NAME Solution

STUDENT ID # ______

INSTRUCTOR ______

INSTRUCTIONS

1. There are 6 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–6.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given but the work on the test booklet may be used in borderline cases.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 10 points. The maximum possible score is 100 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–10 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
   (e) Sign your answer sheet.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your instructor.

1. The angle between the vectors $\vec{i}+\vec{j}$ and $(\sqrt{3}-1)\vec{i}+(\sqrt{3}+1)\vec{j}$ is:

$\vec{u}=\vec{i}+\vec{j}$, $\vec{v}=(\sqrt{3}-1)\vec{i}+(\sqrt{3}+1)\vec{j}$

$$\cos\theta=\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}=\frac{(\sqrt{3}-1)(1)+(\sqrt{3}+1)(1)}{\sqrt{2}\ \sqrt{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}}$$

$$\cos\theta=\frac{2\sqrt{3}}{\sqrt{2}\ \sqrt{8}}=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$$

$$\therefore\ \theta=\frac{\pi}{6}$$

A. 0

$\boxed{\text{B.}}$ $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

E. $\frac{\pi}{2}$

2. The symmetric equations for a line containing the point $(2,-1,4)$ and parallel to the line $x=1-3t$, $y=2+12t$, $z=5$ are:

$P_0=(2,-1,4)$

$\vec{L}=-3\vec{i}+12\vec{j}$

$$\Rightarrow\ \frac{x-2}{-3}=\frac{y+1}{12},\ z=4$$

A. $\frac{x-2}{-3}=\frac{y+1}{12}=z-4$

$\boxed{\text{B.}}$ $\frac{x-2}{-3}=\frac{y+1}{12},\ z=4$

C. $\frac{x+3}{2}=\frac{y-12}{-1}=\frac{z-5}{4}$

D. $\frac{x+3}{2}=\frac{y-12}{-1}=\frac{z-4}{4}$

E. $\frac{x-1}{-3}=\frac{y-2}{12},\ z=5$

3. The equation for the plane containing the lines

$$\frac{x-1}{3} = \frac{y-2}{5} = \frac{z+1}{2} \text{ and } \frac{x-1}{2} = y-2 = \frac{z+1}{-1} \text{ is:}$$

A. $-5x+7y-10z = 19$

B. $x+y+z=2$

C. $2x+y-z=5$

D. $3x+5y+2z=11$

$\boxed{\text{E.}}$ $-x+y-z=2$

$P_0 = (1, 2, -1)$

$$\vec{N} = \vec{L}_1 \times \vec{L}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 5 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

$$\vec{N} = -7\vec{i} + 7\vec{j} - 7\vec{k}$$

$$\therefore -7(x-1) + 7(y-2) - 7(z+1) = 0$$

$$-(x-1) + (y-2) - (z+1) = 0$$

$$-x + y - z = 2$$

4. Which point is not in the domain of f where $f(x,y) = \dfrac{(x^2-y^2)^{3/2}}{\ln|2x-4|}$?

A. $(1,1)$

B. $(1,0)$

$\boxed{\text{C.}}$ $(0,1)$

D. $(-1,1)$

E. $(-1,-1)$

Domain of $f(x,y)$:

$2x - 4 \neq \pm 1 \quad \therefore \quad x \neq 3/2, 5/2$

$x^2 - y^2 \geq 0 \quad$ or $\quad |x|^2 \geq |y^2|$

i.e, $\begin{cases} x \neq 2 \\ |x| \geq |y| \end{cases}$

$\therefore$ $(0,1)$ is not in domain of f

5. Find the speed $\|v(2)\|$ where $\vec{a}(t) = -10\vec{k}$ and $v(1) = \vec{i} - \vec{j} + \vec{k}$.

A. $\sqrt{84}$

$\boxed{\text{B.}}$ $\sqrt{83}$

C. $\sqrt{82}$

D. 9

E. $\sqrt{80}$

$$\vec{a}(t) = -10\vec{k} \Rightarrow \vec{v}(t) = -10t\vec{k} + \vec{C}$$

$$\vec{i} - \vec{j} + \vec{k} = \vec{v}(1) = -10\vec{k} + \vec{C}$$

$$\Rightarrow \vec{i} - \vec{j} + 11\vec{k} = \vec{C}$$

$$\therefore \vec{v}(t) = \vec{i} - \vec{j} + (11 - 10t)\vec{k}$$

$$\|\vec{v}(2)\| = \|\vec{i} - \vec{j} - 9\vec{k}\| = \sqrt{1 + 1 + 81} = \sqrt{83}$$

