1. Determine a so that the line

$$\frac{x-3}{a} = \frac{y+5}{2} = \frac{z+1}{4}$$

is parallel to the plane $2x + 3y - 5z = 14$.

Direction vector $\vec{u} = \langle a, 2, 4 \rangle$
Normal to the plane $\vec{n} = \langle 2, 3, -5 \rangle$
The line and plane are parallel
when $\vec{u} \cdot \vec{n} = 2a + 6 - 20 = 0$.

A. $a = -4$
B. $a = 3$
C. $a = -14$
D. $a = -5$
E. $a = 7$

2. The line through $(3, 2, 1)$ and $(5, 1, 2)$ intersects the plane $x + y + z = 14$ at the point

Direction vector: $\vec{u} = \langle 5, 1, 2 \rangle - \langle 3, 2, 1 \rangle$
$= \langle 2, -1, 1 \rangle$
Parametric equations:
$x = 3 + 2t, \quad y = 2 - t, \quad z = 1 + t$
Intersection point:
$x + y + z = (3 + 2t) + (2 - t) + (1 + t) = 14, \quad t = 4$
$x = 11, \quad y = -2, \quad z = 5$

A. $(11, -2, 5)$
B. $(8, 4, 2)$
C. $(15, 0, -1)$
D. $(10, -1, 5)$
E. $(9, -3, 8)$

3. The vector-valued function $\vec{r}(t) = \vec{i} + (t\cos t)\vec{j} + (t\sin t)\vec{k}$, $0 \le t < \infty$ describes a

$x = 1$, $y = t\cos t$, $z = t\sin t$

The curve belongs to the vertical plane $x = 1$.

In polar coords

$(y, z) = (r\cos\theta, r\sin\theta)$

the curve is a spiral $r = \theta$

A. circle in a horizontal plane

B. circle in a vertical plane

C. spiral in a horizontal plane

D. spiral in a vertical plane

E. ellipse in a vertical plane

4. In spherical coordinates the two equations $\rho = 2$, $\phi = \pi/6$ describe

In cylindrical coords

$z = \rho\cos\phi = \sqrt{3}$

$r = \rho\sin\phi = 1$

This is a unit circle in the horizontal plane $z = \sqrt{3}$

A. a cone

B. a circle

C. a plane

D. a sphere

E. a cylinder

<u>Alternatively</u>, this is an intersection of the sphere $\rho = 2$ and the upper half-cone $\phi = \pi/6$.

5. The curves $\vec{r}_1(t) = \langle t, t^2, 1\rangle$ and $\vec{r}_2(t) = \langle \sin t, \sin 2t, 1\rangle$ intersect at $(0, 0, 1)$ at an angle θ, where $\cos\theta =$

Intersection point at $t=0$.

$\vec{r}_1{}'(t) = \langle 1, 2t, 0\rangle$, $\vec{r}_1{}'(0) = \langle 1, 0, 0\rangle$

$\vec{r}_2{}'(t) = \langle \cos t, 2\cos 2t, 0\rangle$, $\vec{r}_2{}'(0) = \langle 1, 2, 0\rangle$

$$\cos\theta = \frac{\vec{r}_1{}'(0)\cdot\vec{r}_2{}'(0)}{|\vec{r}_1{}'(0)|\,|\vec{r}_2{}'(0)|} = \frac{1}{\sqrt{5}}$$

A. $\frac{1}{\sqrt{5}}$

B. $\frac{1}{3}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{6}$

E. $\frac{1}{\sqrt{6}}$

6. The level surfaces of the function $f(x, y, z) = x - y^2 - z^2$ are

The level surfaces:

$$x - y^2 - z^2 = k$$

$$(x-k) = y^2 + z^2$$

elliptic (circular) paraboloids

A. ellipsoids

B cones

C. cylinders

D. elliptic paraboloids

E. hyperbolic paraboloids

7. Let $f(x,y) = e^{xy}\sin(x^2)$. Then $\frac{\partial^2 f}{\partial x \partial y}(\sqrt{\pi}, 0) =$

$$\frac{\partial f}{\partial y} = x e^{xy} \sin(x^2)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) =$$

$$e^{xy}\sin(x^2) + xy\, e^{xy}\sin(x^2) + 2x^2 e^{xy}\cos(x^2)$$

$$\frac{\partial^2 f}{\partial x \partial y}(\sqrt{\pi}, 0) = e^0 \sin(\pi) + 0 \cdot e^0 \cdot \sin(\pi) + 2\pi e^0 \cos(\pi)$$

$$= -2\pi$$

One can start with $\frac{\partial f}{\partial x}$, too.

