NAME SOLUTIONS

10-DIGIT PUID

REC. INSTR. REC. TIME

LECTURER

INSTRUCTIONS:

1. There are 14 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–14.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes, calculators or any electronic devices may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 1, write 3801).
   (c) On the bottom, under TEST/QUIZ NUMBER, write 01 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10–digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ |x| < \infty \qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1},\ |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n},\ |x| < \infty \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ |x| < 1$$

1. Find a vector that has the direction opposite of $2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ but has length $\sqrt{5}$.

Let $\vec{a} = 2\vec{i} - 4\vec{j} + 5\vec{k}$
and let $\vec{b}$ denote the desired vector.
$|\vec{a}| = \sqrt{4+16+25} = \sqrt{45} = 3\sqrt{5}$
Unit vector in the direction of $\vec{a}$:
$\vec{u} = \frac{1}{3\sqrt{5}}(2\vec{i} - 4\vec{j} + 5\vec{k})$
Unit vector in the direction opposite of $\vec{a}$:
$-\vec{u} = -\frac{1}{3\sqrt{5}}(2\vec{i} - 4\vec{j} + 5\vec{k})$
$\vec{b} = \sqrt{5}(-\vec{u}) = -\frac{2}{3}\vec{i} + \frac{4}{3}\vec{j} - \frac{5}{3}\vec{k}$

A. $-\frac{2}{\sqrt{5}}\mathbf{i} + \frac{4}{\sqrt{5}}\mathbf{j} - \frac{5}{\sqrt{5}}\mathbf{k}$

B. $\frac{2}{\sqrt{5}}\mathbf{i} - \frac{4}{\sqrt{5}}\mathbf{j} + \frac{5}{\sqrt{5}}\mathbf{k}$

C. $\frac{2\sqrt{5}}{\sqrt{11}}\mathbf{i} - \frac{4\sqrt{5}}{\sqrt{11}}\mathbf{j} + \frac{5\sqrt{5}}{\sqrt{11}}\mathbf{k}$

D. $-\frac{2\sqrt{5}}{\sqrt{11}}\mathbf{i} + \frac{4\sqrt{5}}{\sqrt{11}}\mathbf{j} - \frac{5\sqrt{5}}{\sqrt{11}}\mathbf{k}$

E. $-\frac{2}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{5}{3}\mathbf{k}$

2 For what values of b are the vectors $< -2, b, 1 >$ and $< 3, b, -b >$ orthogonal?

$\langle -2, b, 1\rangle \cdot \langle 3, b, -b\rangle = 0$
$-6 + b^2 - b = 0$
$b^2 - b - 6 = 0$
$(b-3)(b+2) = 0$
$b = -2, 3$

A. $b = 1, 3$

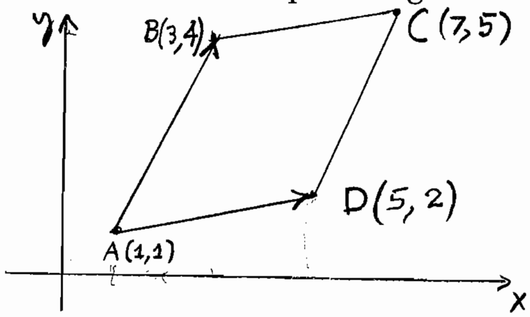
B. $b = -3, 2$

C. $b = -2, 3$

D. $b = 1, 2$

E. $b = 2, 3$

3. Find the area of the parallelogram with vertices $A(1,1)$, $B(3,4)$, $C(7,5)$ and $D(5,2)$.



A. 11

B. 13

C. 10

D. 12

E. 9

$\vec{AB} = 2\vec{i} + 3\vec{j} \qquad \vec{AD} = 4\vec{i} + \vec{j}$

$$\vec{AB} \times \vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix} = -10\vec{k}$$

$$|\vec{AB} \times \vec{AD}| = \sqrt{100} = 10$$

4. Find the area of the region bounded by $y = x^2$ and $y = 2x - x^2$.

A. $\frac{11}{3}$

B. $\frac{10}{3}$

C. $\frac{16}{3}$

D. $\frac{1}{3}$

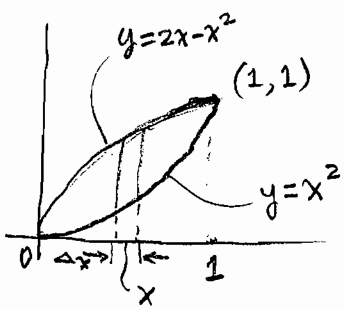
E. $\frac{4}{3}$

Points of intersection:

