}{2}\right]$

D. $(-\infty, \infty)$

E. The series converges only for $x = 0$

Ratio test:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{2^{n+1}x^{n+1}}{5^{n+1}(n+1)} \frac{5^n n}{2^n x^n}\right|$$

$$= \lim_{n\to\infty} \frac{2}{5}\frac{n}{n+1}|x| = \frac{2}{5}|x|$$

$\therefore$ series conv. if $\frac{2}{5}|x| < 1$

or $|x| < \frac{5}{2}$

or $-\frac{5}{2} < x < \frac{5}{2}$

When $x = -\frac{5}{2}$: $\sum_{n=1}^{\infty} \frac{2^n}{5^n n}\left(-\frac{5}{2}\right)^n = \sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$ is convergent by alt. ser. test.

When $x = \frac{5}{2}$: $\sum_{n=1}^{\infty} \frac{2^n}{5^n n}\left(\frac{5}{2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n}$ is divergent

19. Use the Maclaurin series for e^{-x^2} to approximate the integral $\int_0^1 e^{-x^2}\,dx$. The smallest number of terms needed to approximate the integral with error < 0.01 is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\int_0^1 e^{-x^2}dx = \int_0^1 \left(1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \frac{(-x^2)^4}{4!} + \cdots\right)dx$$

$$= \int_0^1 \left(1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24} - \cdots\right)dx$$

$$= \left[x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{9\cdot 24} - \cdots\right]_0^1$$

$$= 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \frac{1}{216} - \cdots$$

$\frac{1}{42} > 0.01$ $\quad \frac{1}{216} < 0.01$ $\quad \therefore$ 4 is the smallest number of terms needed

A. 1
B. 2
C. 3
D. 4
E. 5

20. In the Taylor series for $f(x) = \dfrac{x}{1+x}$ about $a = 1$, the coefficient of $(x-1)^2$ is

The Taylor series of f about $a=1$ is

$$f(1) + \frac{f^{(1)}(1)}{1!}(x-1) + \frac{f^{(2)}(1)}{2!}(x-1)^2 + \cdots$$

$$f(x) = \frac{x}{1+x}$$

$$f^{(1)}(x) = \frac{1+x-x}{(1+x)^2} = (1+x)^{-2}$$

$$f^{(2)}(x) = -2(1+x)^{-3}$$

$$f^{(2)}(1) = -2(2)^{-3} = -\frac{1}{4}$$

$$\frac{f^{(2)}(1)}{2!} = -\frac{1}{8}$$

A. 1
B. -1
C. $-\frac{1}{2}$
D. $-\frac{1}{4}$
E. $-\frac{1}{8}$

21. The graph of the parametric curve $x = 3\cos t,\ y = 2\sin^2 t$ is part of $a(n)$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right) = 1$$

$$y = 2 - \frac{2}{9}x^2 \quad \text{parabola}$$

A. circle
B. ellipse
C. parabola
D. hyperbola
E. line

22. An equation of the tangent line to the parametric curve $x = t^4 + t^2 + 1$, $y = t^2 - t$ at the point corresponding to $t = -1$ is

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-1}{4t^3+2t}$$

when $t=-1$: $\frac{dy}{dx} = \frac{-2-1}{-4+2} = \frac{-3}{-6} = \frac{1}{2}$

$x = 1+1+1 = 3$

$y = 1+1 = 2$

tan. line: $y - 2 = \frac{1}{2}(x-3)$

$2y - 4 = x - 3$

$x - 2y + 1 = 0$

A. $2x + y - 8 = 0$

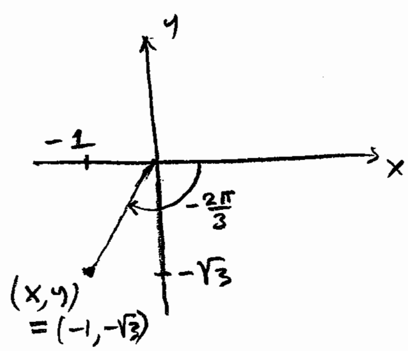
B. $2x - y - 4 = 0$

C. $x - 2y + 1 = 0$

D. $x + 2y - 7 = 0$

E. $x + y - 5 = 0$

23. A point P has Cartesian coordinates $(x, y) = (-1, -\sqrt{3})$. Polar coordinates of P are



$r^2 = 1 + 3 = 4$

$r = \pm 2$

$\tan\theta = \sqrt{3}$

$(r, \theta) = \left(2, -\frac{2\pi}{3}\right)$

or $(r, \theta) = \left(-2, \frac{\pi}{3}\right)$

A. $\left(-2, -\frac{\pi}{3}\right)$

B. $\left(-2, \frac{2\pi}{3}\right)$

C. $\left(-2, \frac{4\pi}{3}\right)$

D. $\left(2, \frac{\pi}{3}\right)$

E. $\left(2, -\frac{2\pi}{3}\right)$

24. The graph of the polar equation

$$r^2 = \frac{1}{2\cos^2\theta + 3\sin^2\theta}$$

is $a(n)$

A. ellipse

B. line

C. circle

D. parabola

E. hyperbola

$r^2(2\cos^2\theta + 3\sin^2\theta) = 1$

$2r^2\cos^2\theta + 3r^2\sin^2\theta = 1$

$2x^2 + 3y^2 = 1$ ellipse

25. The polar form of the complex number $\dfrac{\sqrt{3}-i}{\sqrt{3}+i}$ with argument between 0 and 2π is

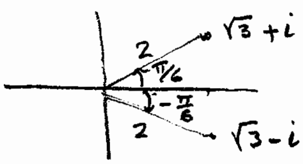
A. $\frac{1}{2}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

B. $\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$

C. $\sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

D. $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

E. $\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$



$\sqrt{3}-i = 2\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]$

$\sqrt{3}+i = 2\left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right]$

$\dfrac{\sqrt{3}-i}{\sqrt{3}+i} = \dfrac{2}{2}\left[\cos\left(-\frac{\pi}{6}-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}-\frac{\pi}{6}\right)\right]$

$= 1\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$

$= \cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)$

Or $\dfrac{\sqrt{3}-i}{\sqrt{3}+i} = \dfrac{(\sqrt{3}-i)(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)} = \dfrac{3-i\sqrt{3}-i\sqrt{3}-1}{3+1} = \dfrac{2-i2\sqrt{3}}{4}$

$= \frac{1}{2}\left(1-i\sqrt{3}\right) = \frac{1}{2}\cdot 2\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$

$= \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$