A. -2π
B. $-2\sqrt{\pi}$
C. 0
D. π
E. $\sqrt{2\pi}$

8. The area of the triangle with vertices $(1,0,1)$, $(1,1,0)$ and $(0,1,1)$ is

Let $P = (1,0,1)$, $Q = (1,1,0)$, $R = (0,1,1)$

$\vec{PQ} = (0,1,-1)$, $\vec{PR} = (-1,1,0)$

$$\text{Area} = \frac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|$$

$$= \frac{1}{2}\left|\langle 1,1,1\rangle\right| = \frac{\sqrt{3}}{2}$$

A. $\frac{1}{2}$
B. $\frac{\sqrt{3}}{2}$
C. $\sqrt{3}$
D. 1
E. $\sqrt{5}$

9. A particle has acceleration $\vec{a}(t) = 6t\vec{j} + 2\vec{k}$. The initial position is $\vec{r}(0) = \vec{j}$ and the initial velocity is $\vec{v}(0) = \vec{i} - \vec{j}$. The distance from the position of the particle at time $t = 1$ to the point $(2, 2, 3)$ is

$\vec{v} = \int \vec{a}\, dt + \vec{C}_1 = \langle 0, 3t^2, 2t \rangle + \vec{C}_1$

$\vec{C}_1 = \vec{v}(0) = \langle 1, -1, 0 \rangle$

$\vec{r} = \int \vec{v}\, dt + \vec{C}_2 = \langle t, t^3 - t, t^2 \rangle + \vec{C}_2$

$\vec{C}_2 = \vec{r}(0) = \langle 0, 1, 0 \rangle$

$\vec{r}(1) = \langle 1, 1, 1 \rangle$

$|\langle 2, 2, 3 \rangle - \langle 1, 1, 1 \rangle| = |\langle 1, 1, 2 \rangle| = \sqrt{6}$

A. 3

B. $\sqrt{7}$

C. $\sqrt{6}$

D. 4

E. 2

10. The curvature of the curve defined by the intersection of the cylinder $x^2 + y^2 = 1$ with the plane $y + z = 2$ at $(0, 1, 1)$ is

(*you may use the formula* $\kappa = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$)

Parametric equations:

$x = \cos t, \quad y = \sin t, \quad z = 2 - \sin t$

Vector equation: $\vec{r} = \langle \cos t, \sin t, 2 - \sin t \rangle$

$\vec{r}\,' = \langle -\sin t, \cos t, -\cos t \rangle$

$\vec{r}\,'' = \langle -\cos t, -\sin t, \sin t \rangle$

$\vec{r} = \langle 0, 1, 1 \rangle$ corresponds to $t = \pi/2$

$\vec{r}\,'(\pi/2) = \langle -1, 0, 0 \rangle, \quad \vec{r}\,''(\pi/2) = \langle 0, -1, 1 \rangle$

Since $\vec{r}\,'(\pi/2) \cdot \vec{r}\,''(\pi/2) = 0, \quad |\vec{r}\,'(\pi/2) \times \vec{r}\,''(\pi/2)| = |\vec{r}\,'(\pi/2)||\vec{r}\,''(\pi/2)| = \sqrt{2}$

Since $|\vec{r}\,'(\pi/2)| = 1, \quad \kappa = \sqrt{2}/1^3 = \sqrt{2}$

A. 1

B. $\frac{1}{2}$

C. $\sqrt{2}$

D. $\frac{\sqrt{2}}{2}$

E. 2