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$x(x-1) = 0 \longrightarrow x = 0, 1$



Area of typical approximating rectangle

$$\Delta A = [(2x - x^2) - x^2]\Delta x$$

$$A = \int_0^1 (2x - x^2 - x^2)\,dx = \left[x^2 - 2\frac{x^3}{3}\right]_0^1$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

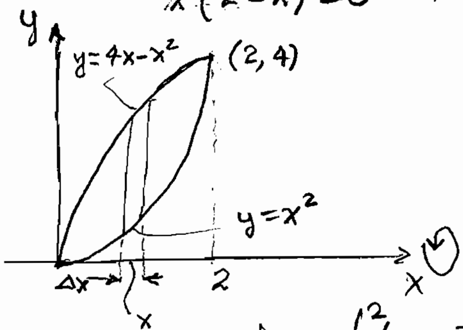
5. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 4x - x^2$ about the x–axis.

Points of intersection:

$4x - x^2 = x^2$

$4x - 2x^2 = 0$

$x(2-x) = 0 \rightarrow x = 0, 2$



Volume of typical approximating washer

$\Delta V = \left[\pi(4x-x^2)^2 - \pi(x^2)^2\right]\Delta x$

$V = \pi\int_0^2 (16x^2 - 2\cdot 4x x^2 + x^4 - x^4)\,dx$

$= \pi\int_0^2 (16x^2 - 8x^3)\,dx = \pi\left[16\frac{x^3}{3} - 8\frac{x^4}{4}\right]_0^2$

$= \pi\left[16\cdot\frac{8}{3} - 8\cdot\frac{16}{4}\right] = \pi\, 16\left[\frac{8}{3} - 2\right] =$

$= \pi(16)\frac{2}{3} = \frac{32\pi}{3}$

A. $\frac{11}{3}\pi$

B. $\frac{10}{3}\pi$

C. $\frac{16}{3}\pi$

D. $\frac{8}{3}\pi$

E. $\frac{32}{3}\pi$

6. It took 2700 J of work to stretch a spring from its natural length of $2m$ to a length of $5m$. Find the spring's force constant.

$W = \int_0^3 kx\,dx$

$2700 = k\frac{x^2}{2}\Big|_0^3$

$2700 = k\frac{9}{2}$

$300 = \frac{k}{2}$

$k = 600$

A. 150

B. $\frac{800}{3}$

C. 400

D. $\frac{1400}{3}$

E. 600

7. Evaluate $\int_1^e \frac{\ln x}{x^2}\, dx.$ $=$

$u = \ln x \quad dv = x^{-2}dx$

$du = \frac{1}{x}dx \quad v = -x^{-1} = -\frac{1}{x}$

$= -\frac{1}{x}\ln x\Big|_1^e + \int_1^e \frac{1}{x}\frac{1}{x}dx$

$= -\frac{1}{e} + \left[-\frac{1}{x}\right]_1^e$

$= -\frac{1}{e} - \frac{1}{e} + 1 = \frac{e-2}{e}$

A. $\frac{1-e^3}{2e^3}$

B. $\frac{3-3e^4}{e^4}$

C. $\frac{e-2}{e}$

D. $\frac{e^2-2}{e^2}$

E. $\frac{4-3e^3}{e^3}$

8. Evaluate $\int_{\pi/2}^{3\pi/4} \frac{\cos^3\theta}{\sin\theta}\, d\theta.$ $= \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \frac{1-\sin^2\theta}{\sin\theta}\cos\theta\, d\theta =$

$u = \sin\theta \quad du = \cos\theta\, d\theta$

$\theta = \frac{\pi}{2} \to u = 1$

$\theta = \frac{3\pi}{4} \to u = \frac{1}{\sqrt{2}}$

$= \int_1^{\frac{1}{\sqrt{2}}} \frac{1-u^2}{u}du$

$= \int_1^{\frac{1}{\sqrt{2}}} \left(\frac{1}{u} - u\right)du$

$= \left[\ln|u| - \frac{u^2}{2}\right]_1^{\frac{1}{\sqrt{2}}} = \ln\frac{1}{\sqrt{2}} - \frac{\frac{1}{2}}{2} - \left(0 - \frac{1}{2}\right)$

