NAME Solution

STUDENT ID # ______________________

RECITATION INSTRUCTOR ______________________

RECITATION TIME ______________________

DIRECTIONS

1) Fill in the above information. Also write your name at the top of each page of the exam.
2) The test has 9 pages, including this one.
3) Problems 1 through 6 are multiple choice; circle the correct answer.
4) Problems 7 through 10 are problems to be worked out. Write your answer in the box provided. YOU MUST SHOW SUFFICIENT WORK TO JUSTIFY YOUR ANSWERS. CORRECT ANSWERS WITH INCONSISTENT WORK MAY NOT RECEIVE CREDIT.
5) Points for each problem are given in parenthesis in the left margin.
6) No books, notes, or calculators may be used on this test.

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(10) 1) Parametric equations for the line that contains the point $(1, -2, 3)$ and is perpendicular to the plane $3x - 4y + 2z = 8$ are:

A. $x = 1 + 3t,\ y = -2 - 4t,\ z = 3 + 2t$

B. $x = 3 + t,\ y = -4 + 2t,\ z = 2 + 3t$

C. $x = 8 + 3t,\ y = 8 - 4t,\ z = 8 + 2t$

D. $x = -1 + 3t,\ y = 2 - 4t,\ z = -3 + 2t$

E. $x = -1 - 3t,\ y = 2 + 4t,\ z = -3 - 2t$

Point : $(1,-2,3)$

Direction : $3\vec{i} - 4\vec{j} + 2\vec{k}$

$x = 1 + 3t$

$y = -2 - 4t$

$z = 3 + 2t$

(10) 2) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{(x^2+y^2)} \cdot (y+2)$ is equal to:

A. 0

B. 1

C. 2

D. 4

E. Does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} \cdot (y+2) =$$

$$\left(\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}\right) \cdot \left(\lim_{(x,y)\to(0,0)} (y+2)\right) =$$

$$1 \cdot (0+2) = 2$$

(10) 3) Symmetric equations for the line tangent to the curve $\overrightarrow{r(t)} = t^2\vec{i} + (3t-4)\vec{j} + (2-t^2)\vec{k}$ at the point $(4, 2, -2)$ are given by:

$\underline{pt}: (4, 2, -2) = (t^2, 3t-4, 2-t^2)$

$t^2 = 4, \; (3t-4=2$

$-2 = 2-t^2$

$\Rightarrow 3t = 6$

$t = 2$

$\underline{\text{Direction}}: \overrightarrow{r'}(2) = (2t\vec{i} + 3\vec{j} - 2t\vec{k})\big|_2$

$= 4\vec{i} + 3\vec{j} - 4\vec{k}$

$\frac{x-4}{4} = \frac{y-2}{3} = \frac{z+2}{-4}$

A. $\dfrac{x-4}{4} = \dfrac{y-2}{3} = \dfrac{z-2}{-4}$

B. $x = 4$ and $\dfrac{y-2}{2} = \dfrac{z+2}{3}$

C. $\dfrac{x-4}{4} = \dfrac{y+2}{-3} = \dfrac{z-2}{-4}$

D. $\dfrac{x-4}{4} = \dfrac{y-2}{3} = \dfrac{z+2}{-4}$

E. $\dfrac{x-4}{4} = \dfrac{y-3}{2} = \dfrac{z+4}{-2}$

(10) 4) Let S be the level surface of $f(x, y, z) = x^2 - y^2 - \frac{z^2}{4}$ corresponding to $c = 1$. The intersection of S with the xy plane is:

$x^2 - y^2 - \frac{z^2}{4} = 1$

$+ \; z = 0$ (xy-plane)

$\Rightarrow x^2 - y^2 = 1$

A. two lines

B. a circle

C. a parabola

D. an ellipse

E. a hyperbola

5) An object has acceleration $\overrightarrow{a(t)} = e^t\vec{i} + 2\vec{k}$, initial velocity $\overrightarrow{v(0)} = \vec{i}$, and initial position $\overrightarrow{r(0)} = 2\vec{j}$. Find the position vector of the object at time $t = 1$.