6. Find the length of the curve $\vec{r}(t) = t^2\vec{i} + 2t\vec{j} + \ln t\vec{k}$, $\quad 1 \le t \le e$.

$\boxed{\text{A.}}$ e^2

B. $2e + 1$

C. $2e - 2$

D. $2e + 2$

E. $e^2 - 1$

$$\vec{r}\,'(t) = 2t\vec{i} + 2\vec{j} + \frac{1}{t}\vec{k}$$

$$L = \int_1^e \|\vec{r}\,'(t)\|dt = \int_1^e \sqrt{(2t)^2 + 4 + \left(\frac{1}{t}\right)^2}\, dt$$

$$= \int_1^e \sqrt{\left(2t + \frac{1}{t}\right)^2}\, dt = \int_1^e \left|2t + \frac{1}{t}\right| dt = \int_1^e \left(2t + \frac{1}{t}\right) dt$$

$$= \left(t^2 + \ln t\right)\Big|_{t=1}^{e} = (e^2 + \ln e) - (1 + \ln 1)$$

$$= e^2 + 1 - 1 - 0 = e^2$$

7. The intersection of the surface $-10x^2 + x + 2y^2 + z^2 = 4$ with a plane parallel to the yz-plane is

A. a circle

B. a parabola

C. a hyperbola

$\boxed{\text{D.}}$ a ellipse

E. two lines

Any plane || yz plane is $x = C$

$\therefore -10c^2 + c + 2y^2 + z^2 = 4$

$2y^2 + z^2 = 4 + 10c^2 - c$

(ellipse, since $4 + 10c^2 - c > 0$)

8. Let $z = \sqrt{x^2 + y^2}$, $x = uv$, $y = u^2 - v^2$. Find $\frac{z_u}{z_v}$.

A. $\dfrac{xv + 2yu}{xu + 2yv}$

B. $\dfrac{u^2v + 2v^3}{\sqrt{x^2 + y^2}}$

C. $\dfrac{2v^3 + u^2v}{2u^3 + uv^2}$

D. $\dfrac{uv}{u^2 - v^2}$

$\boxed{\text{E.}}$ $\dfrac{2u^3 - uv^2}{2v^3 - u^2v}$

$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$; $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$

$\therefore \frac{\partial z}{\partial u} = \left(\frac{xv}{\sqrt{x^2+y^2}}\right) + \left(\frac{2uy}{\sqrt{x^2+y^2}}\right)$

$\therefore \frac{\partial z}{\partial v} = \left(\frac{xu}{\sqrt{x^2+y^2}}\right) + \left(\frac{-2vy}{\sqrt{x^2+y^2}}\right)$

$\Rightarrow \frac{z_u}{z_v} = \frac{xv + 2uy}{xu - 2vy} = \frac{(uv)v + 2u(u^2 - v^2)}{uv(u) - 2v(u^2 - v^2)} = \frac{2u^3 - uv^2}{2v^3 - u^2v}$

9. Suppose that $\sqrt{x^3y + x^2y^2} = 10$ implicitly defines y as a function of x. Find $\dfrac{dy}{dx}$.

$x^3y + x^2y^2 = 100$

let $f(x,y) = x^3y + x^2y^2 - 100$

$$\Rightarrow \frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}$$

$$= -\frac{3x^2y + 2xy^2}{x^3 + 2x^2y}$$

A. $\dfrac{3x^2y + 2xy^2}{2\sqrt{x^3y + x^2y^2}}$

$\boxed{\text{B.}}$ $-\dfrac{3x^2y + 2xy^2}{x^3 + 2x^2y}$

C. $\dfrac{\sqrt{x^3y + x^2y^2} - 10}{x^3 - 3xy^2}$

D. $\dfrac{x^3y + 2x^2y}{3x^2y + 2xy^2}$

E. $\dfrac{x^2y + 2xy^2 - 10}{\sqrt{x^3y + x^2y^2}}$

10. The directional derivative at $(-1, 1)$ of $f(x, y) = e^{-\frac{x^2}{2} - \frac{y^3}{3}}$ in the direction of $\vec{a} = -2\vec{i} + \vec{j}$ is

$$\nabla f = \left(-x e^{-\frac{x^2}{2} - \frac{y^3}{3}}\right)\vec{i} + \left(-y^2 e^{-\frac{x^2}{2} - \frac{y^3}{3}}\right)\vec{j}$$

$$\nabla f(-1,1) = e^{-\frac{5}{6}}\vec{i} - e^{-\frac{5}{6}}\vec{j}$$

$$\vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{1}{\sqrt{5}}(-2\vec{i} + \vec{j})$$

$$\therefore D_{\vec{u}} f(-1,1) = \nabla f(-1,1) \cdot \vec{u}$$

$$= \frac{1}{\sqrt{5}}\left\{-2e^{-\frac{5}{6}} - e^{-\frac{5}{6}}\right\} = -\frac{3}{\sqrt{5}} e^{-\frac{5}{6}}$$

A. $\dfrac{1}{5}e^{-\frac{5}{6}}$

B. $-\dfrac{1}{\sqrt{5}}\vec{i} - \dfrac{3}{\sqrt{5}}\vec{j}$

$\boxed{\text{C.}}$ $-\dfrac{3}{\sqrt{5}}e^{-\frac{5}{6}}$

D. $2e^{\frac{7}{6}}$

E. $-\dfrac{2}{5}\vec{i} + \dfrac{1}{\sqrt{5}}e^{-\frac{5}{6}}\vec{j}$