$= \ln 2^{-1/2} - \frac{1}{4} + \frac{1}{2}$

$= -\frac{1}{2}\ln 2 + \frac{1}{4}$

A. $\frac{3}{4} - \frac{1}{2}\ln 2$

B. $\frac{1}{2}\ln 2 - \frac{3}{4}$

C. -1

D. $\frac{1}{4} - \frac{1}{2}\ln 2$

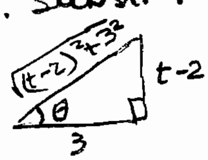
E. $\frac{1}{2}\ln 2 - \frac{1}{4}$

9. A trigonometric substitution can be used to convert the definite integral $\displaystyle\int_2^5 \frac{dt}{\sqrt{t^2-4t+13}}$ into which of the following definite integrals?

$$t^2-4t+13 = t^2-4t+4+9 = (t-2)^2+3^2$$

$$\int_2^5 \frac{dt}{\sqrt{t^2-4t+13}} = \int_2^5 \frac{dt}{\sqrt{(t-2)^2+3^2}} =$$

trig. subst. : $t-2 = 3\tan\theta$; $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$



$dt = 3\sec^2\theta\, d\theta$

$\sqrt{(t-2)^2+3^2} = 3\sec\theta$

$t=2 \longrightarrow \theta=0$

$t=5 \longrightarrow \tan\theta = 1 \rightarrow \theta=\frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} \frac{3\sec^2\theta}{3\sec\theta}\, d\theta = \int_0^{\frac{\pi}{4}} \sec\theta\, d\theta$$

A. $\displaystyle\int_0^{\pi/4} \sec\theta\, d\theta$

B. $\displaystyle\int_0^{\pi/4} \cos\theta\, d\theta$

C. $\displaystyle\int_0^{\pi/3} \sin\theta\, d\theta$

D. $\displaystyle\int_0^{\pi/3} \cos\theta\, d\theta$

E. $\displaystyle\int_0^{\pi/3} \sec\theta\, d\theta$

10. Compute $\displaystyle\int_0^2 \frac{8x-4}{x^2-2x-3}\, dx$.

$$x^2-2x-3 = (x-3)(x+1)$$

$$\frac{8x-4}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$8x-4 = Ax + A + Bx - 3B$$

$A+B=8$, $A-3B=-4$ $\rightarrow$ $4B=12$, $B=3$, $A=5$

$$\int_0^2 \frac{8x-4}{x^2-2x-3}\, dx = \int_0^2 \left(\frac{5}{x-3}+\frac{3}{x+1}\right) dx$$

$$= \Big[5\ln|x-3| + 3\ln|x+1|\Big]_0^2 = 5\ln 1 + 3\ln 3 - 5\ln 3 - 3\ln 1$$

$$= -2\ln 3$$

A. $-8\ln 3$

B. $-5\ln 3$

C. $-3\ln 3$

D. $-2\ln 3$

E. $-\ln 3$

11. Find whether the series $\sum_{n=1}^{\infty} 3\left(-\frac{1}{2}\right)^n$ converges or diverges, and find its sum if it converges.

$$\sum_{n=1}^{\infty} 3\left(-\tfrac{1}{2}\right)^n = 3\left(-\tfrac{1}{2}\right)\underbrace{\sum_{n=1}^{\infty}\left(-\tfrac{1}{2}\right)^{n-1}}_{\text{geometric series with } r=-\frac{1}{2} \text{ conv. because } |-\frac{1}{2}|<1} =$$

$$= 3\left(-\tfrac{1}{2}\right)\frac{1}{1-\left(-\frac{1}{2}\right)}$$

$$= 3\left(-\tfrac{1}{2}\right)\frac{1}{\frac{3}{2}}$$

$$= -1$$

A. Diverges.

B. Converges and sum = 0

C. Converges and sum = 2

D. Converges and sum = −1

E. Converges and sum = −3

12. Which of these improper integrals converge?

I. $\int_0^3 \frac{1}{x-2}\,dx.$

II. $\int_0^3 \frac{1}{\sqrt{3-x}}\,dx.$

III. $\int_3^{\infty} \frac{1}{\sqrt{x}}\,dx.$

(I) $\int_0^3 \frac{1}{x-2}dx = \int_0^2 \frac{1}{x-2}dx + \int_2^3 \frac{1}{x-2}dx$

$\int_0^2 \frac{1}{x-2}dx = \lim_{t\to 2^-}\int_0^t \frac{1}{x-2}dx$

$= \lim_{t\to 2^-}\left[\ln|x-2|\right]_0^t = \lim_{t\to 2^-}\left[\ln|t-2| - \ln 2\right]$

$= -\infty \to$ div.