A. $(e-1)\vec{i} - 2\vec{j} + \vec{k}$

B. $(e-1)\vec{i} + 2\vec{j} + \vec{k}$

C. $e\vec{i} - 2\vec{j}$

D. $e\vec{i} + 2\vec{j} + \vec{k}$

E. $e\vec{i} + 2\vec{j} - \vec{k}$

$\overrightarrow{a(t)} = e^t\vec{i} + 2\vec{k}$

$\Rightarrow \overrightarrow{v(t)} = e^t\vec{i} + 2t\vec{k} + \vec{c}$

$\Rightarrow \vec{i} = \overrightarrow{v(0)} = \vec{i} + \vec{c}$

So $\vec{c} = \vec{0}$

$\therefore \overrightarrow{v(t)} = e^t\vec{i} + 2t\vec{k}$

$\Rightarrow \overrightarrow{r(t)} = e^t\vec{i} + t^2\vec{k} + \vec{d}$

so $2\vec{j} = \overrightarrow{r(0)} = \vec{i} + \vec{d}$

$\therefore \vec{d} = -\vec{i} + 2\vec{j}$

$\overrightarrow{r(t)} = (e^t - 1)\vec{i} + 2\vec{j} + t^2\vec{k}$

$\overrightarrow{r(1)} = (e-1)\vec{i} + 2\vec{j} + \vec{k}$

(10) 6) Let $f(x,y) = \ln(x^2+y^2)$ with $x = g(t)$ and $y = h(t)$. Assuming that $g(0) = 1$, $h(0) = 3$, $g'(0) = 2$, and $h'(0) = 4$, the value of $\frac{d}{dt}(f(g(t),\ h(t))$ when $t = 0$ is:

$$\frac{df}{dt}\Big|_{t=0} = f_x\Big|_{(1,3)} \frac{dx}{dt}\Big|_{t=0} + f_y\Big|_{(1,3)} \frac{dy}{dt}\Big|_{t=0}$$

$$= \frac{2x}{x^2+y^2}\Big|_{(1,3)} \cdot 2 + \frac{2y}{x^2+y^2}\Big|_{(1,3)} \cdot 4$$

$$= \frac{2}{10} \cdot 2 + \frac{6}{10} \cdot 4$$

$$= \frac{28}{10} = \frac{14}{5}$$

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{7}{5}$

E. $\frac{14}{5}$

P_1 P_2 P_3

7. Consider the plane containing the points $(0, 1, 2)$, $(1, 2, 3$, and $(2, 1, 0)$.

(5) a) Find a vector $\vec{n}$ which is perpendicular to the plane. (Put your answer in the box below.)

$$\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = (\vec{i} + \vec{j} + \vec{k}) \times (2\vec{i} + 0\vec{j} - 2\vec{k})$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 0 & -2 \end{vmatrix} = \vec{i}(-2) - \vec{j}(-2-2) + \vec{k}(-2)$$

$$= -2\vec{i} + 4\vec{j} - 2\vec{k}$$

Answer to 7.a) $\vec{n} = -2\vec{i} + 4\vec{j} - 2\vec{k}$

(5) b) Find the equation for the plane.

$$-2(x-0) + 4(y-1) - 2(z-2) = 0$$

$$-2x + 4y - 2z - 4 + 4 = 0$$

$$-x + 2y - z = 0$$

Answer to 7.b) $-x + 2y - z = 0$

8. Consider the curve given by:
$\overrightarrow{r(t)} = t^2\vec{i} + 2t\vec{j} + (\ln t)\vec{k},\ 1 \le t \le e.$

(6) a) Write down an integral that gives the arclength L of this curve (including limits of integration).

$$\|\vec{r'(t)}\| = \|2t\vec{i} + 2\vec{j} + \tfrac{1}{t}\vec{k}\|$$
$$= \sqrt{4t^2 + 4 + \tfrac{1}{t^2}}$$

