Fall 1999.

MA 261 **EXAM 1** Name Solution

1. Find the equation of the plane containing $(0,1,2)$ and whose normal is perpendicular to both $\bar{a}=\bar{i}+\bar{j}$, $\bar{b}=\bar{j}-\bar{k}$.

A. $x+y+z=3$
B. $-x+y+z=3$
C. $x-y-z=3$
D. $x+y+z=-3$
E. None of the above

$\vec{N}=\vec{a}\times\vec{b}=-\vec{i}+\vec{j}+\vec{k}=(-1,1,1)$

Equation of the plane with the normal $\vec{N}$ containing $(0,1,2)$:

$(-1)\cdot x+1\cdot(y-1)+1\cdot(z-2)=0$

$-x+y+z=3$

2. The distance between the plane
$2x+y+2z=4$
and the point $(1,7,2)$ is

A. 1
B. 2
C. 3
D. 4
E. None of the above

Let $P=(1,7,2)$.

Choose a point P_0 in the plane, e.g., $P_0=(1,0,1)$.

$\vec{N}=(2,1,2)$ is a normal vector to the plane

The distance $D=\|pr_N \overrightarrow{P_0P}\|=\dfrac{|\vec{N}\cdot\overrightarrow{P_0P}|}{\|\vec{N}\|}=\dfrac{9}{3}=3$

3. A unit tangent vector to the graph of $y = 2x^3$ at $(1, 2)$ is given by

Parametric equations:
$$\begin{cases} x = t \\ y = 2t^3 \end{cases}$$
The point $(1,2)$ corresponds to $t=1$.
Tangent vector $\vec{T} = (x', y') = (1, 6t^2)$
At $t=1$, $\vec{T} = (1, 6)$, $\|\vec{T}\| = \sqrt{37}$
Unit tangent vector $\vec{T}/\|\vec{T}\|$
equals $\dfrac{\vec{i} + 6\vec{j}}{\sqrt{37}}$

A. $\dfrac{\bar{i} + 6\bar{j}}{\sqrt{37}}$

B. $\dfrac{\bar{i} + 4\bar{j}}{\sqrt{17}}$

C. $\dfrac{\bar{i} - \bar{j}}{\sqrt{2}}$

D. $\dfrac{2\bar{i} + 3\bar{j}}{\sqrt{13}}$

E. $\dfrac{\bar{i} + 2\bar{j}}{\sqrt{5}}$

4. A particle is moving with acceleration $4\bar{j} + 6t\bar{k}$. If the position at time $t = 1$ is $\bar{r}(1) = \bar{i} + 3\bar{j} + \bar{k}$ and the velocity at time $t = 0$ is $\bar{v}(0) = \bar{i} + \bar{j}$, then the position at time $t = 2$ is

$\vec{a}(t) = 4\vec{j} + 6t\vec{k}$
$\vec{v}(t) = \int \vec{a}(t)\,dt + \vec{C} =$
$\quad 4t\vec{j} + 3t^2\vec{k} + \vec{C}$
Since $\vec{v}(0) = \vec{i} + \vec{j}$, $\vec{C} = \vec{i} + \vec{j}$, $\vec{v}(t) = \vec{i} + (4t+1)\vec{j} + 3t^2\vec{k}$
$\vec{r}(t) = \int \vec{v}(t)\,dt + \vec{C}_1 = t\vec{i} + (2t^2+t)\vec{j} + t^3\vec{k} + \vec{C}_1$
Since $\vec{r}(1) = \vec{i} + 3\vec{j} + \vec{k}$, $\vec{C}_1 = 0$.
Hence $\vec{r}(2) = 2\vec{i} + 10\vec{j} + 8\vec{k}$