A. All

B. Only (I)

C. Only (II)

D. Only (III)

E. (I) and (II)

(II) $\int_0^3 \frac{1}{\sqrt{3-x}}dx = \lim_{t\to 3^-}\int_0^t \frac{1}{\sqrt{3-x}}dx = \lim_{t\to 3^-}\left[-2\sqrt{3-x}\right]_0^t$

$= \lim_{t\to 3^-}\left[-2\sqrt{3-t} + 2\sqrt{3}\right] = 2\sqrt{3}$

∴ conv.

(III) $\int_3^{\infty}\frac{1}{\sqrt{x}}dx = \lim_{t\to\infty}\int_3^t \frac{1}{\sqrt{x}}dx$

$= \lim_{t\to\infty}\left[2\sqrt{x}\right]_3^t = \lim_{t\to\infty}\left[2\sqrt{t} - 2\sqrt{3}\right] = \infty$

∴ div.

13. The curve $y = x^2 + 1$, $0 \le x \le 2$, is rotated about the x–axis. The area of the surface is given by

$S = \int 2\pi y \, ds$

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

$= \sqrt{1 + (2x)^2} \, dx$

$S = \int_0^2 2\pi (x^2+1)\sqrt{1+4x^2} \, dx$

A. $\int_0^2 2\pi(x^2+1)\sqrt{1+4x^2}\,dx$

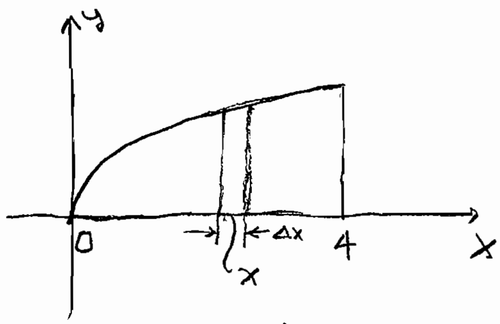
B. $\int_0^2 2\pi x\sqrt{1+4x^2}\,dx$

C. $\int_0^2 \pi(x^2+1)^2\,dx$

D. $\int_0^2 2\pi x(x^2+1)\,dx$

E. $\int_0^2 (x^2+1)\,dx$

14. Consider the lamina bounded by the graph of $y = \sqrt{x}$, the x–axis and the line $x = 4$, with density $\rho = 1$. The x–coordinate $\overline{x}$ of the center of mass of the lamina is



A. 1

B. 2

C. 3

D. $\frac{5}{2}$

E. $\frac{12}{5}$

$m\overline{x} = M_y$

$m = \int_0^4 \sqrt{x}\,dx = \left.\frac{x^{3/2}}{\frac{3}{2}}\right|_0^4 = \frac{2}{3}\cdot 8 = \frac{16}{3}$

$M_y = \int_0^4 x\sqrt{x}\,dx = \left.\frac{x^{5/2}}{\frac{5}{2}}\right|_0^4 = \frac{2}{5}\cdot 32 = \frac{64}{5}$

$\frac{16}{3}\overline{x} = \frac{64}{5} \qquad \overline{x} = \frac{3}{5}\cdot\frac{64}{16} = \frac{12}{5}$

15. Let $a = \lim_{n\to\infty} ne^{-n}$ and $b = \lim_{n\to\infty} \frac{n!}{(2n)!}$. Then

$$a = \lim_{n\to\infty} \frac{n}{e^n} = \lim_{x\to\infty} \frac{x}{e^x} \overset{L'H}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{1}{e^x} = 0$$

$$b = \lim_{n\to\infty} \frac{n!}{(2n)!} = \lim_{n\to\infty} \frac{1\cdot 2\cdots n}{1\cdot 2\cdots n\,(n+1)\cdots(2n)}$$

$$= \lim_{n\to\infty} \frac{1}{(n+1)(n+2)\cdots(2n)} = 0$$

A. $a = 1$ and $b = \frac{1}{2}$
B. $a = 0$ and $b = \frac{1}{2}$
C. $a = 1$ and $b = 0$
D. $a = 0$ and $b = 0$
E. $a = e^{-1}$ and $b = 0$

16. The series $\sum_{n=0}^{\infty} (-1)^n \frac{\tan^{-1} n}{1+n^2}$ is

A. absolutely convergent
B. conditionally convergent
C. divergent since $\lim_{n\to\infty} (-1)^n \frac{\tan^{-1} n}{1+n^2} \neq 0$
D. divergent even though $\lim_{n\to\infty} (-1)^n \frac{\tan^{-1} n}{1+n^2} = 0$
E. divergent by the ratio test

abs. conv. ? : $\sum_{n=0}^{\infty} \left| (-1)^n \frac{\tan^{-1} n}{1+n^2} \right| = \sum_{n=0}^{\infty} \frac{|\tan^{-1} n|}{1+n^2}$ conv. ?