Answer to 8.a) $L = \int_1^e \sqrt{4t^2 + 4 + (\tfrac{1}{t})^2}\, dt$

(4) b) Compute the integral in 8.a) to get the exact value of the arclength L.

$$\sqrt{4t^2 + 4 + \tfrac{1}{t^2}} = \sqrt{\frac{4t^4 + 4t^2 + 1}{t^2}} = \sqrt{\frac{(2t+1)^2}{t^2}}$$
$$= \frac{|2t+1|}{|t|}$$
$$\int_1^e \sqrt{4t^2 + 4 + \tfrac{1}{t^2}}\, dt = \int_1^e \frac{(2t+1)}{t}\, dt \quad \int_1^e (2 + \tfrac{1}{t})\, dt$$
$$= (2t + \ln(t))\Big]_1^e = (2e + 1) - (2)$$
$$= 2e - 1$$

Answer to 8.b) $L = 2e -$

9. Let $f(x,y) = x^2 e^{xy}$.

(6) a) Find $\dfrac{\partial^2 f}{\partial x \partial y}$.

$$\frac{\partial f}{\partial y} = x^3 e^{xy}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 3x^2 e^{xy} + y x^3 e^{xy}$$

$$= x^2 e^{xy}(3 + xy)$$

Answer to 9.a) $\boxed{\dfrac{\partial^2 f}{\partial x \partial y} = x^2 e^{xy}(3 + xy)}$

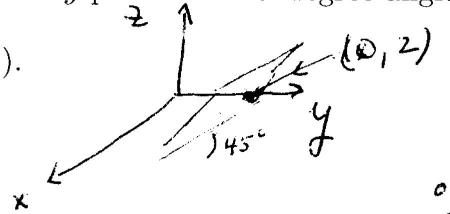
(4) b) What is $\dfrac{\partial^2 f}{\partial x \partial y}$ at the point $(1,0)$?

$$\frac{\partial^2 f}{\partial x \partial y}(1,0) = 1 e^0 (3 + 0) = 3$$

Answer to 9.b) $\boxed{\dfrac{\partial^2 f}{\partial x \partial y}(1,0) = 3}$

10. A function $f(x, y)$ is positive if $y > 2$, negative if $y < 2$. The graph of f is a plane which intersects the xy plane at a 45–degree angle.

(3) a) Find $\frac{\partial f}{\partial x}(0, 2)$.

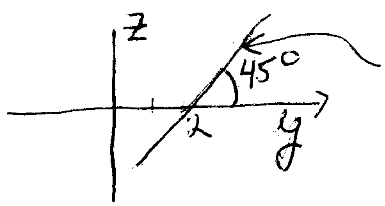


$f(x,2) \equiv 0$ for all x

$\therefore \frac{\partial f}{\partial x}(0,2) = 0$

Answer to 10.a) $\boxed{\frac{\partial f}{\partial x}(0, 2) = 0}$

(3) b) Find $\frac{\partial f}{\partial y}(0, 2)$.



$z = f(x, y)$

$z = y + 2$

$x = 0$.

$\frac{\partial z}{\partial y}(0,2) = \frac{\partial f}{\partial y}(0,2)$

$= \frac{\partial}{\partial y}(y+2) = 1$

Answer to 10.b) $\boxed{\frac{\partial f}{\partial y}(0, 2) = 1}$

(4) c) Find the directional derivative of f at $(0, 2)$ in the direction $\vec{i} - \vec{j}$.

$\vec{u} = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$, $\quad \frac{\partial f}{\partial x}(0,2) = 0 \quad \frac{\partial f}{\partial y}(0,2) = 1$

$D_{\vec{u}} f(0,2) = 0 \cdot \frac{1}{\sqrt{2}} + 1\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$

Answer to 10.c) $\boxed{-\frac{1}{\sqrt{2}}}$