A. $4\bar{i} + 10\bar{j} + 10\bar{k}$

B. $\bar{i} + 4\bar{j} + 10\bar{k}$

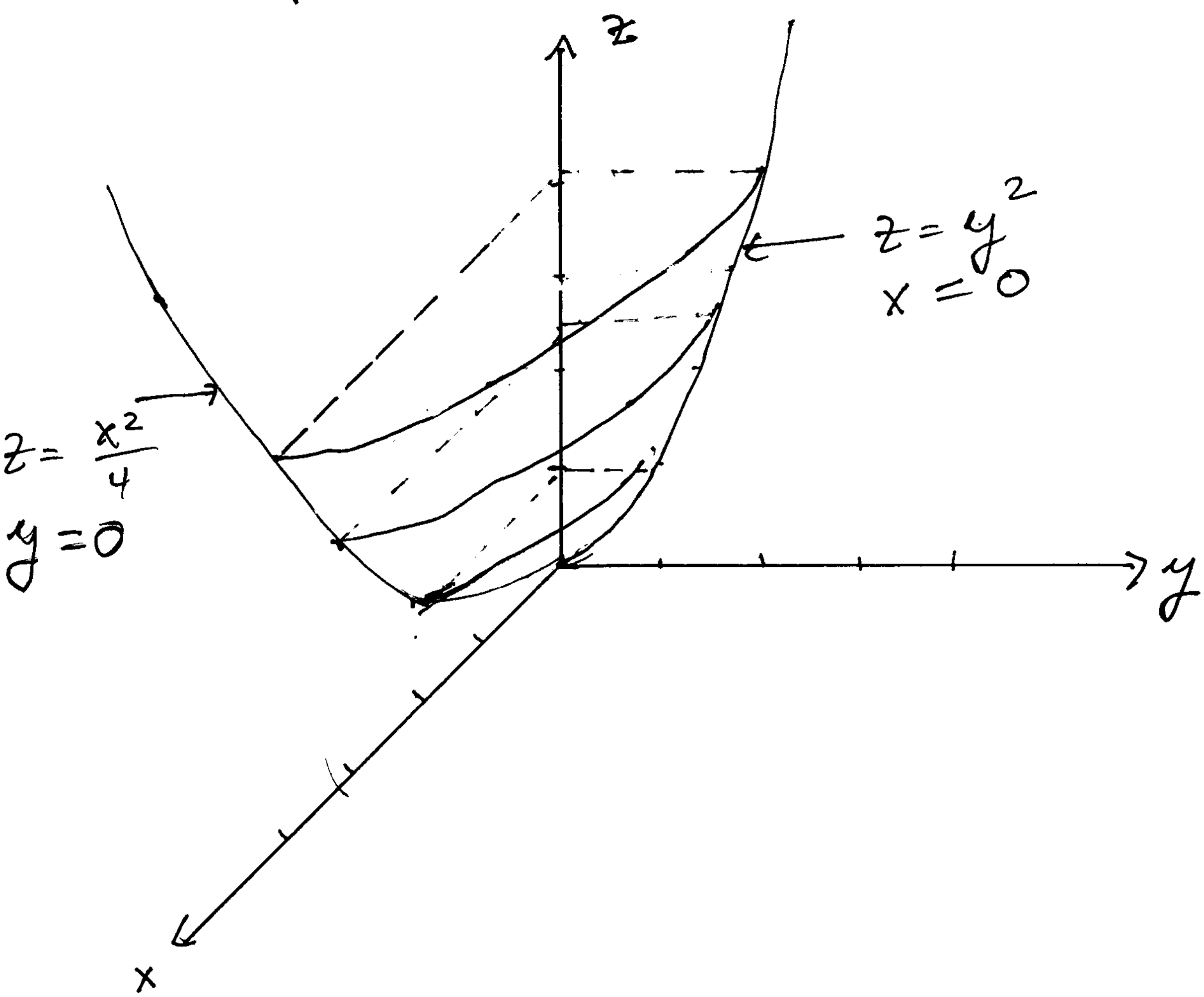
C. $\bar{i} + \frac{8}{3}\bar{j} + 4\bar{k}$

D. $2\bar{i} + 10\bar{j} + 8\bar{k}$

E. $2\bar{i} + 8\bar{j} + 8\bar{k}$

5. Which of the following surfaces represents the graph of $z = \frac{x^2}{4} + y^2$ in the 1st octant.

Elliptic Paraboloid.