Yes: compare with $\sum_{n=1}^{\infty} \frac{\frac{\pi}{2}}{n^2}$ which is conv. (p-series $p=2>1$)

and $\frac{|\tan^{-1} n|}{1+n^2} \le \frac{\frac{\pi}{2}}{n^2}$ for all $n \ge 1$

17. Which of the following series converge?

(I) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n^2} = \sum_{n=2}^{\infty} \frac{1}{n^{3/2}}$ conv. p-series $p=\frac{3}{2}>1$

(II) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ $\quad \frac{n^n}{n!} = \frac{n\cdot n\cdot n\cdots\cdot n}{1\cdot 2\cdot 3\cdots\cdot n} \geq 1$ for all $n\geq 1$.

$\therefore \lim_{n\to\infty} \frac{n^n}{n!} \neq 0 \quad \therefore$ div. by the test for divergence

(III) $\sum_{n=1}^{\infty} \frac{1}{5^n-2}$ $\quad$ Compare with $\sum_{n=1}^{\infty} \frac{1}{5^n}$ which conv (geom. ser. $r=\frac{1}{5}, |\frac{1}{5}|<1$)

Limit comp. test ; $\lim_{n\to\infty} \frac{\frac{1}{5^n-2}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{5^n}{5^n-2} = 1 \quad \therefore$ conv

(IV) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[4]{n}}$ $\quad$ Alternating series test:

$b_n = \frac{1}{\sqrt[4]{n}}$

b_n decreasing

$\lim_{n\to\infty} b_n = 0$

$\therefore$ conv

A. (III) only

B. (III) and (IV) only

C. All

D. (I), (III) and (IV) only

E. (II), (III) and (IV) only

18. Use a Maclaurin series and the Estimation Theorem for alternating series to approximate $\sin\left(\frac{1}{2}\right)$ using the fewest number of terms necessary so that the error is less than 0.001.

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

$\sin\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{1}{8\cdot 6} + \frac{1}{32\cdot 120} - \cdots$

$< \frac{1}{1000}$

$\frac{1}{2} - \frac{1}{48} = \frac{24-1}{48}$

$= \frac{23}{48}$

A. $\frac{1}{2}$

B. $\frac{23}{48}$

C. $\frac{3}{4}$

D. $\frac{33}{40}$

E. $\frac{41}{60}$

19. Consider the power series $\sum_{n=1}^{\infty} \frac{2^n}{n} x^n$. The radius of convergence R and the interval of convergence of this series are

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{2^{n+1} x^{n+1}}{(n+1)} \frac{n}{2^n x^n} \right|$$

$$= 2 \frac{n}{n+1} |x| \longrightarrow 2|x| \text{ as } n \to \infty$$

$\therefore$ series conv. if $2|x| < 1$

or $|x| < \frac{1}{2}$, or $-\frac{1}{2} < x < \frac{1}{2}$

$R = \frac{1}{2}$

When $x = -\frac{1}{2}$: $\sum_{n=1}^{\infty} \frac{2^n}{n} \frac{(-1)^n}{2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ conv. by alt. ser. test.

When $x = \frac{1}{2}$: $\sum_{n=1}^{\infty} \frac{2^n}{n} \frac{1}{2^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ div. p-series $p=1$.

Interval of convergence $[-\frac{1}{2}, \frac{1}{2})$

A. $R = \frac{1}{2}$, $\left(-\frac{1}{2}, \frac{1}{2}\right)$

B. $R = \frac{1}{2}$, $\left[-\frac{1}{2}, \frac{1}{2}\right)$

C. $R = \frac{1}{2}$, $\left[-\frac{1}{2}, \frac{1}{2}\right]$

D. $R = 1$, $(-1, 1)$

E. $R = 1$, $[-1, 1)$

20. The interval of convergence for the series $\sum_{n=1}^{\infty} \frac{10^n}{n!} (x-1)^n$ is

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{10^{n+1} (x-1)^{n+1}}{(n+1)!} \frac{n!}{10^n (x-1)^n} \right|$$

$$= 10 \frac{1}{n+1} |x-1| \longrightarrow 0 \text{ as } n \to \infty$$

$\therefore$ series conv for all x.

A. $(0, 2)$

B. $[0, 2)$

C. $(9, 11)$

D. $[9, 11)$

E. $(-\infty, \infty)$

21. In the Taylor series expansion for $f(x) = \dfrac{x-1}{x-2}$ about $a = 1$, the coefficient of $(x-1)^{10}$ is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!}(x-1)^n$$

$$f(x) = \frac{x-1}{x-2}$$

$$f^{(1)}(x) = \frac{(x-2)-(x-1)}{(x-2)^2} = -(x-2)^{-2} \qquad f^{(1)}(1) = -1$$

$$f^{(2)}(x) = 2(x-2)^{-3} \qquad f^{(2)}(1) = -2$$

$$f^{(3)}(x) = -2\cdot 3(x-2)^{-4} \qquad f^{(3)}(1) = -2\cdot 3 = -3!$$

$$\ldots\ldots$$

$$f^{(10)}(1) = -10!$$

$$\frac{f^{(10)}(1)}{10!} = \frac{-10!}{10!} = -1$$

A. -2

B. -1

C. 0

D. 1

E. 2

22. Evaluate $\displaystyle\lim_{x\to 0} \frac{\cos(x^2) - 1 + \frac{x^4}{2}}{x^8}$.

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots$$

$$\lim_{x\to 0} \frac{\cos(x^2) - 1 - \frac{x^4}{2}}{x^8}$$

$$= \lim_{x\to 0} \frac{1 - \frac{x^4}{2} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots - 1 + \frac{x^4}{2}}{x^8}$$

$$= \lim_{x\to 0} \frac{\frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots}{x^8} = \frac{1}{4!} = \frac{1}{24}$$

A. $\frac{1}{2}$

B. $\frac{1}{8}$

C. $\frac{1}{6}$

D. $\frac{1}{120}$

E. $\frac{1}{24}$

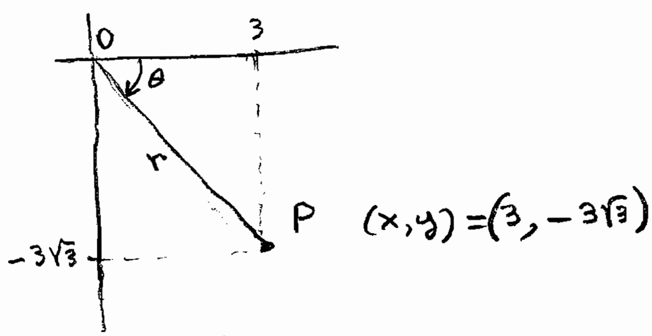
23. Find the slope of the tangent line to the curve described by $x = \ln\ t,\ y = 1 + t^2$ at $t = 1$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{\frac{1}{t}} = 2t^2$$

When $t = 1$: $\frac{dy}{dx} = 2$

A. 0
B. 3
C. -1
D. 2
E. $\frac{1}{3}$

24. A point P has Cartesian coordinates $(x, y) = (3, -3\sqrt{3})$. Which of the following gives polar coordinates of P?



A. $\left(-6, \frac{\pi}{3}\right)$
B. $\left(6, -\frac{\pi}{3}\right)$
C. $\left(6, \frac{\pi}{6}\right)$
D. $\left(-6, \frac{\pi}{6}\right)$
E. $\left(6, -\frac{\pi}{6}\right)$

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-3\sqrt{3})^2}$$

$$= \sqrt{9 + 27} = \sqrt{36} = 6$$

$$\tan\theta = \frac{y}{x} = \frac{-3\sqrt{3}}{3} = -\sqrt{3}$$

$$\theta = -\frac{\pi}{3}$$

25. Identify the curve $r^2 = r\tan(\theta)\sec(\theta)$ by finding the Cartesian equation for it.

A. ellipse

B. line

C. circle

D. parabola

E. hyperbola

$x = r\cos\theta \qquad y = r\sin\theta$

$r^2 = r\tan\theta\sec\theta$

$r^2 = r\dfrac{\sin\theta}{\cos\theta}\dfrac{1}{\cos\theta}$

$(r\cos\theta)^2 = r\sin\theta$

$x^2 = y$

parabola