6. If $f(x,y) = \dfrac{3x^2+yx}{x^2+y^2}$, $(x,y) \neq (0,0)$, let ℓ be the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the y-axis, and let m be the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the line $y = x$. Then

$$\ell = \lim_{y\to 0} f(0,y) = \lim_{y \to 0} \frac{0}{y^2} = 0$$

$$m = \lim_{x \to 0} f(x,x) = \lim_{x\to 0} \frac{4x^2}{2x^2} = 2$$

A. $\ell = 3, \quad m = 2$

B. $\ell = 0, \quad m = 2$

C. $\ell = 0, \quad m = \frac{3}{2}$

D. $\ell = 3, \quad m = 3$

E. $\ell = \frac{1}{2}, \quad m = \frac{1}{2}$

7. Find a value of a for which the function $z = 4\cos(x + ay)$ satisfies
$\dfrac{\partial^2 z}{\partial y^2} = 9\,\dfrac{\partial^2 z}{\partial x^2}$.

$$\frac{\partial z}{\partial x} = -4\sin(x+ay)$$

$$\frac{\partial^2 z}{\partial x^2} = -4\cos(x+ay)$$

$$\frac{\partial z}{\partial y} = -4a\sin(x+ay)$$

$$\frac{\partial^2 z}{\partial y^2} = -4a^2\cos(x+ay)$$

$$a^2 = 9, \quad a = \pm 3$$

A. $a = 2$

B. $a = 0$

C. $a = \frac{1}{2}$

D. $a = 1$

E. $a = 3$

8. Find the maximal directional derivative of
$f(x, y, z) = e^x + e^y + e^{2z}$
at $(1, 1, -1)$.

$\|\nabla f\| = \|e^x \vec{i} + e^y \vec{j} + 2e^{2z} \vec{k}\| =$

$\sqrt{e^{2x} + e^{2y} + 4e^{4z}}$

At $(1, 1, -1)$, $\|\nabla f\| = \sqrt{e^2 + e^2 + 4e^{-4}}$

A. $e\sqrt{3 - 2e}$
B. $\sqrt{2e^2 + 4e^{-4}}$
C. $\frac{1}{e}\sqrt{2 - 4e^{-3}}$
D. $\sqrt{2e^2 + e^{-4}}$
E. $\sqrt{e^2 + 2e^{-4}}$

9. Find symmetric equations of the line containing $(1, 2, 3)$ and perpendicular to the plane $2x + 3y - z = 8$.

Normal vector to the plane $\vec{N} = (2, 3, -1)$
Symmetric equations of the line containing $(1, 2, 3)$ and parallel to $(2, 3, -1)$ are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{-1}$$

$$\boxed{\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{-1}}$$

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10. Find the length of the curve
$\bar{r}(t) = \frac{t^2}{2}\,\bar{\imath} + 7\bar{\jmath} + \frac{t^3}{3}\,\bar{k}$, $0 \le t \le 2$.

$\vec{r}\,'(t) = t\,\vec{i} + t^2\,\vec{k}$

$\| \vec{r}\,'(t) \| = \sqrt{t^2 + t^4}$

$L = \int_0^2 \| \vec{r}\,'(t) \| \, dt = \int_0^2 t\sqrt{1+t^2}\, dt = \frac{1}{2}\int_1^5 \sqrt{u}\, du = \frac{1}{3} u^{3/2} \Big|_1^5$

$1+t^2 = u,\ 2t\,dt = du$

$\boxed{\frac{1}{3}\left(5^{3/2} - 1\right)}$

11. (a) Complete the following definition of f_y at $(0,0)$:

$$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$$

(b) If $f(x,y) = \begin{cases} \frac{x+y^3}{3x^2+4y^2}, & (x,y) \ne (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$, compute $f_y(0,0)$ by evaluating the above limit.

$$f_y(0,0) = \lim_{h \to 0} \frac{h^3/4h^2}{h} = \frac{1}{4}$$

$\boxed{f_y(0,0) = 1/4}$

12. A right circular cylinder has a radius and altitude that vary with time. At a certain instant the altitude is increasing at 0.5 ft/sec and the radius is decreasing at 0.2 ft/sec. How fast is the volume changing if at this time the radius is 20 feet and the altitude is 60 feet.

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r \cdot r' \cdot h + \pi r^2 h' =$$

$$2\pi \cdot 20 \cdot (-0.2) \cdot 60 + \pi \cdot 20^2 \cdot 0.5 =$$

$$\pi(-480 + 200) = -280\pi \ ft^3/sec$$

$$\boxed{\frac{dV}{dt} = -280\pi \ ft^3/sec}$$